EPFL

Fall 2025

ME-445 AERODYNAMICS
04 - Thin airfoil theory - addendum

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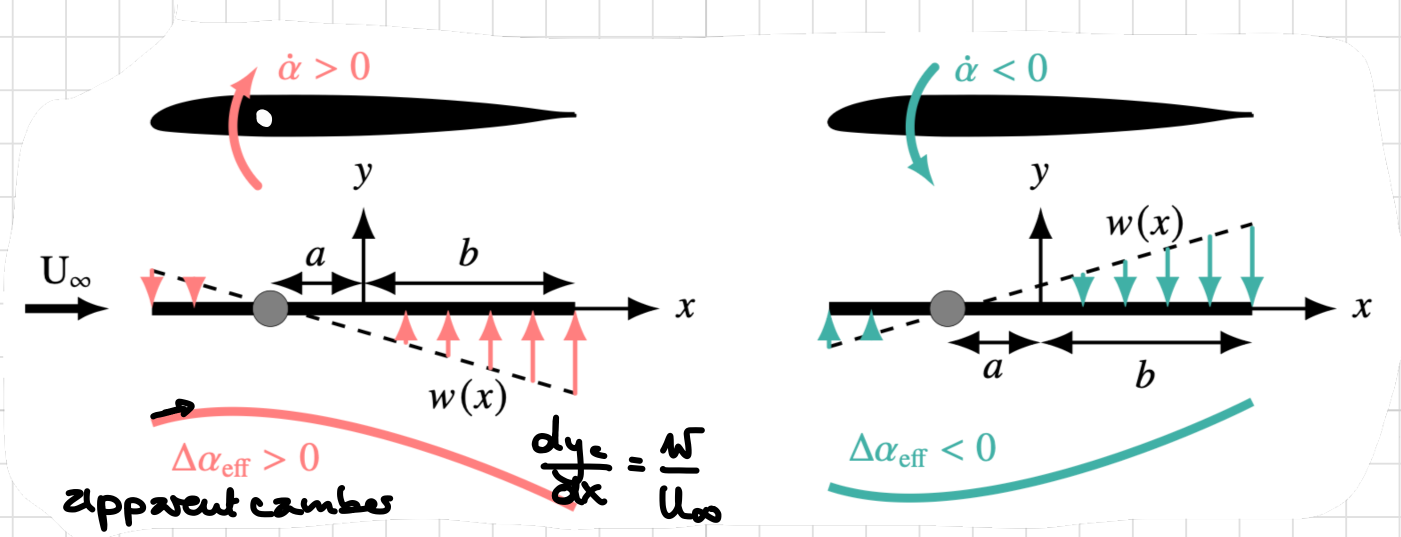
Applications of thin airfoil theory:

- quasi-steady modelling of pitching airfoil
- multi-element airfoils.

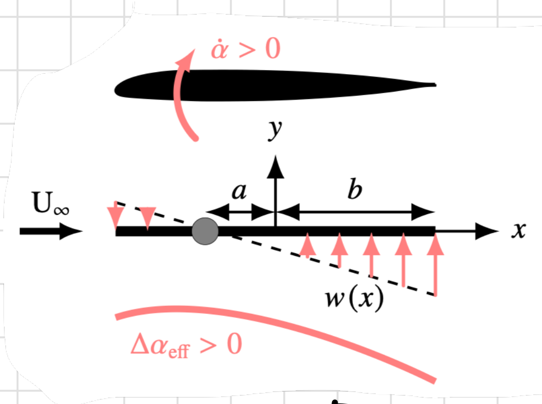
Pitching airfoil: quasi-steady modelling:



pitch rate: $\dot{\alpha}$ $\left(\frac{d\alpha}{dt}\right)$

$$w(x) = -\dot{\alpha}(x - ab) \; ; \; b = \frac{c}{2}$$

for pitch axis at quarter chord: $a = -1/2$



$$w(x) = -\dot{\alpha}\,(x - ab)$$

$$\frac{dy_c}{dx} = \frac{w(x)}{U_\infty}$$

$$A_0 = \frac{1}{\pi}\int_0^{\pi} \frac{dy_c}{dx}\, d\vartheta = \frac{1}{\pi}\int_0^{\pi} \frac{-\dot{\alpha}(x - ab)}{U_\infty}\, d\vartheta$$

$$x = -b\cos\vartheta$$
$$b = \frac{c}{2}$$

$$= \frac{1}{\pi U_\infty}\int_0^{\pi} -\dot{\alpha}\left(-\frac{c}{2}\cos\vartheta - \frac{ac}{2}\right) d\vartheta$$

$$= \frac{\dot{\alpha} c}{2\pi U_\infty}\int_0^{\pi} (a + \cos\vartheta)\, d\vartheta = \frac{\dot{\alpha} c}{2 U_\infty}\, a$$

$$A_1 = \frac{2}{\pi}\int_0^{\pi} \frac{w(x)}{U_\infty}\cos\vartheta \, d\vartheta = \frac{2}{\pi U_\infty}\int_0^{\pi} \dot{\alpha}\left(-\frac{c}{2}\cos\vartheta - \frac{ac}{2}\right)\cos\vartheta \, d\vartheta$$

$$= \frac{\dot{\alpha} c}{U_\infty \pi}\int_0^{\pi}\left(\cos^2\vartheta + a\cos\vartheta\right) d\vartheta = \frac{\dot{\alpha} c}{2U_\infty}$$

$$C_\ell = 2\pi\alpha + \pi\left(A_1 - 2A_0\right) = 2\pi\alpha + \pi\left(\frac{\dot{\alpha} c}{2U_\infty} - 2a\frac{\dot{\alpha} c}{2U_\infty}\right)$$

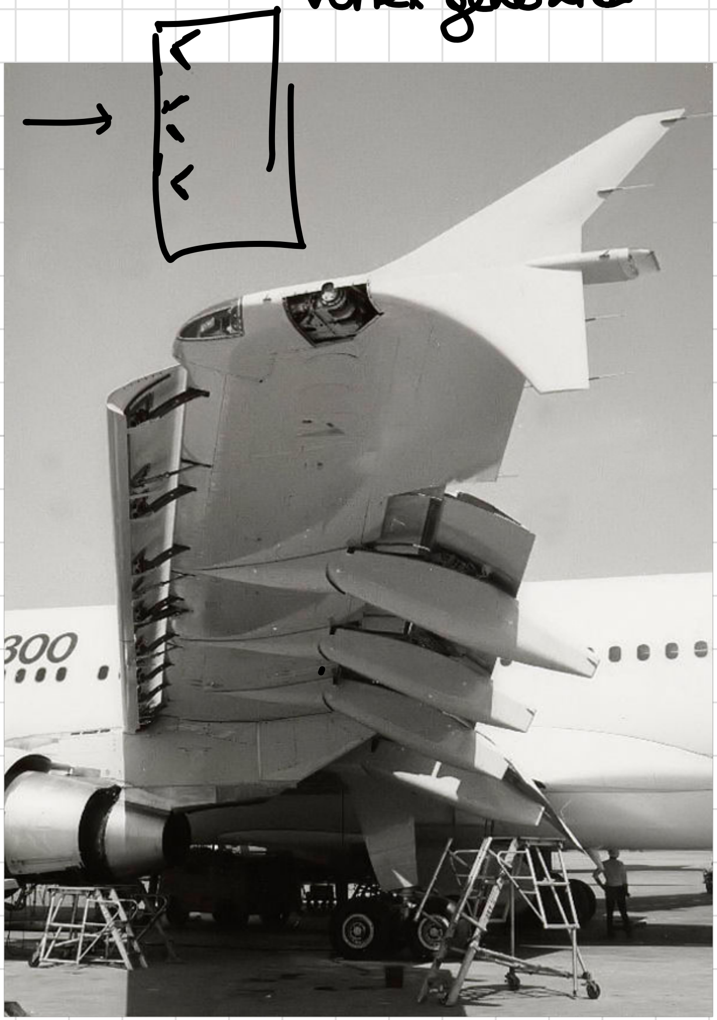
$$= 2\pi\left[\alpha + \frac{\dot{\alpha} c}{4U_\infty}(1-2a)\right]$$

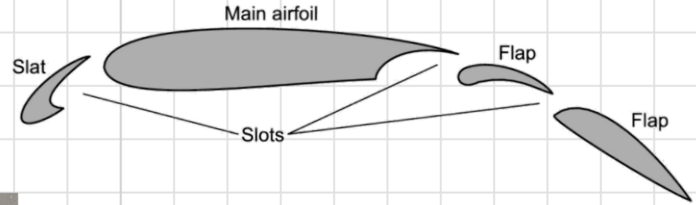
for $a = -1/2$: $C_\ell = 2\pi\left[\alpha + \frac{\dot{\alpha} c}{2U_\infty}\right]$ quasi-steady model

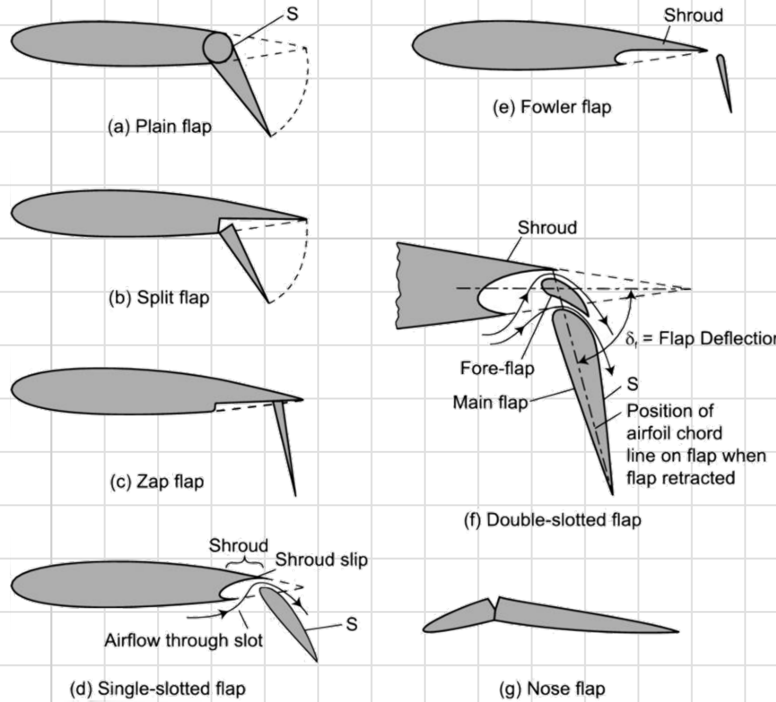
$\underbrace{\alpha + \frac{\dot{\alpha} c}{2U_\infty}}_{\alpha_{effective}}$

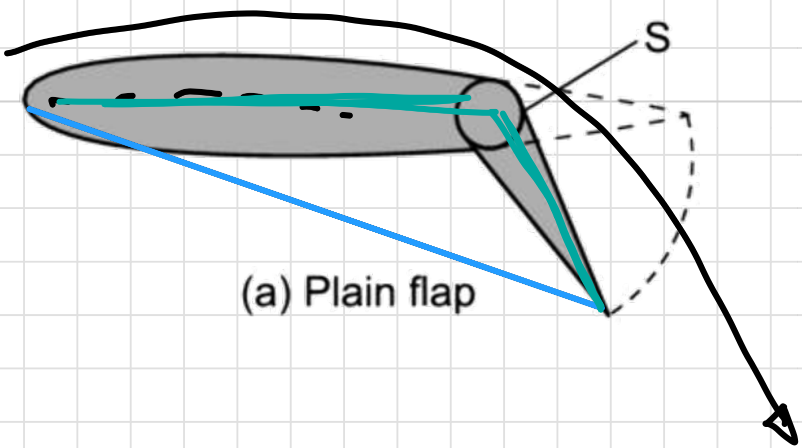
Multi-element airfoils

vortex generator





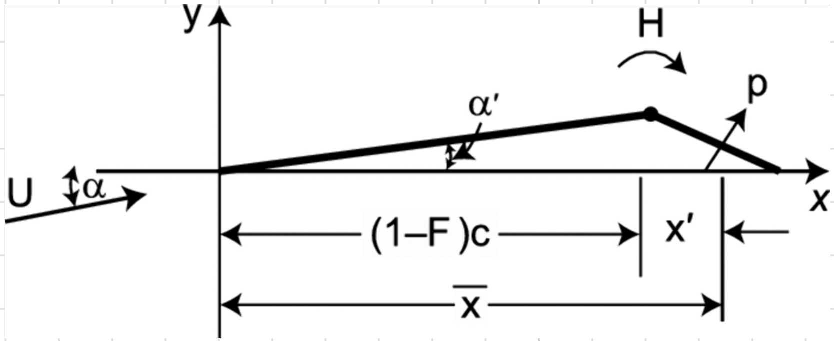




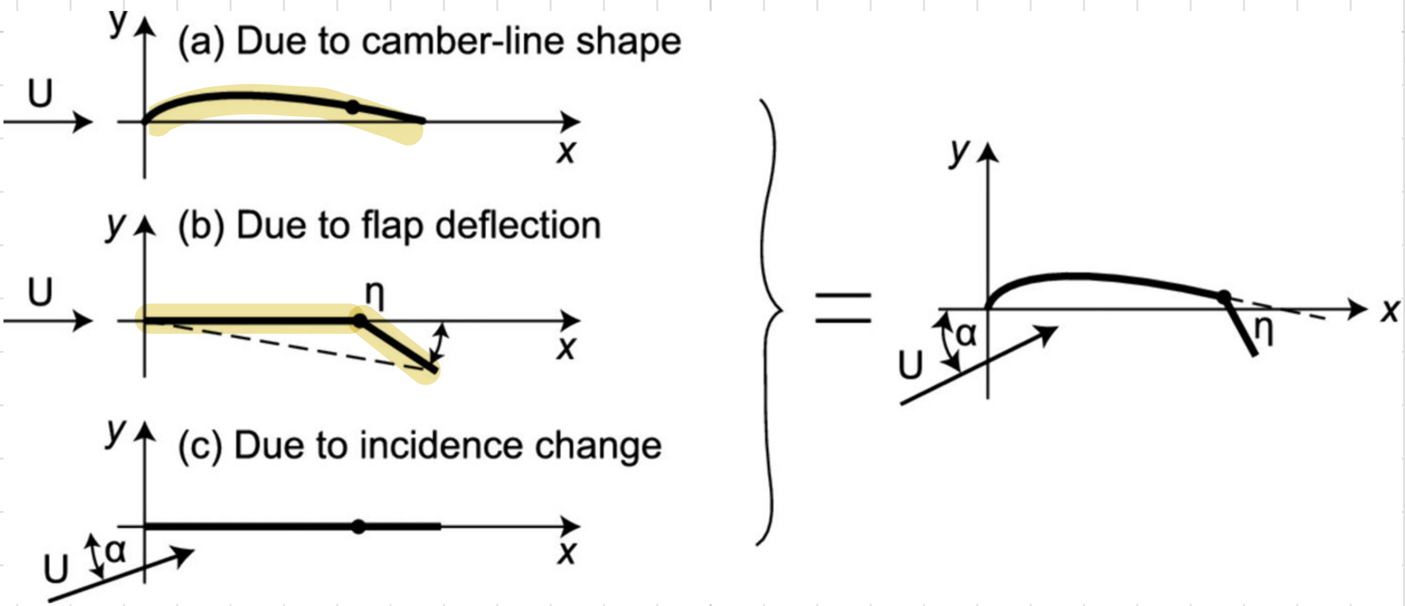
flap down

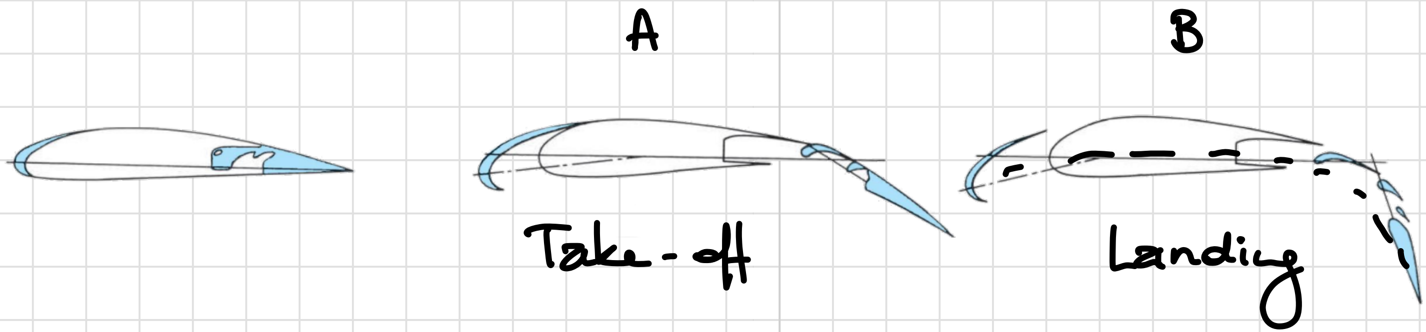
⇒ increase in 'camber'

⇒ increase in lift

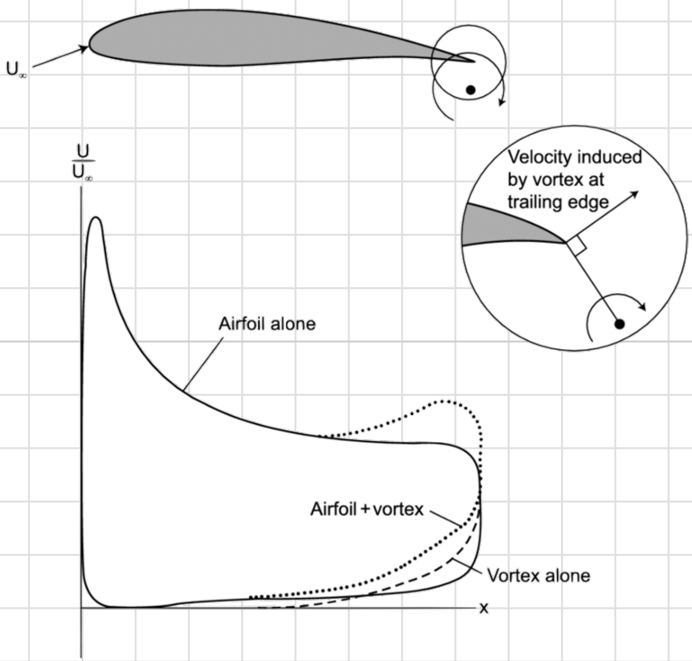


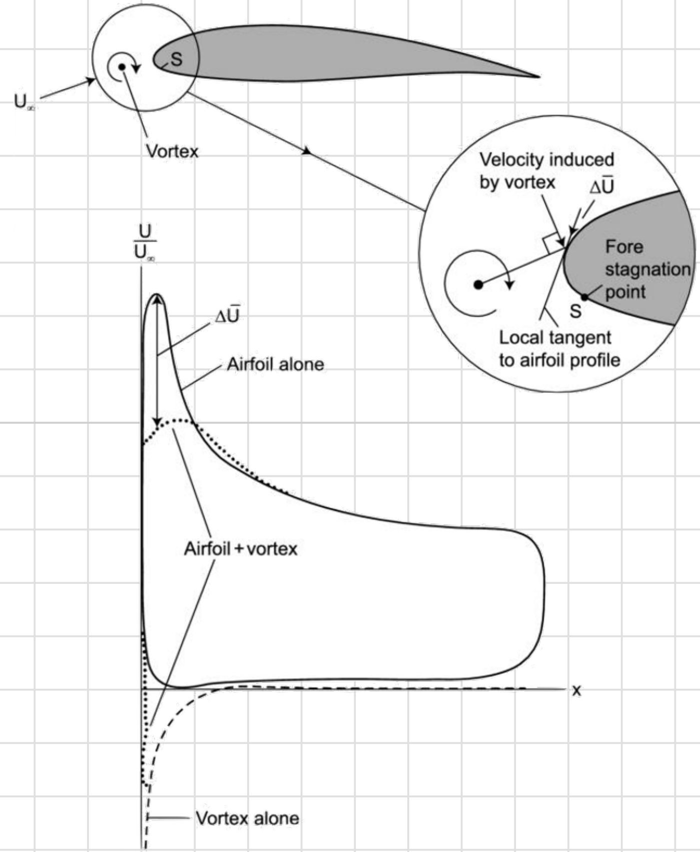
$$\frac{dy_c}{dx}(\eta)$$



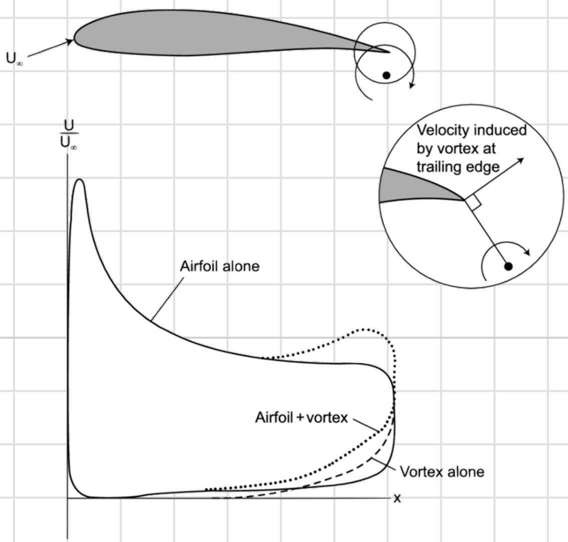

A
B
Take-off
Landing

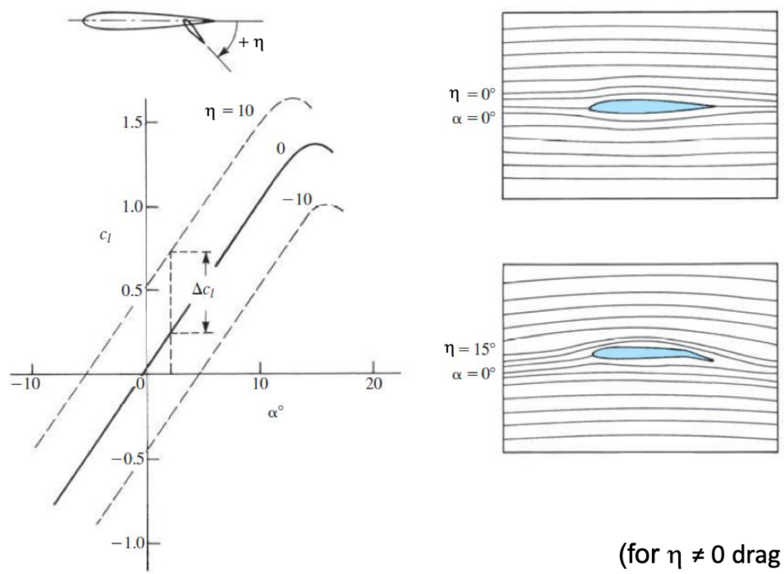
Multi-element airfoils





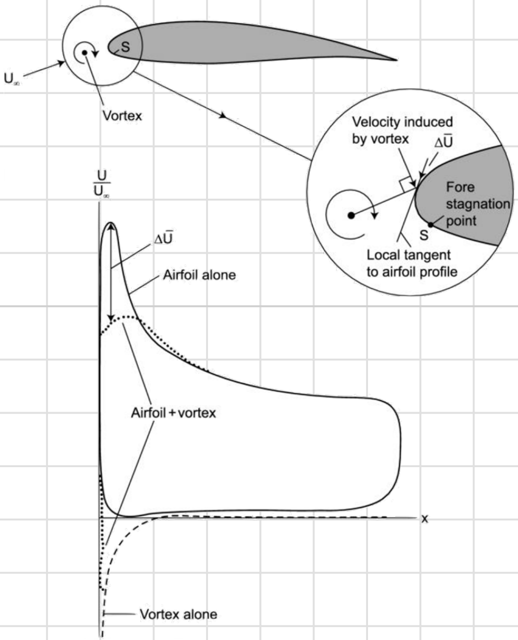
Multi-element airfoils

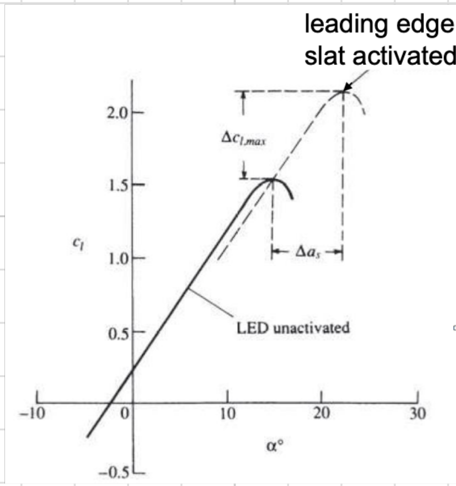


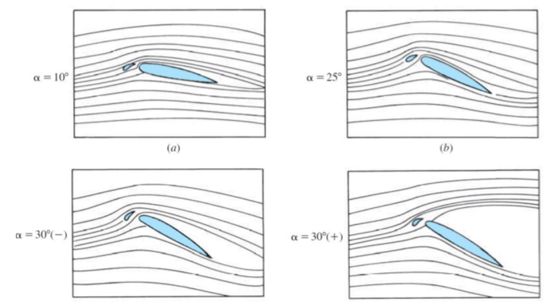


(for η ≠ 0 drag

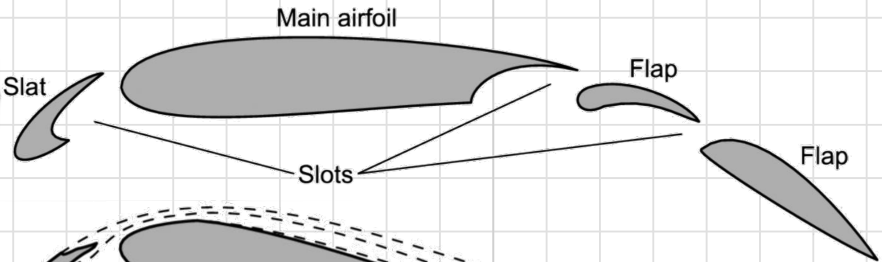
Multi-element airfoils

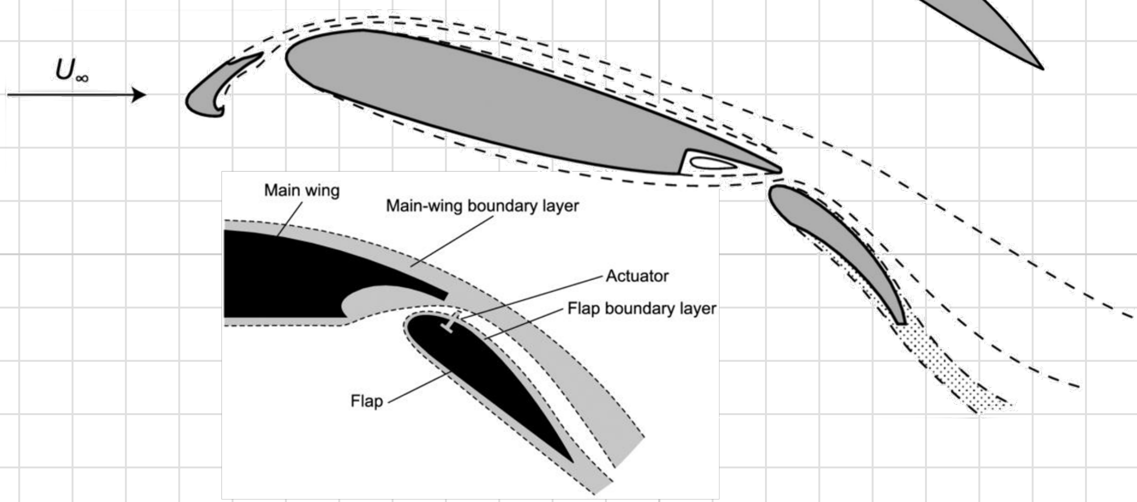




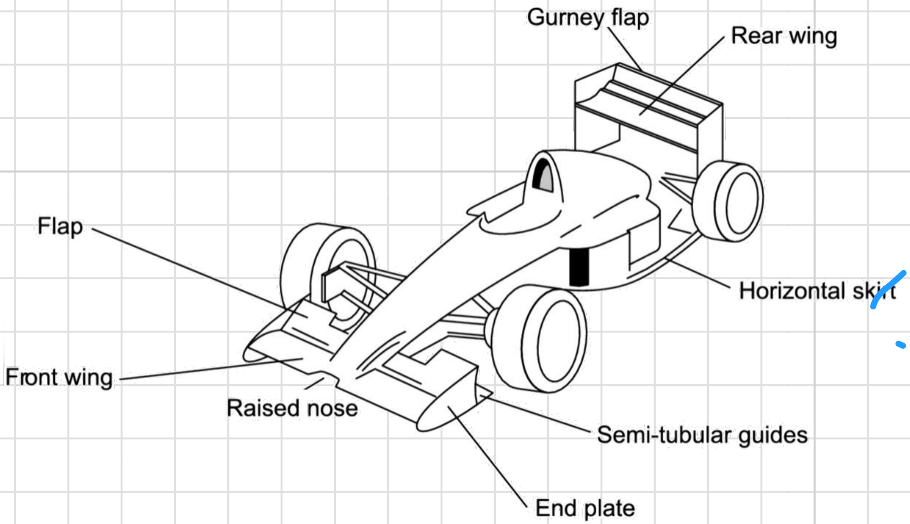


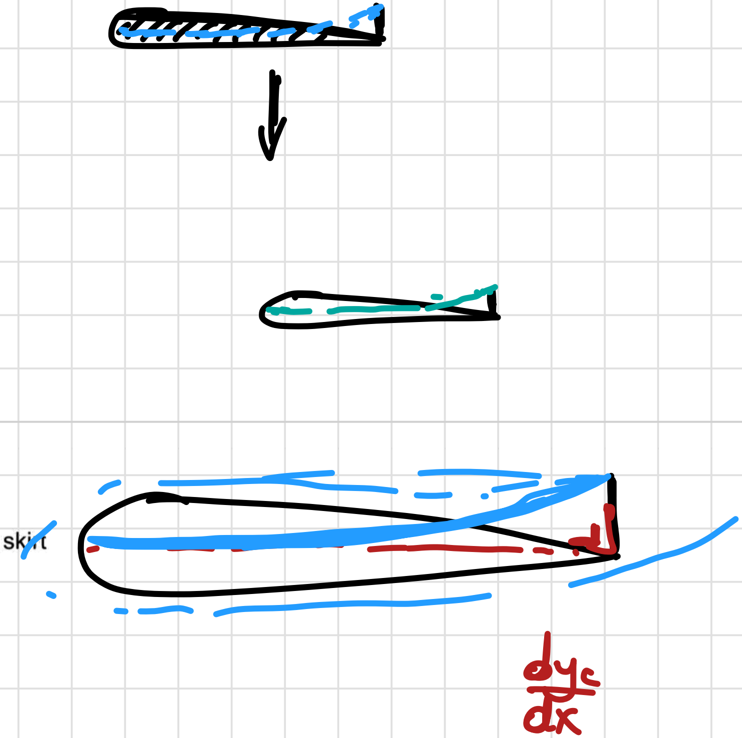
Multi-element airfoils



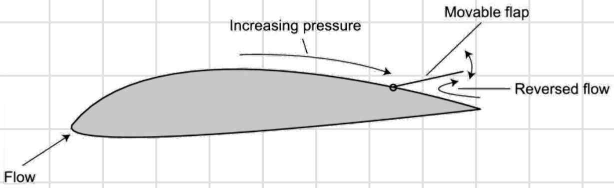


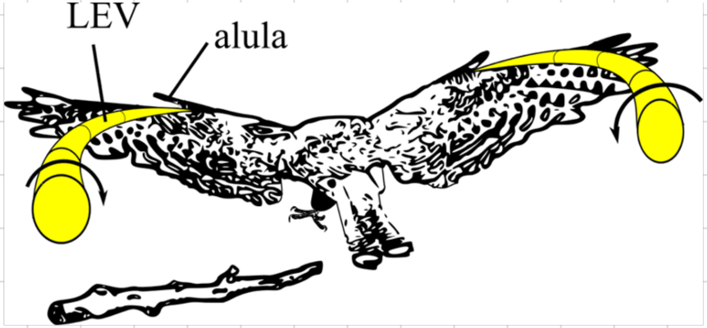
Gurney flaps





Bio-inspired flaps





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