Fall 2025

ME-445 AERODYNAMICS

03 - Potential flow theory

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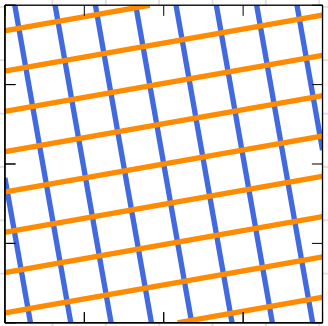
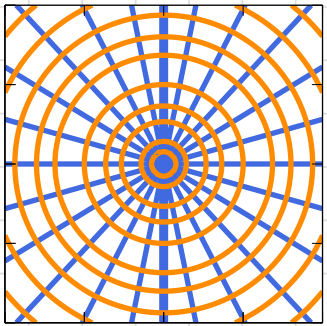
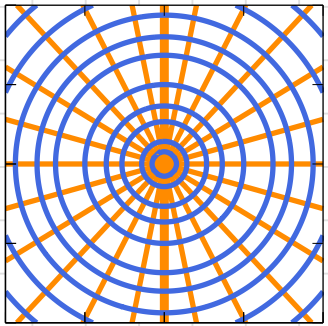
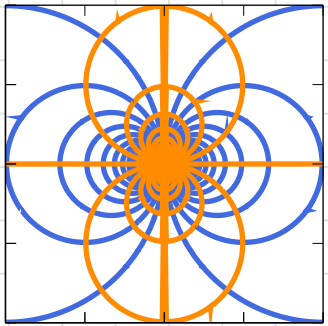
Elementary potential flows

streamlines
equipotential lines

Summary

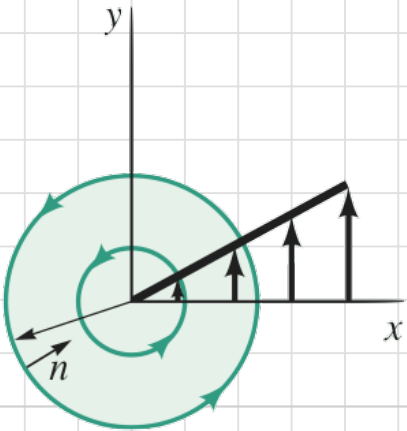
	w	ϕ	ψ
a. Uniform parallel flow	$U_\infty e^{-i\alpha} z$	$U_\infty (x\cos\alpha + y\sin\alpha)$	$U_\infty (y\cos\alpha - x\sin\alpha)$
b. Potential vortex	$-\frac{i\gamma}{2\pi}\ln z$	$\frac{\gamma}{2\pi}\theta$	$-\frac{\gamma}{2\pi}\ln r$
c. Point source or sink	$\frac{Q}{2\pi}\ln z$	$\frac{Q}{2\pi}\ln r$	$\frac{Q}{2\pi}\theta$
d. Source-sink doublet	$\frac{\mu}{2\pi z}e^{i\alpha}$	$\frac{\mu}{2\pi r}\cos(\theta-\alpha)$	$-\frac{\mu}{2\pi r}\sin(\theta-\alpha)$

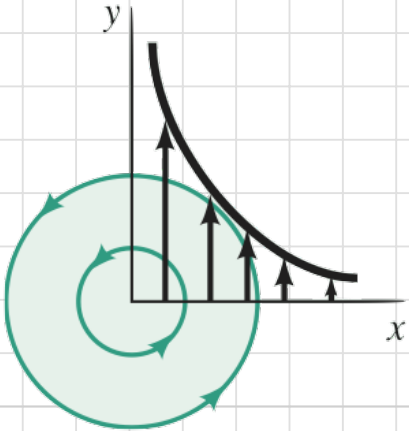





Solid body rotation vs potential or point vortex

$$\nabla \cdot \vec{u} = \frac{1}{r}\frac{\partial (r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}$$

$$\nabla \times \vec{u} = \left(0, 0, \frac{1}{r}\left[\frac{\partial (r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta}\right]\right)$$





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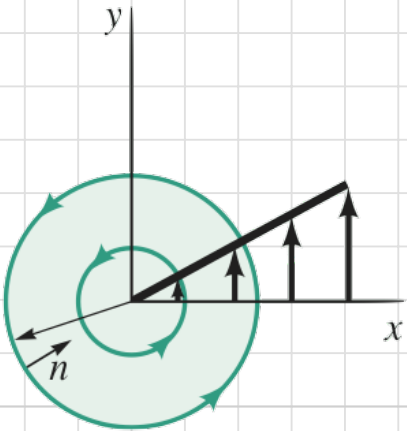
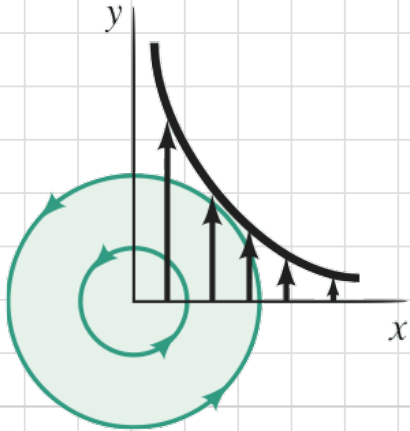
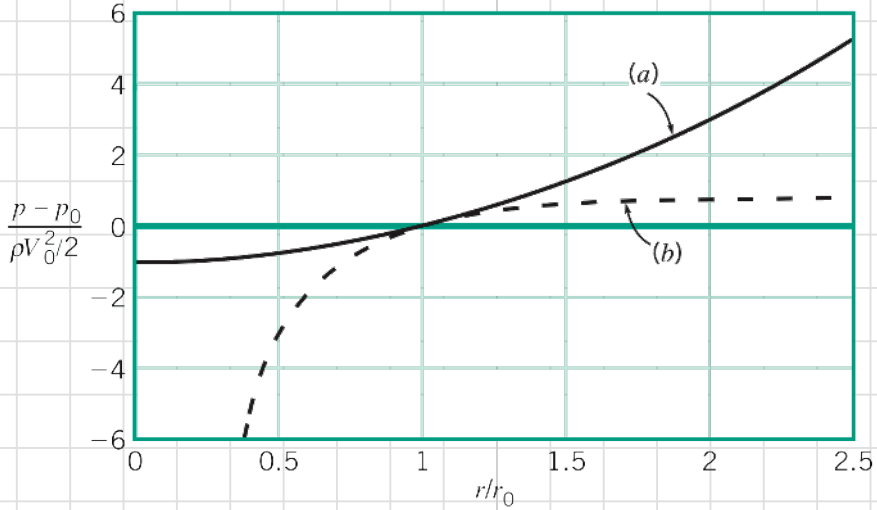
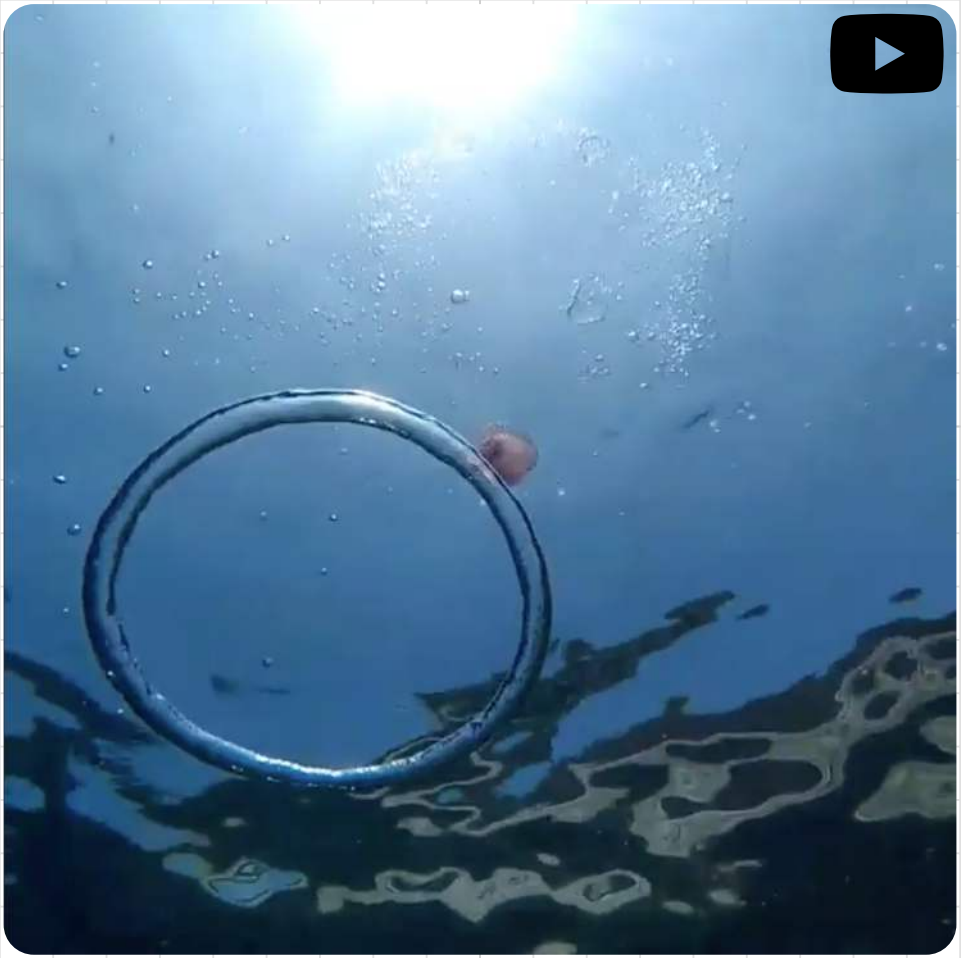






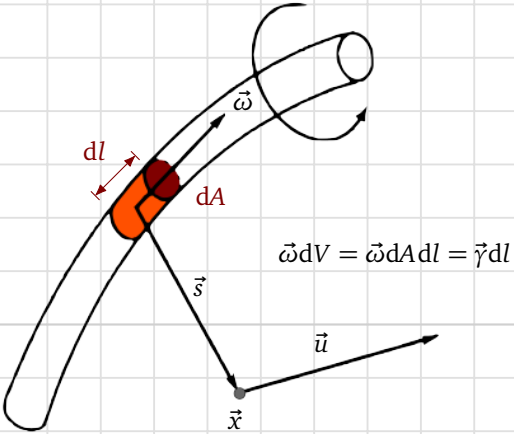
Point vortices vs real vortices

Induced velocity - Biot-Savart



Biot-Savart

determines the velocity field associated with given vorticity field

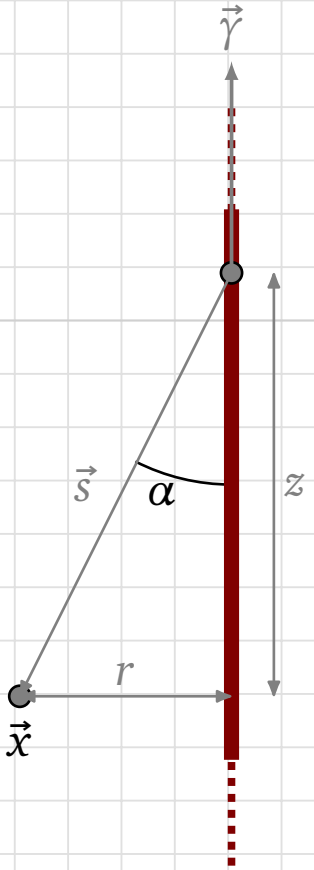


$$\vec{\omega}\mathrm{d}V = \vec{\omega}\mathrm{d}A\mathrm{d}l = \vec{\gamma}\mathrm{d}l$$

$$\vec{u}(\vec{x}) = \frac{1}{4\pi}\iiint_V \frac{\vec{\omega}\times\vec{s}}{s^3}\mathrm{d}V = \frac{1}{4\pi}\oint_L \frac{\vec{\gamma}\times\vec{s}}{s^3}\mathrm{d}l$$

Biot-Savart

Induced velocity by point vortices



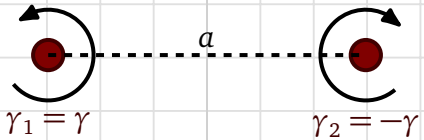
$$u(\vec{x}) = \frac{\gamma r}{4\pi} \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{(r^2 + z^2)^{3/2}} = \frac{\gamma}{2\pi r}$$

semi-infinite vortex line:

$$u(\vec{x}) = \frac{\gamma r}{4\pi} \int_{0}^{\infty} \frac{\mathrm{d}z}{(r^2 + z^2)^{3/2}} = \frac{\gamma}{4\pi r}$$

Interaction of point vortices

What would be the trajectory of this vortex pair?



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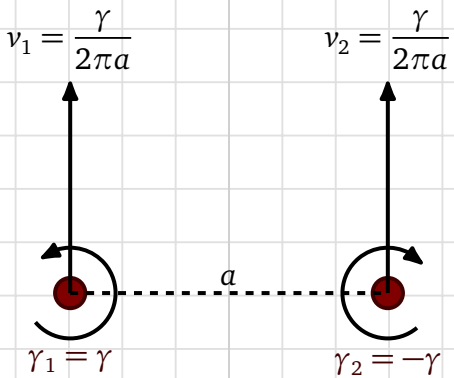

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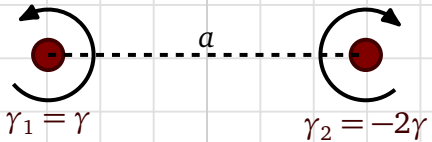


Interaction of point vortices with walls

What would happen to this vortex?

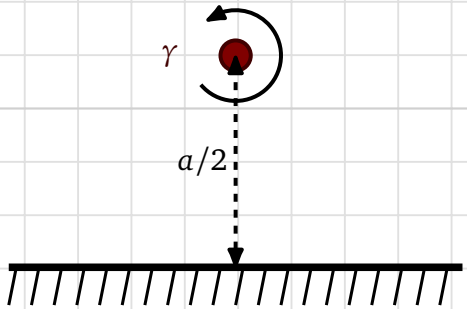
(A) it splits in 2 parts

(B) it disintegrates

(C) it moves to the left

(D) it moves to the right

(E) it bounces back up



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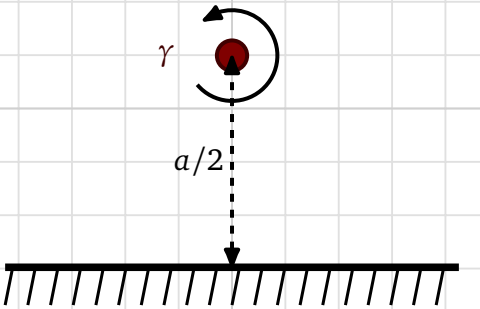
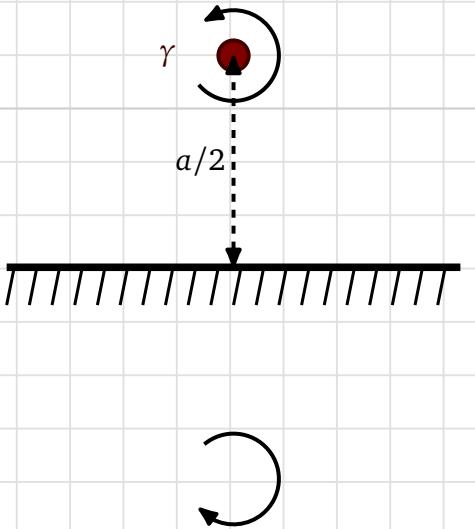


Interaction of point vortices with walls

What about the pressure at the wall?



Interaction of point vortices with walls