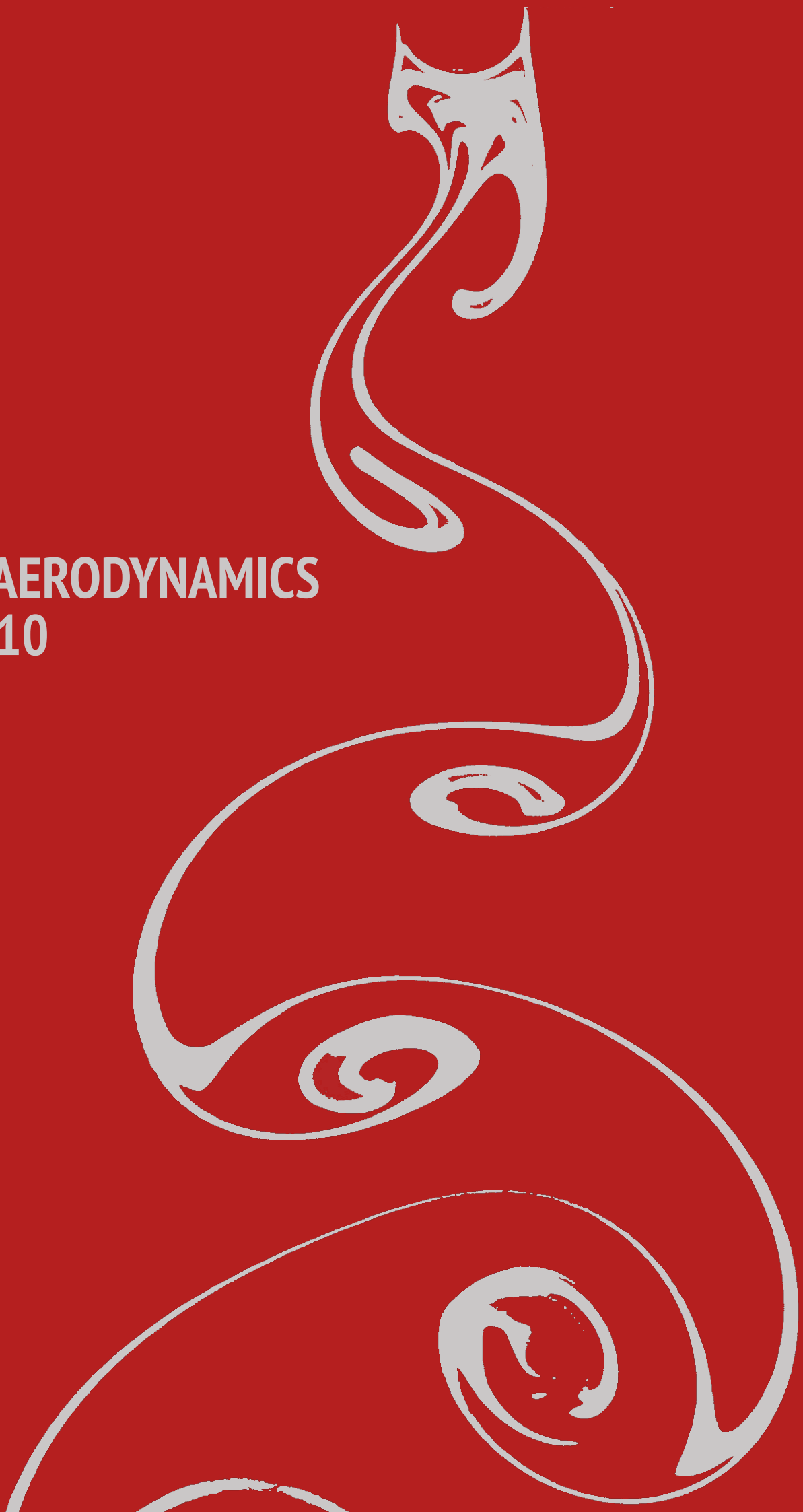


ME-445 AERODYNAMICS
Exercise 10
Week 8



Formula sheet

Cylindrical coordinates

$$\nabla \vec{u} = \left(\frac{\partial v_r}{\partial r}, \frac{1}{r} \frac{\partial v_\theta}{\partial \theta}, 0 \right)$$

$$\nabla \cdot \vec{u} = \frac{1}{r} \frac{\partial(rv_r)}{\partial r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta}$$

$$\nabla \times \vec{u} = \left(0, 0, \frac{1}{r} \left[\frac{\partial(rv_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta} \right] \right)$$

Potential flow

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r} \frac{\partial \psi}{\partial \theta}, \quad v_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$$

Uniform parallel flow $w = \phi + i\psi = U_\infty e^{-i\alpha} z$

Potential vortex in z_0 $w = -\frac{i\gamma}{2\pi} \ln(z - z_0)$

Point source or sink in z_0 $w = \frac{Q}{2\pi} \ln(z - z_0)$

Source-sink doublet in z_0 $w = \frac{\mu}{2\pi(z - z_0)}$

$$\frac{dw}{dz} = u - iv$$

Milne-Thomson circle theorem:

$$g(z) = w(z) + \overline{w\left(\frac{a^2}{z}\right)}$$

Thin airfoil theory

For a camber line with:

$$\frac{dy_c}{dx} = A_0 + \sum_{n=1}^{\infty} A_n \cos n\theta$$

$$\frac{x}{c} = \frac{(1 - \cos \theta)}{2}$$

we know:

$$k = 2U_\infty \left[(\alpha - A_0) \frac{\cos \theta + 1}{\sin \theta} + \sum_{n=1}^{\infty} A_n \sin n\theta \right]$$

$$A_0 = \frac{1}{\pi} \int_0^\pi \frac{dy_c}{dx} d\theta$$

$$A_n = \frac{2}{\pi} \int_0^\pi \frac{dy_c}{dx} \cos n\theta d\theta$$

$$C_l = 2\pi\alpha + \pi(A_1 - 2A_0)$$

$$C_{m,1/4} = -\frac{\pi}{4}(A_1 - A_2)$$

$$x_{cp} = \frac{1}{4} + \frac{\pi}{4C_l}(A_1 - A_2)$$

Finite wings with $AR=b^2/S$

Sign convention:

if induced velocity points downward: $w(y) > 0, \alpha_i(y) > 0$

if induced velocity points upward: $w < 0, \alpha_i < 0$

Prandtl's lifting-line theory

$$U_\infty \alpha_i(y_0) = w(y_0) = -\frac{1}{4\pi} \int_{-b/2}^{b/2} \frac{(d\Gamma/dy)}{y - y_0} dy$$

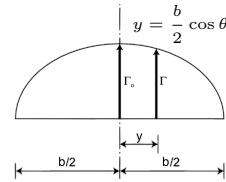
$$\alpha(y_0) = \alpha_{\text{eff}}(y_0) + \alpha_i(y_0)$$

Elliptical loading $\Gamma(y) = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$

$$w = \frac{\Gamma_0}{2b}$$

$$\alpha_i = \frac{C_L}{\pi AR}$$

$$C_{D,i} = \frac{C_L^2}{\pi AR}$$



General loading $\Gamma(\theta) = 2bU_\infty \sum_{n=1}^{\infty} A_n \sin n\theta$

$$w(\theta) = U_\infty \sum_{n=1}^{\infty} n A_n \frac{\sin n\theta}{\sin \theta}$$

$$C_L = \pi A_1 AR$$

$$C_{D,i} = \frac{C_L^2}{\pi AR} (1 + \delta) \text{ with } \delta = \sum_{n=2}^{\infty} n (A_n/A_1)^2$$

Boundary Layer

Flat plate **laminar** boundary layer

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}} \text{ boundary layer growth}$$

$$C_f = \frac{1.328}{\sqrt{Re_x}} \text{ skin friction drag coefficient}$$

Flat plate **turbulent** boundary layer

$$\frac{\delta}{x} = \frac{0.37}{Re_x^{1/5}} \text{ boundary layer growth}$$

$$C_f = \frac{0.074}{Re_x^{1/5}} \text{ skin friction drag coefficient}$$

Miscellaneous

θ	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

water

kinematic viscosity $\nu = 1 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$

density $\rho = 1000 \text{ kg m}^{-3}$

air

kinematic viscosity $\nu = 1.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$

density $\rho = 1.2 \text{ kg m}^{-3}$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$\int_0^{\pi} \cos \theta d\theta = 0$$

$$\int_0^{\pi} \sin \theta d\theta = 2$$

$$\int_0^{\pi} \cos^2 \theta d\theta = \int_0^{\pi} \sin^2 \theta d\theta = \frac{\pi}{2}$$

$$\int_0^{\pi} \frac{\cos n\theta}{\cos \theta - \cos \theta_1} d\theta = \pi \frac{\sin n\theta_1}{\sin \theta_1} \quad n = 0, 1, 2, \dots$$

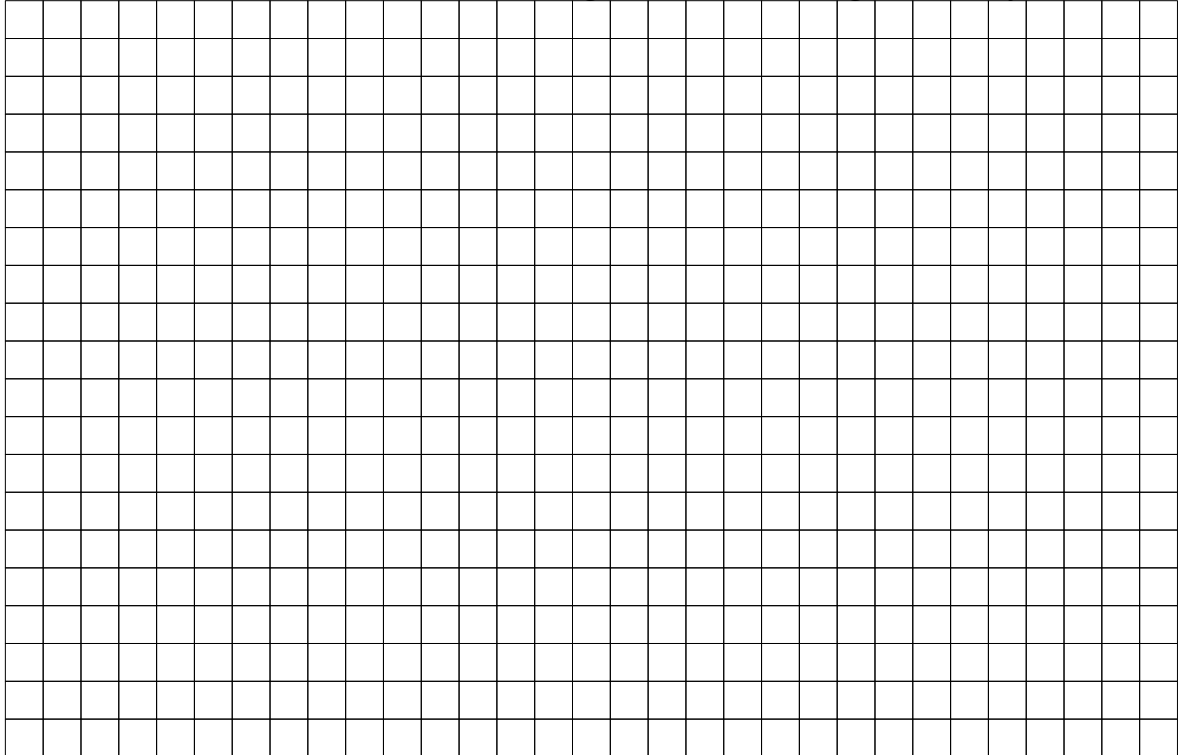
$$\int_0^{\pi} \frac{\sin n\theta \sin \theta}{\cos \theta - \cos \theta_1} d\theta = -\pi \cos n\theta_1 \quad n = 1, 2, 3, \dots$$

1. Consider a wing with an aspect ratio $AR = 10$. Its non-dimensional circulation distribution is

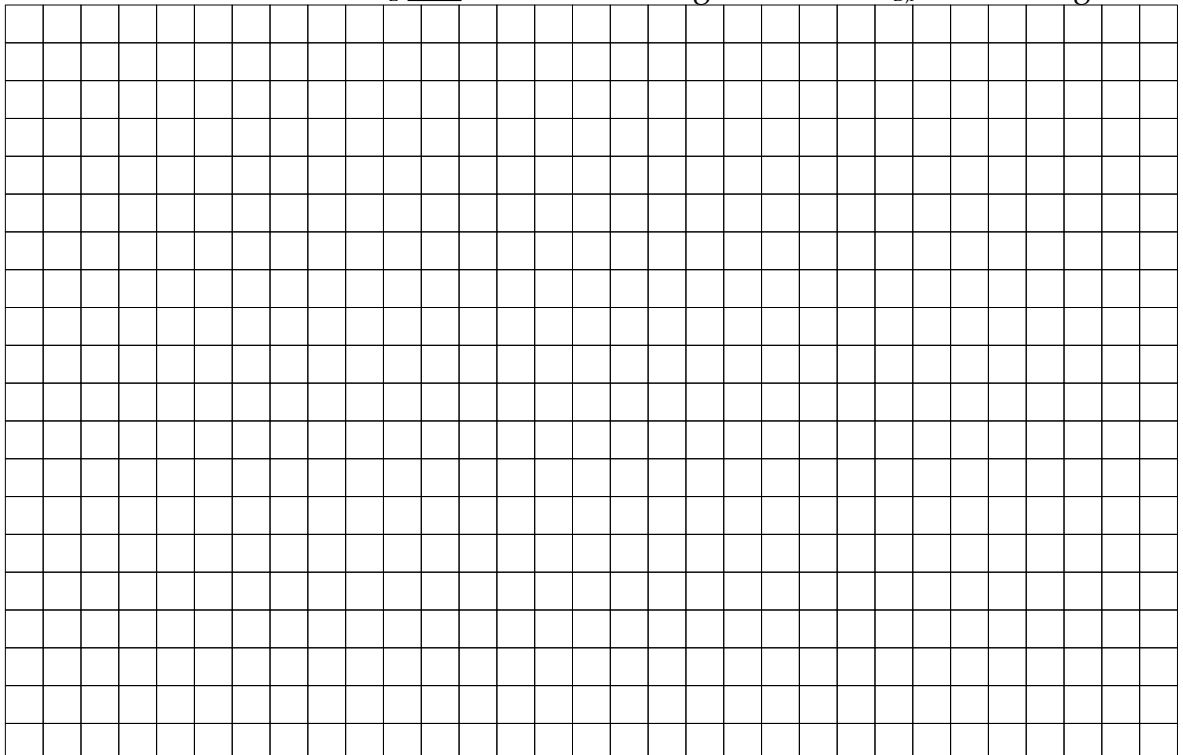
$$\frac{\Gamma(\theta)}{bU_\infty} = \frac{2\epsilon}{\pi AR} (3 \sin \theta + \sin 3\theta)$$

with $\frac{y}{b} = \frac{1}{2} \cos \theta$, b the wing span from tip to tip, and ϵ a constant value.

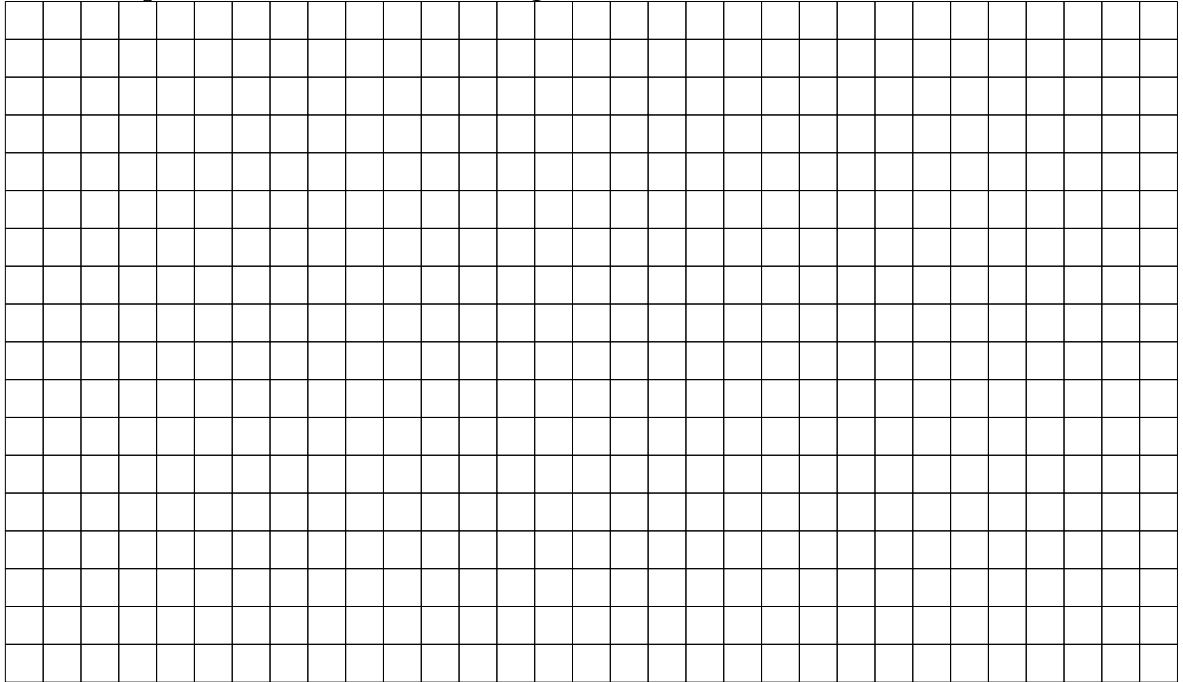
(a) Determine the Fourier coefficients according to Prandtl's lifting line theory.



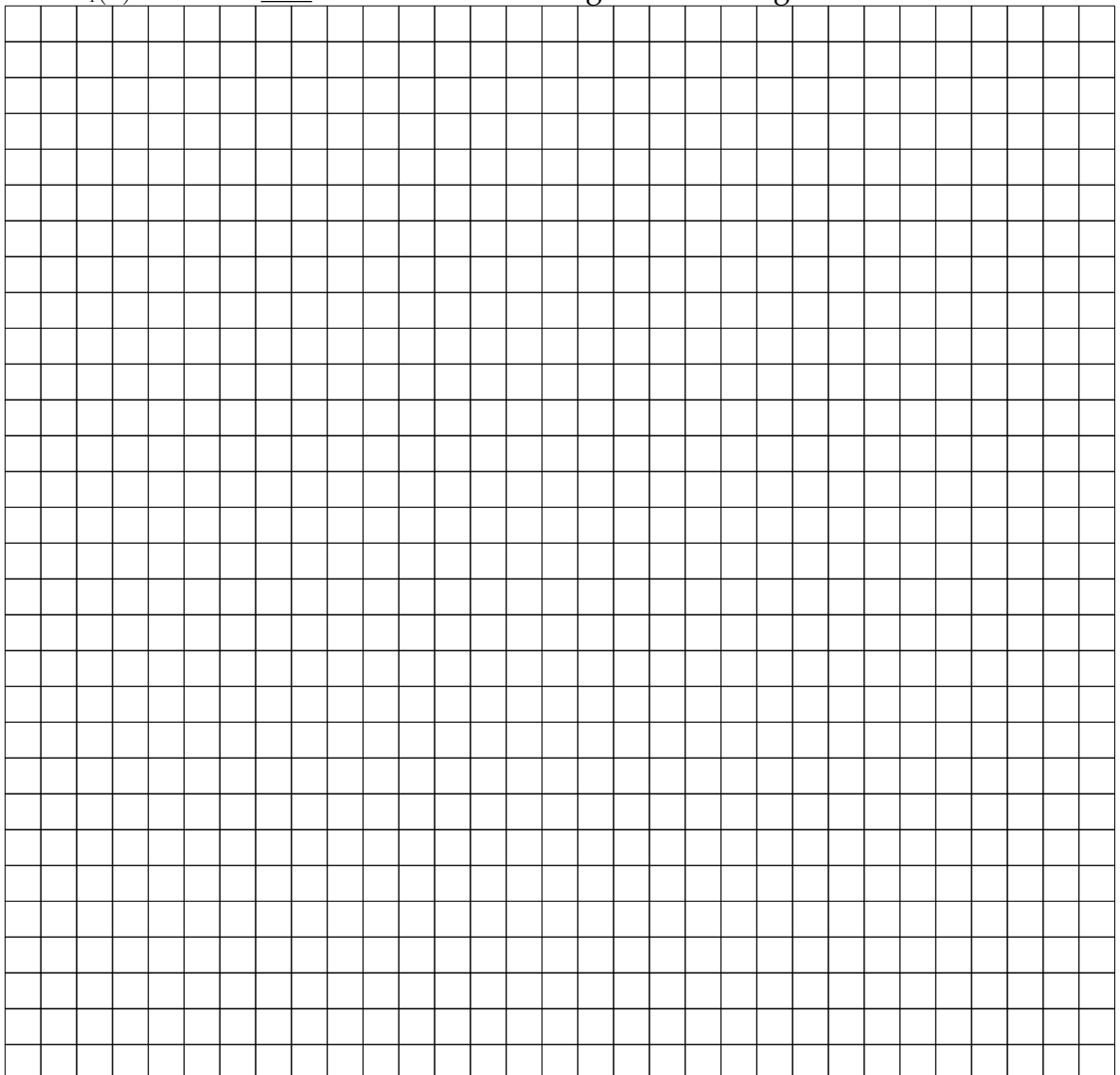
(b) Calculate the lift coefficient C_L and the induced drag coefficient C_{Di} of this wing.



(c) Write the expression for the induced angle of attack α_i as a function of θ .



(d) Sketch $\alpha_i(\theta)$ for $\epsilon > 0$ and for $\epsilon < 0$. Do not forget to add a legend.



(e) The wing is formed by thin airfoils with the same airfoil profiles. At which span-wise location would this wing starts to stall first for $\epsilon > 0$ and where for $\epsilon < 0$?

Motivate your answer.

