ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE
SECTION DE GENIE MECANIQUE
6th & 8th Semester, Fall 2025

ME-435: Aeroelasticity & Fluid Structure Interaction

Chapter 1: Introduction

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About the course

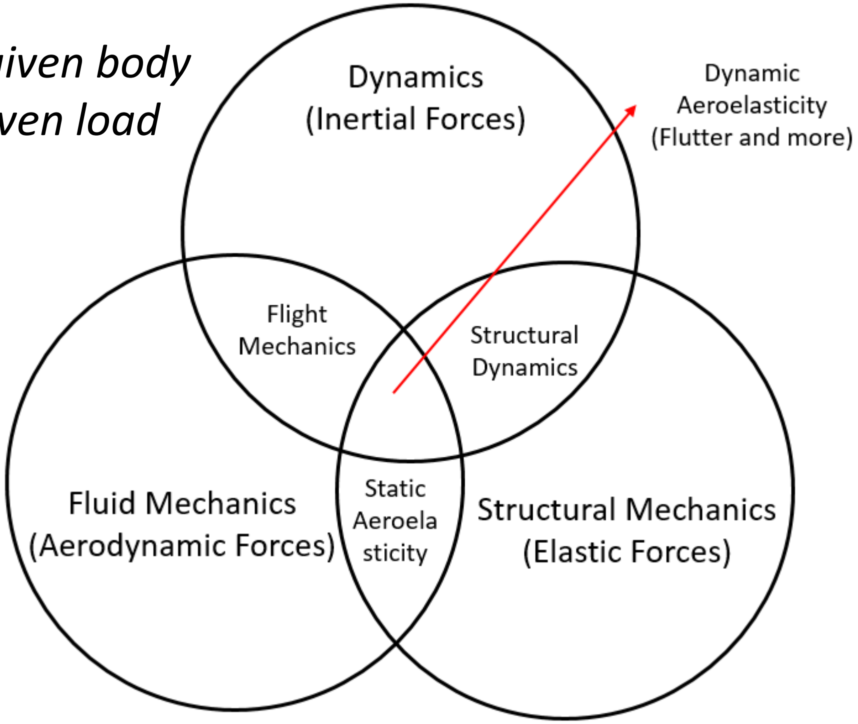
- ***COURSE MATERIAL:***
  - ***Slides (PDF, Moodle server)***
  - ***Videos (Kaltura)***

- ***EXAM***
  - ***Written exam (3 hours)***
    Without access to any written or digital document

DEFINITION

***Aeroelasticity** is the interdisciplinary field of study concerned with the interaction between the deformation of an elastic structure in an airstream and the resulting aerodynamic forces*

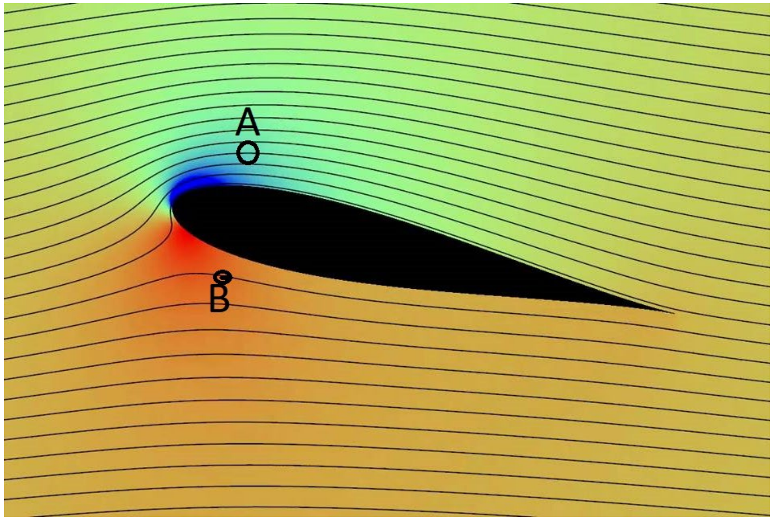
- *Dynamic Aeroelasticity:*
  - *Fluid Mechanics → Forces acting on a given body*
  - *Elasticity → Shape of a body under a given load*
  - *Dynamics → Effect of inertial forces*



DEFINITION

***Fluid-Solid Interaction (FSI)** is a broader field of study concerned with the interaction between the deformation of an elastic structure in any stationary or flowing fluid and the resulting dynamic forces*

*In **classical fluid mechanics**, the solid is simply seen as a boundary, which does not interact with the fluid*



*In **classical solid mechanics**, the fluid is simply seen as a load applied to the structure*

Fluid dynamics framework

- **Continuum mechanics:**
  - Fluids are treated in macroscopic way as continuous mass: Kinematic of individual molecules is not considered
  - Fluid particle = Large number of molecules Within the particle: statistical variation of fluid properties negligible
  - Gas may be considered as continuous medium if the length scale is large compared to the mean free path (MFP):
    - Under normal conditions:
      - The mean free path for a gas ~ 70 nm
      - Nb of molecules within 1 $\mu m^3 \sim 2.7\ 10^7$
    - As the altitude is increased, the mean free path increases and becomes of the same order as a satellite diameter. In this case, the continuum mechanics hypothesis is no more valid.
  - For liquids, the hypothesis is always valid (smaller MFP)

Fluid dynamics framework

- **Physical properties of liquids and gas:**
  - Density: $\rho = \frac{dM}{dV}$
    - dM = Mass of the molecules contained in a fluid particle of volume dV
    - Variation of the density: more likely to occur with a gas than with a liquid
    - If the density of a moving fluid particle remains “constant” ⇒ incompressible flow
  - Viscosity
    - Velocity gradient in a flowing fluid → shear forces
    - Newtonian fluids: the shear stress (tangential force / unit surface) is proportional to the velocity gradient:

$$\tau = \mu \frac{\partial u}{\partial y} = \rho \nu \frac{\partial u}{\partial y}$$

μ: dynamic viscosity ν: kinematic viscosity

    - Shear forces small in comparison with pressure → Ideal fluid, with no internal friction (inviscid flows)

Fluid dynamics framework

- **Mass conservation (Continuity equation):**
  - **Integral form (control volume V bound by a surface S):**

$$\frac{\partial}{\partial t}\iiint_V \rho dV = -\oiint_S \rho\,\boldsymbol{v}.\boldsymbol{n}dS$$

  - **Differential form (using divergence theorem)**:

$$0 = \frac{\partial \rho}{\partial t} + \nabla(\rho\boldsymbol{v}) = \frac{\partial \rho}{\partial t} + (\boldsymbol{v}.\nabla)\rho + \rho(\nabla.\boldsymbol{v}) = \frac{D\rho}{Dt} + \rho(\nabla.\boldsymbol{v})$$

where $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

  - Incompressible flow:
    - Constant density of any fluid particle along its trajectory: $\frac{D\rho}{Dt} = 0$

$$\rightarrow \boldsymbol{\nabla.v = 0}$$

Fluid dynamics framework

- **Momentum conservation (Navier Stokes equations):**
  - Viscous and incompressible flows:

$$\rho\frac{D\boldsymbol{v}}{Dt} = -\nabla p + \rho f + \mu\nabla^2\boldsymbol{v}$$

$$\rho\frac{\partial\boldsymbol{v}}{\partial t} + \rho(\boldsymbol{v}.\nabla)\boldsymbol{v} = -\nabla p + \rho f + \mu\nabla^2\boldsymbol{v}$$

  - Inviscid and incompressible flow (Euler equation):

$$\rho\frac{D\boldsymbol{v}}{Dt} = -\nabla p + \rho f$$

$$\rho\frac{\partial\boldsymbol{v}}{\partial t} + \rho(\boldsymbol{v}.\nabla)\boldsymbol{v} = -\nabla p + \rho f$$

Fluid dynamics framework

- **Reynolds number:**
  - Reynolds experiment:
    - Water flow visualization in a pipe of a diameter D (dye injection)
    - $Re = \frac{U\,D}{\nu}$, represents inertia over viscosity forces ratio
    - Velocity gradually increased :

Re < 2000 : laminar flow

2000 < Re < 4000 :
Transition to turbulence

Re > 4000 : Turbulent flow

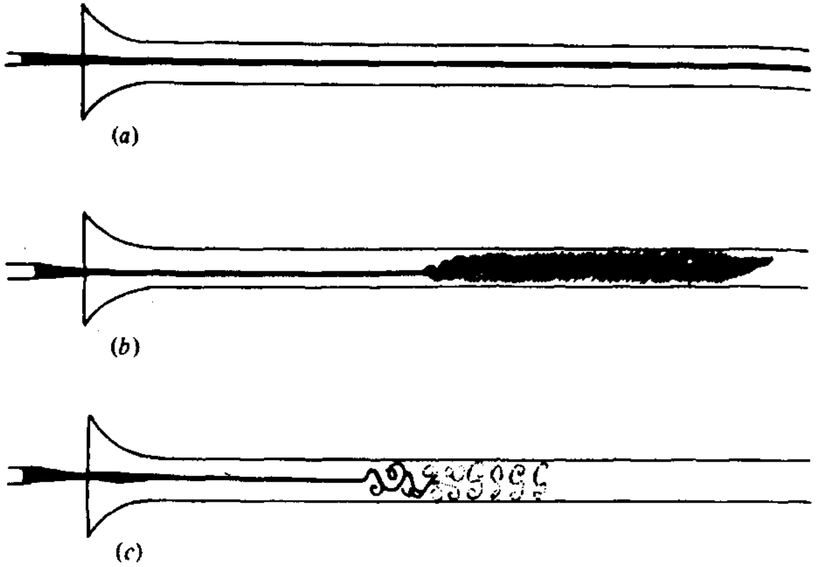


Fig. 9.2. Reynolds's drawings of the flow in his dye experiment.

Fluid dynamics framework

- **Reynolds number:**
  - Reynolds experiment:
    - Water flow visualization in a pipe of a diameter D (dye injection)
    - $Re = \frac{U\,D}{\nu}$, represents inertia over viscous forces ratio
    - Velocity gradually increased :

D = .001 m , Transition to turbulence for U = 2 m/s

D = 1 m, Transition to turbulence for U = 2 mm/s

Fluid dynamics framework

- **Reynolds number:**
  - Momentum conservation of viscous flow over a flat plate

$$\rho \frac{D\boldsymbol{v}}{Dt} = -\nabla p + \cancel{\rho f} + \mu \nabla^2 \boldsymbol{v} \qquad \boldsymbol{v} = (u, v)$$

  - Non dimensional form

$$u^* = \frac{u}{U_\infty}\ ; v^* = \frac{v}{U_\infty}\ ; p^* = \frac{p}{\rho {U_\infty}^2}\ ; x^* = \frac{x}{L}\ ; y^* = \frac{y}{L}; t^* = \frac{tU_\infty}{L}$$

$$\frac{D\boldsymbol{v}^*}{Dt^*} = -\nabla p^* + \frac{1}{Re} \nabla^2 \boldsymbol{v}^*$$

Reynolds number is the **similarity** parameter

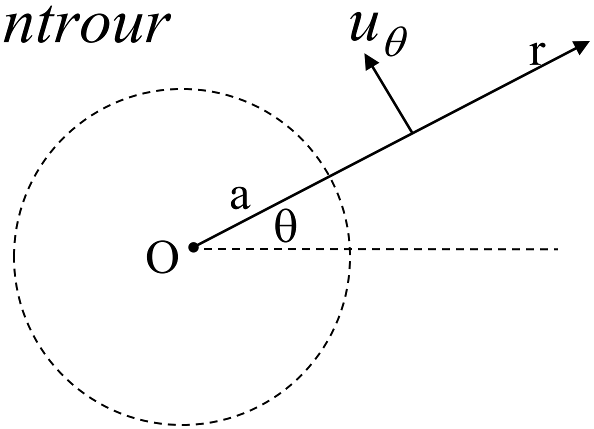
Fluid dynamics framework

Vortices

- **Velocity field of a free vortex (Γ: vortex intensity)**

$$\vec{u} = (0, u_\theta, 0) \quad avec \quad u_\theta(r) = \frac{\Gamma}{2\pi r}$$

$$\Gamma = \int u_\theta(r) dl = cste \quad \forall C, closed\ controur$$



- **Irrotational flow**

$$rot(\vec{u}) = \vec{\nabla} \times \vec{u} = \left(0, 0, \frac{1}{r}\frac{\partial}{\partial r}(r u_\theta)\right) = (0,0,0) \quad \forall r > 0$$

Fluid dynamics framework

Vortices

- **Momentum conservation (radial equilibrium):**

$$\frac{\partial p}{\partial r} = \frac{\rho {u_\theta}^2}{r}$$

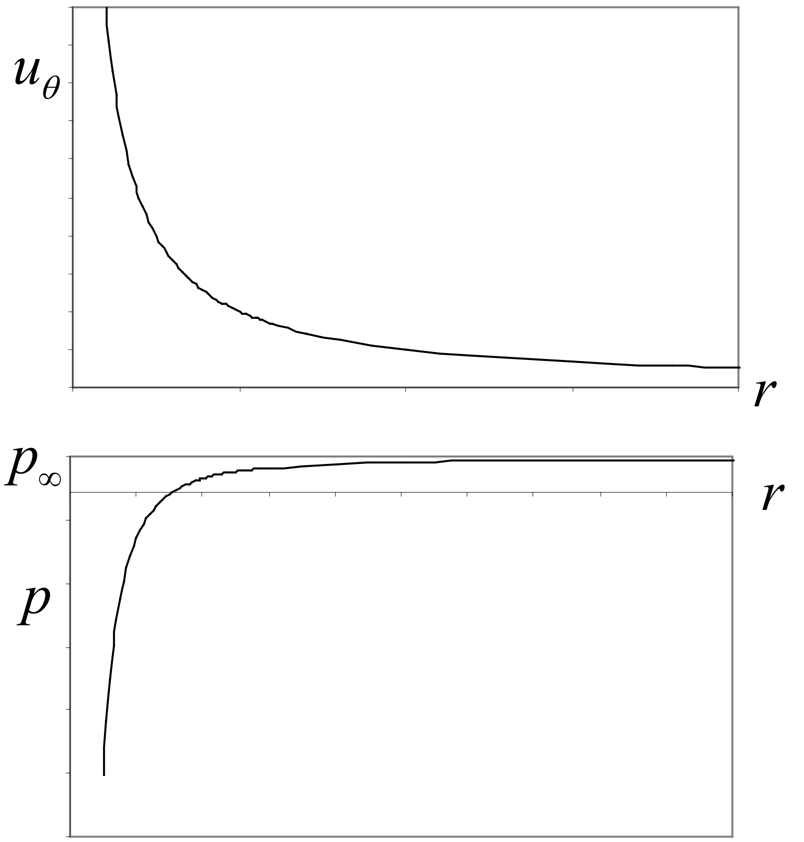
- **Integration:**

$$p(r) = p_\infty - \frac{\rho \Gamma^2}{8\pi^2 r^2}$$

$$r \to 0 \quad \Rightarrow \quad u_\theta(r) \to +\infty$$

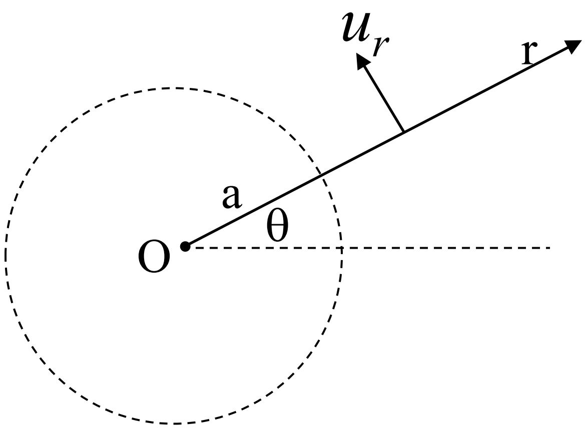
$$r \to 0 \quad \Rightarrow \quad p(r) \to -\infty$$

- **Viscous effects ?**



Fluid dynamics framework

Vortices



- **Rankine vortex model:**
  - **Free vortex zone (I)**
    - **Inviscid flow**
  - **Solid rotation zone (II)**
    - **Viscosity term dominant (viscous core)**

$$Zone\ I\ (r \geq a) \qquad u_\theta(r) = \frac{\Gamma}{2\pi r} \qquad (\Gamma = cste)$$

$$Zone\ II\ (r \leq a) \qquad u_\theta(r) = \omega r \qquad (\omega = cste)$$

- **Continuity of the velocity at *r* = *a*:**

$$\omega a = \frac{\Gamma}{2\pi a} \quad \Rightarrow \quad \Gamma = 2\pi\omega a^2$$

Fluid dynamics framework

Vortices

- **Rankine vortex model:**
  - **Momentum conservation (radial equilibrium):**

$$\frac{\partial p}{\partial r} = \frac{\rho {u_\theta}^2}{r}$$

  - **Solution:**

$$r \geq a \quad p(r) = p_\infty - \frac{\rho \Gamma^2}{8\pi^2 r^2}$$

$$r \leq a \quad p(r) = p_\infty - \frac{\rho \Gamma^2}{8\pi^2 a^2}\left(2 - \frac{r^2}{a^2}\right)$$

Fluid dynamics framework

Vortices

- **Rankine vortex model:**
  - **Non-dimensional form:**
    - **Case of a vortex produced at the tip of a foil ($\boldsymbol{U_\infty}$ is the upstream velocity)**

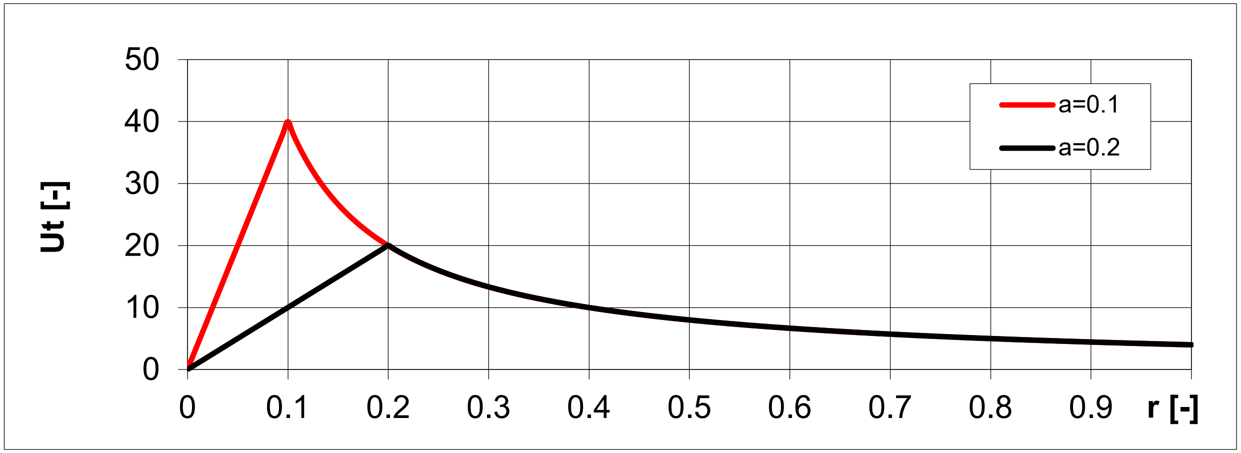
$$r \geq a \qquad c_p(r) = -\frac{\Gamma^2}{4\pi^2 r^2 {U_\infty}^2}$$

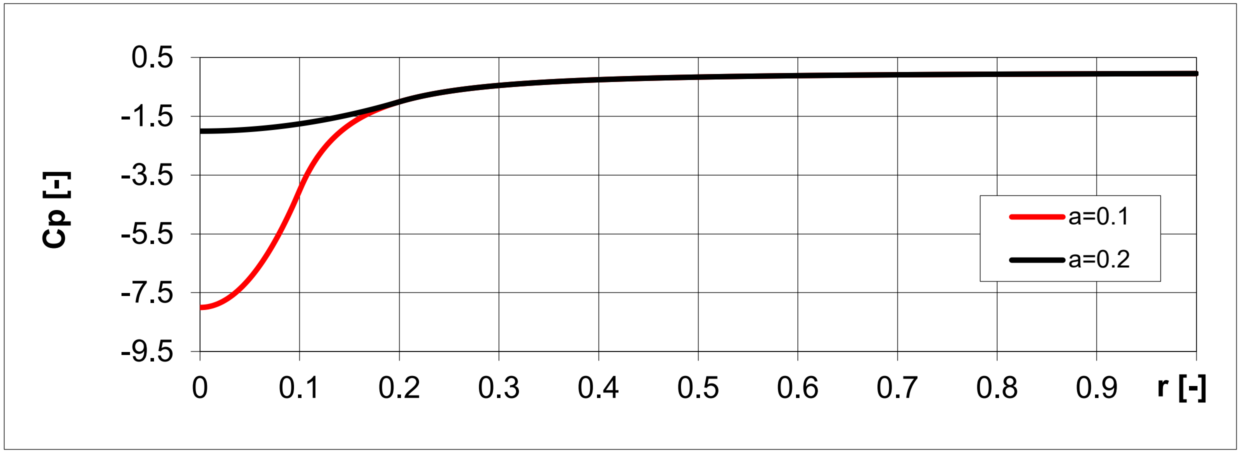
$$r \leq a \qquad c_p(r) = -\frac{\Gamma^2\left(2 - \dfrac{r^2}{a^2}\right)}{4\pi^2 a^2 {U_\infty}^2}$$

Fluid dynamics framework

Vortices

- **Rankine vortex model:**
  - **Effect of the viscous core size**





Fluid dynamics framework

Vortices

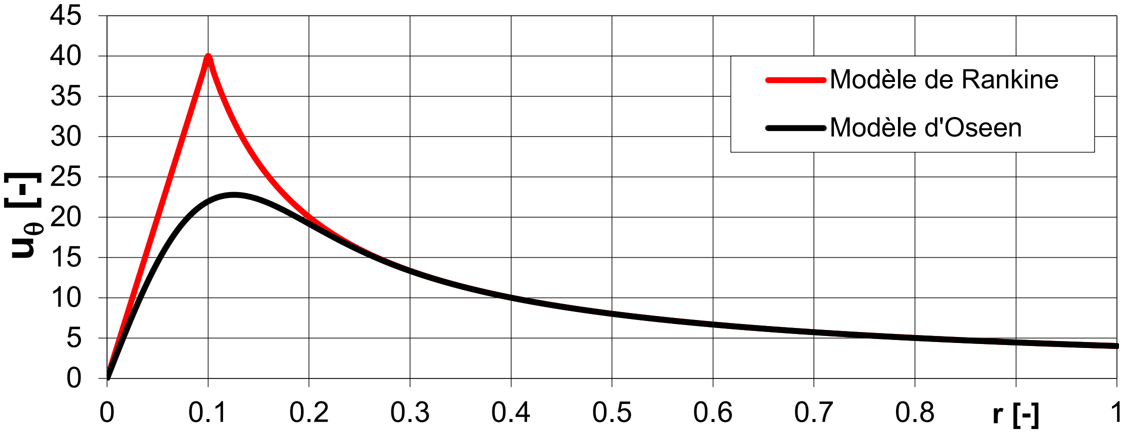
- **Rankine model incompatible with the physics :**
  - **Discontinuity of the velocity derivative at r = a**

- **Oseen Model (more realsitic)**

$$u_\theta(r) = \frac{\Gamma}{2\pi r}\left(1 - e^{-(r/\lambda)^2}\right) \qquad \forall r \geq 0 \qquad\qquad \lambda = \frac{a}{1.12}$$

  - **a: Location of maximum tangential velocity (viscous core size)**

$$r \to +\infty \quad \Rightarrow \quad u_\theta(r) \approx \frac{\Gamma}{2\pi r} \; (Free\ vortex) \quad and \quad u_\theta(0) = 0$$



Fluid dynamics framework

Tip vortices are commonly observed in a variety of industrial applications

- **Airplane wings, wind turbines, compressors, …**
- **Difficult to predict (trajectory, intensity, viscous core size)**

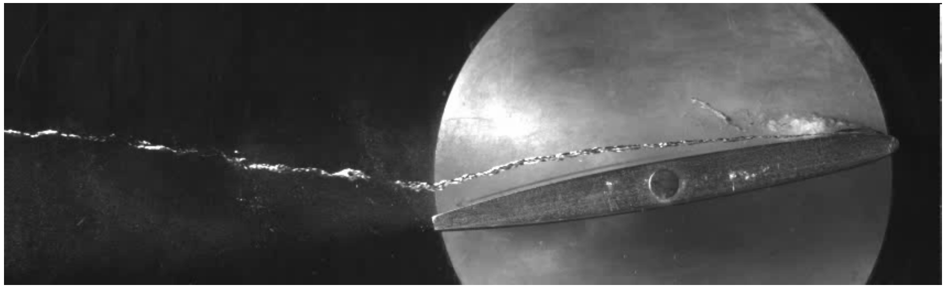
Influence of the gap width on tip leakage vortex in EPFL cavitation tunnel

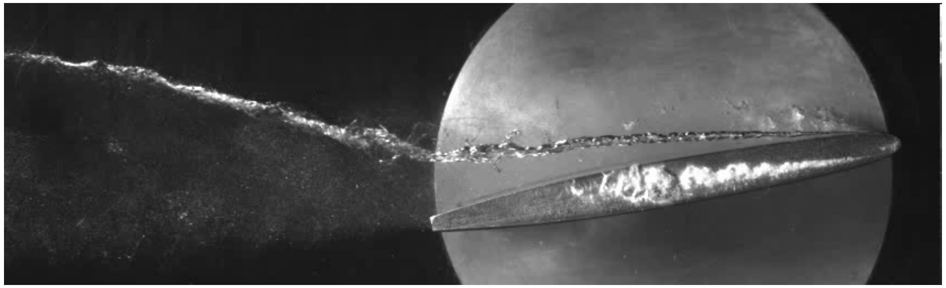
20’000 frames/s, inlet velocity = 10 m/s, incidence=10°, inlet pressure=1 bar

gap=15 [mm]

Matthieu Dreyer, PhD student, EPFL 2013

gap=7 [mm]





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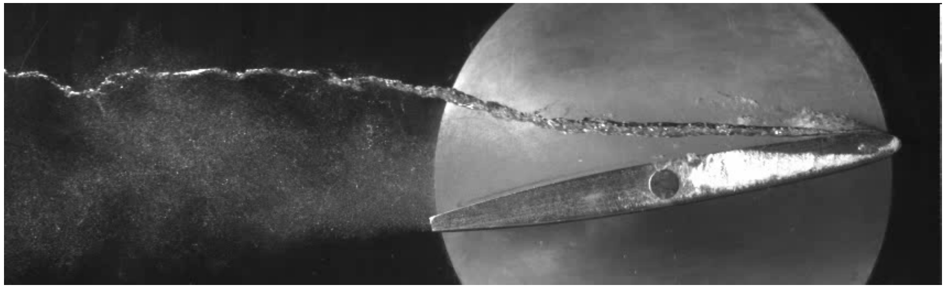
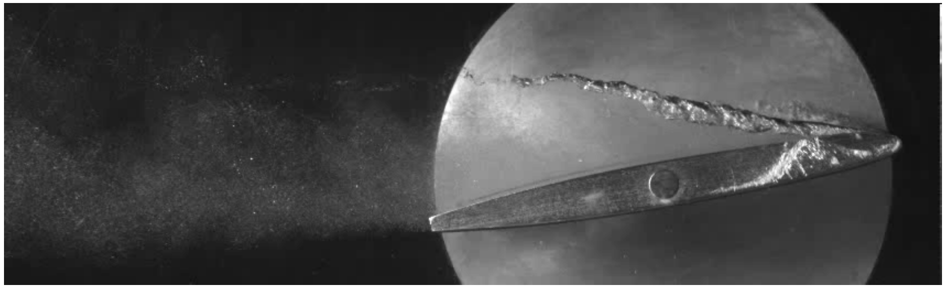
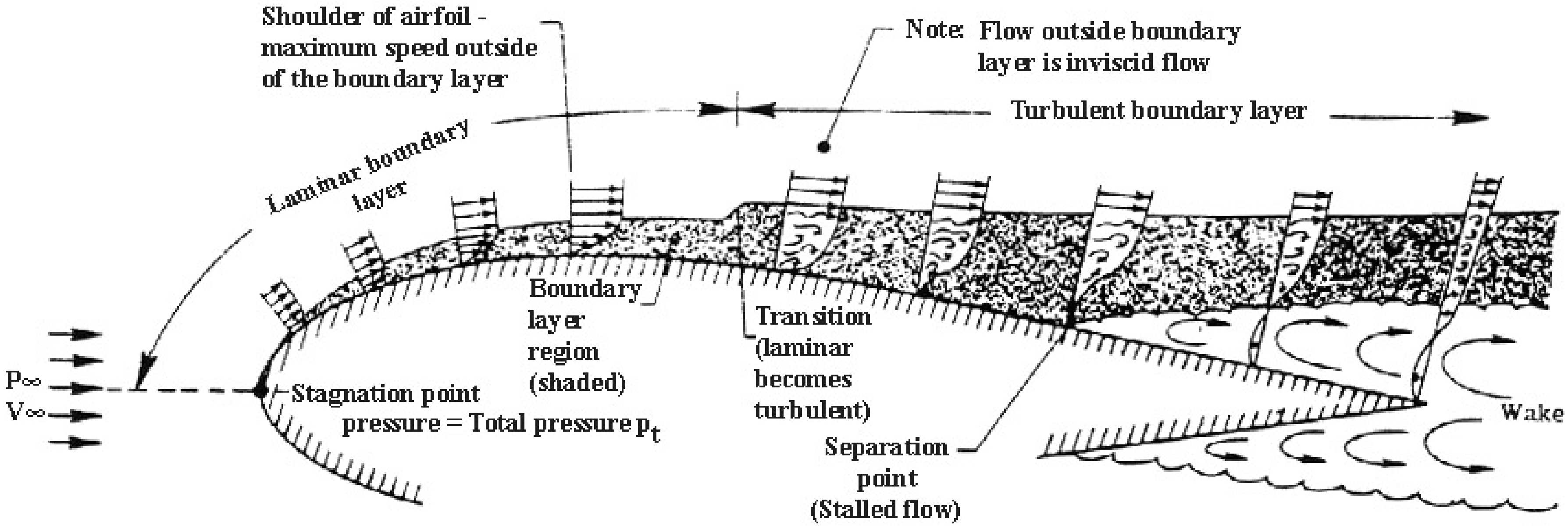




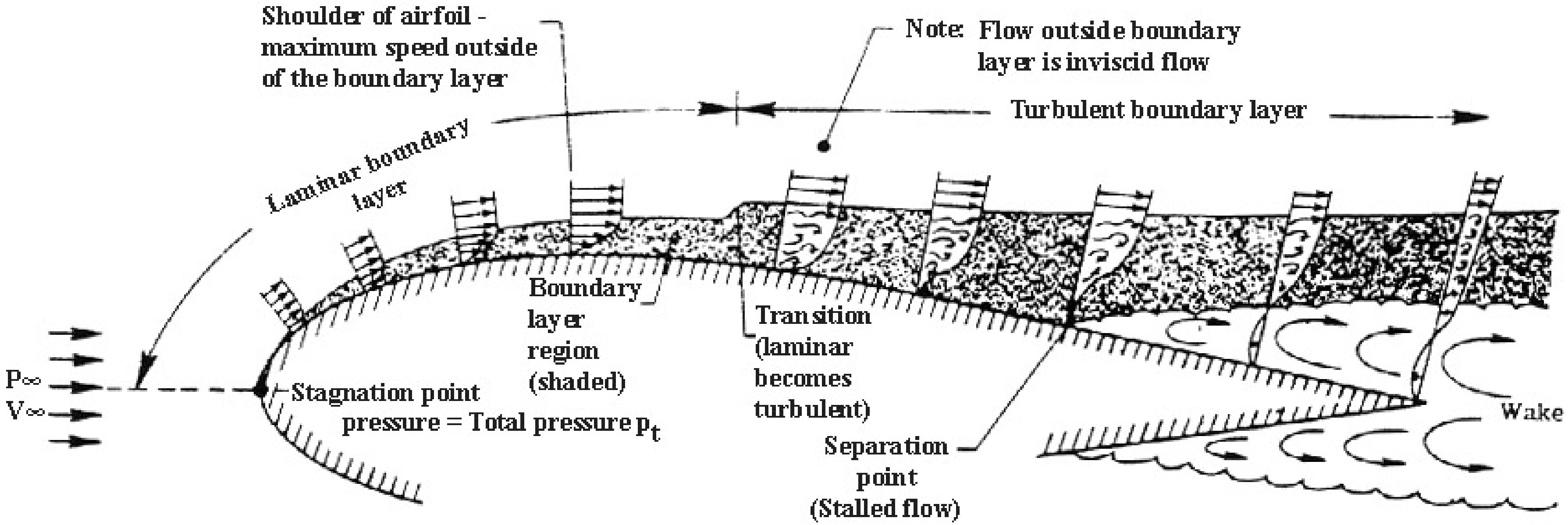
Fluid dynamics framework

- **Boundary layer:**
  - **Definition: the fluid layer, next to a solid surface, where the viscous forces are dominant**
    - **Velocity at the wall = 0 (No slip condition)**
    - **The flow outside the boundary layer is inviscid**



Fluid dynamics framework

- **Boundary layer:**
  - **Boundary layer states:**
    - **Laminar : fluid layers slide on each other without being mixed**
    - **Transition to turbulence**
    - **Turbulent : chaotic motion of the fluid within the BL**
    - **Separated : due to adverse pressure gradient (decelerated flow)**



Fluid dynamics framework

- **Boundary layer:**
  - **Boundary layer thickness:**

$$Thickness : \delta = y\big|_{u=0.99U_e}$$

$$Displacement\,thickness : \quad \delta_1 = \int_0^{\delta} \left(1 - \frac{u}{U_e}\right) dy$$

$$Momentum\;thickness : \qquad \delta_2 = \int_0^{\delta} \left(1 - \frac{u}{U_e}\right)\frac{u}{U_e}\; dy$$

$$Form\;\; factor : \qquad H_{12} = \frac{\delta_1}{\delta_2}$$

Fluid dynamics framework

- **Boundary layer:**
  - **Wall shear stress:**

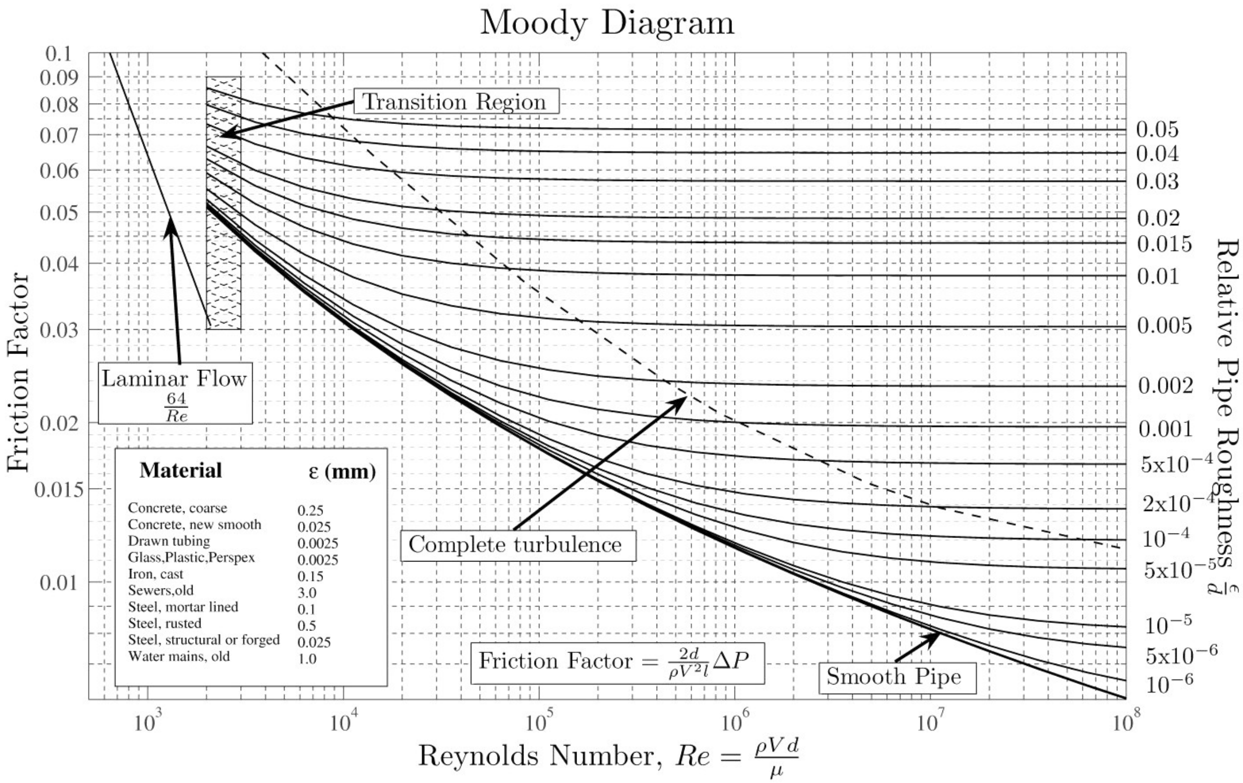
$$\tau_w(x) = \mu\left(\frac{\partial u}{\partial y}\right)\Bigg|_{y=0}$$

  - **Skin Friction Coefficient:**

$$C_f(x) = \frac{\tau_w(x)}{\frac{1}{2}\rho U_\infty^{\,2}}$$

Fluid dynamics framework

- **Boundary layer:**
  - **Friction factor of a flow in a pipe (Moody's diagram):**
    - **Mainly dependent on Reynolds number and relative roughness**
    - **Fully turbulent regime: Friction no more depends on Re number**



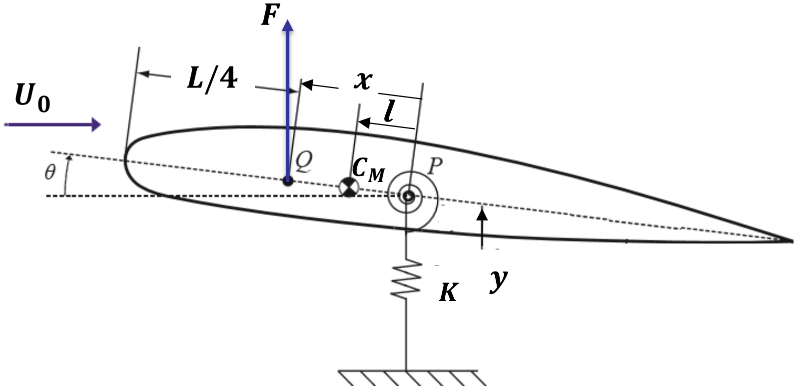
Material	ε (mm)
Concrete, coarse	0.25
Concrete, new smooth	0.025
Drawn tubing	0.0025
Glass,Plastic,Perspex	0.0025
Iron, cast	0.15
Sewers,old	3.0
Steel, mortar lined	0.1
Steel, rusted	0.5
Steel, structural or forged	0.025
Water mains, old	1.0

Examples of Fluid-Structure Interaction

Dynamic instability of a wing : Flutter

Flutter: Large vibration involving 2 vibration modes of a foil in a flow

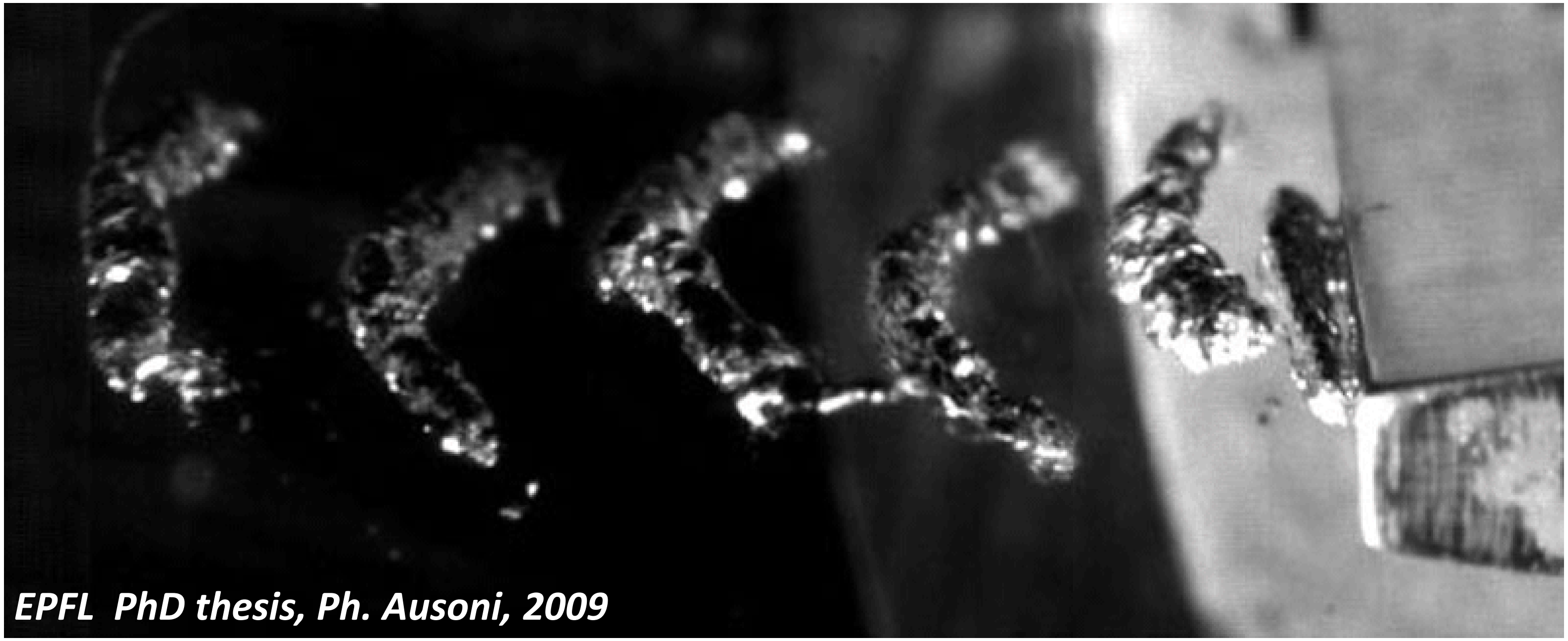
Twist and plunge modes approximation



Examples of Fluid-Structure Interaction

Vibration induced by wake instabilities

- ***Example: Blunt trailing edge hydrofoil in water stream***
  - ***Naca009, 100 mm chord, 150 mm span, 3.2 mm trailing edge thickness***
  - ***Blunt trailing edge → Alternate vortex shedding (Kàrmàn)***
  - ***Excitation of the 1st torsional mode***
  - ***Lock-in: shedding frequency locked on resonance frequency in a wide range of flow velocities (!)***

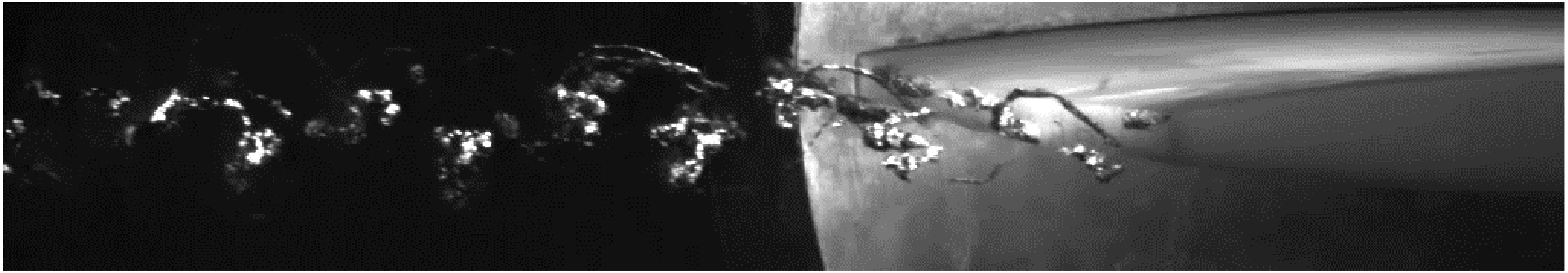


EPFL PhD thesis, Ph. Ausoni, 2009

Examples of Fluid-Structure Interaction

Vibration induced by wake instabilities

- ***Example: Deformable hydrofoil (POM) in water stream***
  - ***Naca009, 100 mm chord, 150 mm span, 3.2 mm trailing edge thickness***
  - ***Blunt trailing edge → Alternate vortex shedding (Kàrmàn)***
  - ***Excitation of the 2nd bending mode → alteration of shedding mechanism***



Examples of Fluid-Structure Interaction

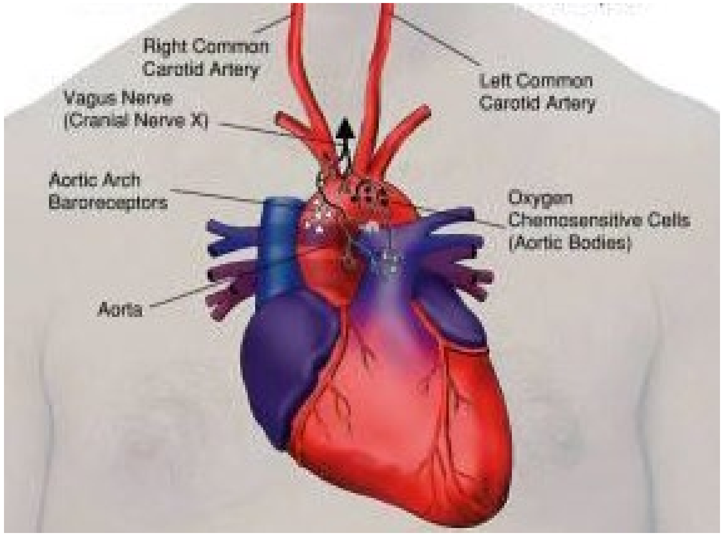
Vibration induced by wake instabilities

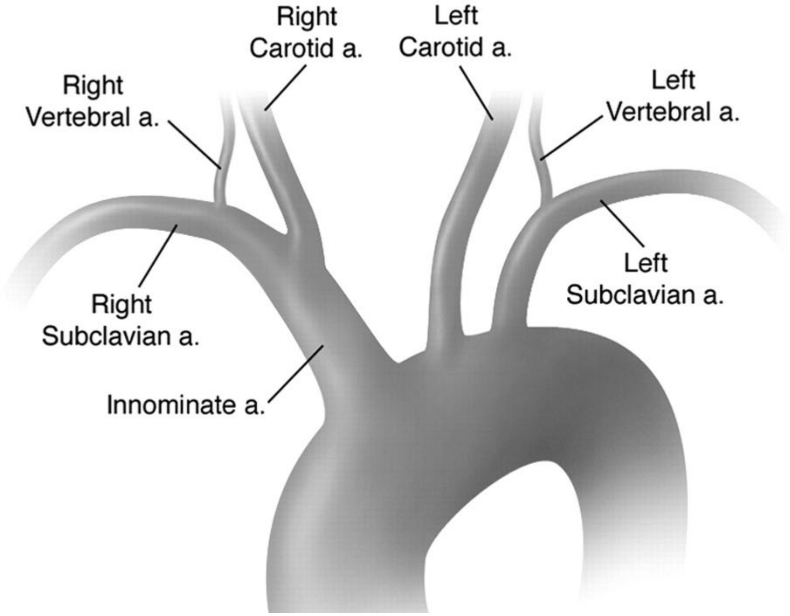
- ***Example: Kaplan Turbine Model***
    - ***Alternate vortex shedding (Kàrmàn)***
    - ***Possible excitation of the blades → risk of premature cracks***
    - ***Lack of scale-up rules for fluid-interaction interaction in turbines***

Examples of Fluid-Structure Interaction

Blood flow in a human aorta

- ***Flow visualization in a <u>rigid</u> aorta model***
  - ***Reverse flow in the arteries is due to the rigidity of the aorta***
  - ***In reality aorta is flexible and adapts its shape to avoid reverse flows***

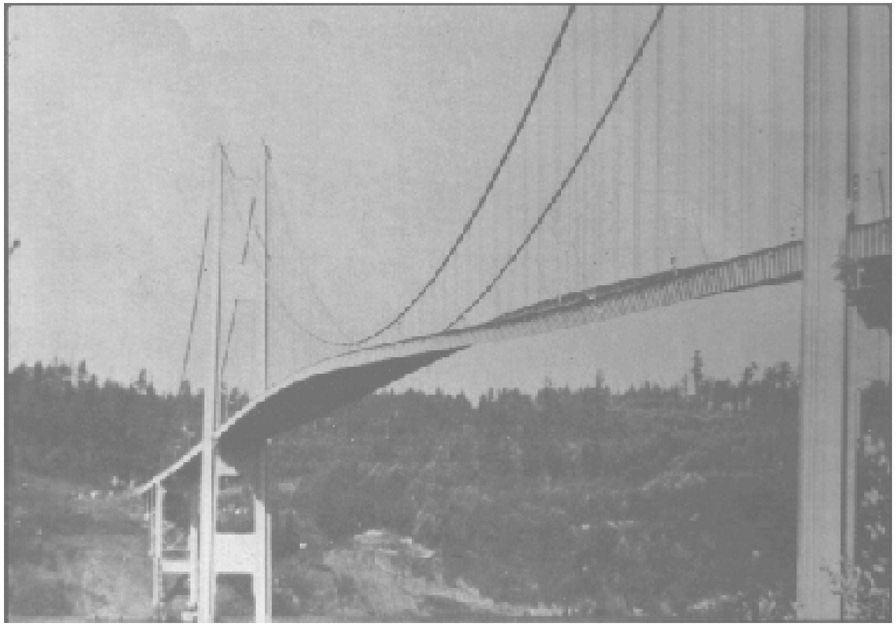




Examples of Fluid-Structure Interaction

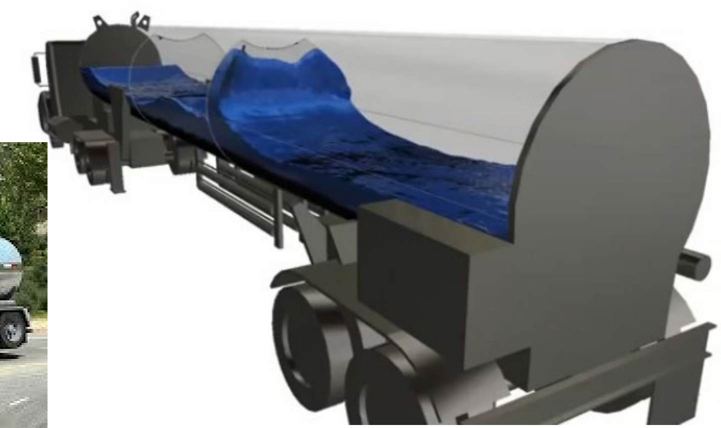
Collapse of Tacoma Narrows bridge

- **Nov. 7th 1940 (after 4 months of operation)**
- **65 km/h wind speed**
- **Root cause: Aero-elastic coupling**
  - **Vortex shedding matches resonance**



Examples of Fluid-Structure Interaction

- ***Sloshing Dynamics***
  - *Oscillating motion of the free surface of a liquid within a partially filled container*
  - *Variety of examples:*
    - *Liquid transportation (trucks and tankers)*
    - *Ballast, or fuel tanks of large ships*
    - *Aircraft and rocket fuel tanks*
    - *Food processing*
    - *Bioreactors*
    - *…*



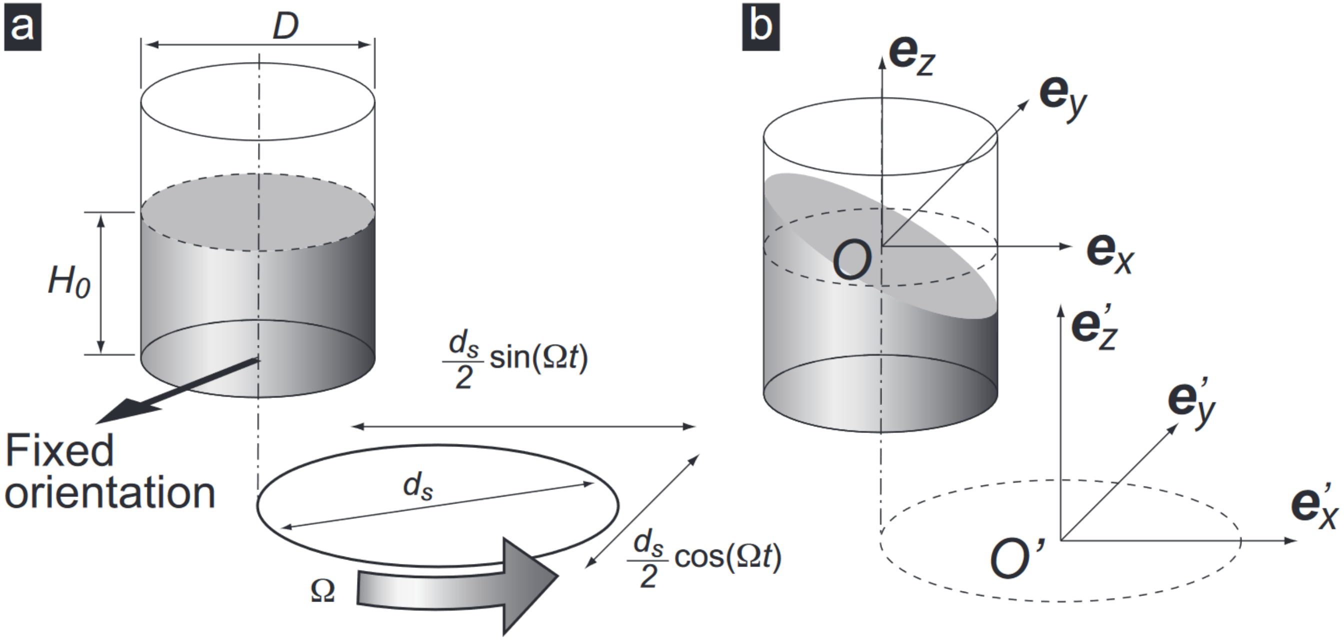
Sloshing Dynamics

- ***Sloshing Dynamics:***
  - *Orbital sloshing as a tool to improve mixing and gas exchange*
    - *Bioreactors for cell cultivation*
    - *Food processing*

Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container - Assumptions :***
  - ***Upright rigid cylinder***
    - ***Diameter:D, liquid height at rest:H_0, Excentricity: ds, Rotational speed: Ω, Interface position: ξ***
  - ***Incompressible, inviscid and irrotational liquid flow***
  - ***Small oscillation amplitude of the interface***
- ***Non-dimensional numbers:***

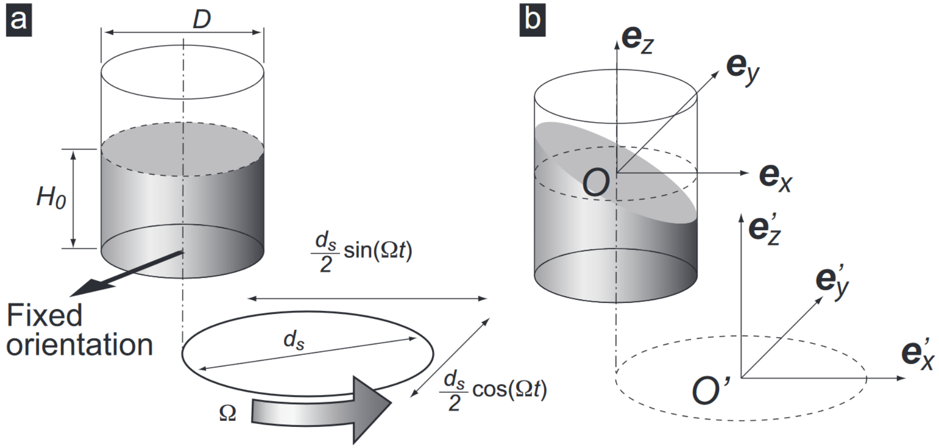
$$\begin{cases} \tilde{d}_s = \dfrac{d_s}{D} \\ \tilde{H}_0 = \dfrac{H_0}{D} \\ Fr = \sqrt{\dfrac{d_s\Omega^2}{g}} \\ \tilde{\xi} = \dfrac{\xi}{D} \end{cases}$$



Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container***
  - ***Rich variety of wave shapes***
  - ***Complex flow dynamics induced by the rotating wave***

M. Reclari, EPFL PhD Thesis, 2014




27/11/11 Why swirling wine in a glass makes it taste better - Telegraph

The Telegraph

Why swirling wine in a glass makes it taste better

Wine buffs who swirl their drink in a glass are using the sophisticated physics of wav
technology to unleash the flavour, scientists say.

Swirling a glass of wine churns the liquid as it travels, drawing in oxygen from the air and intensifying the smell Photo: ALAMY

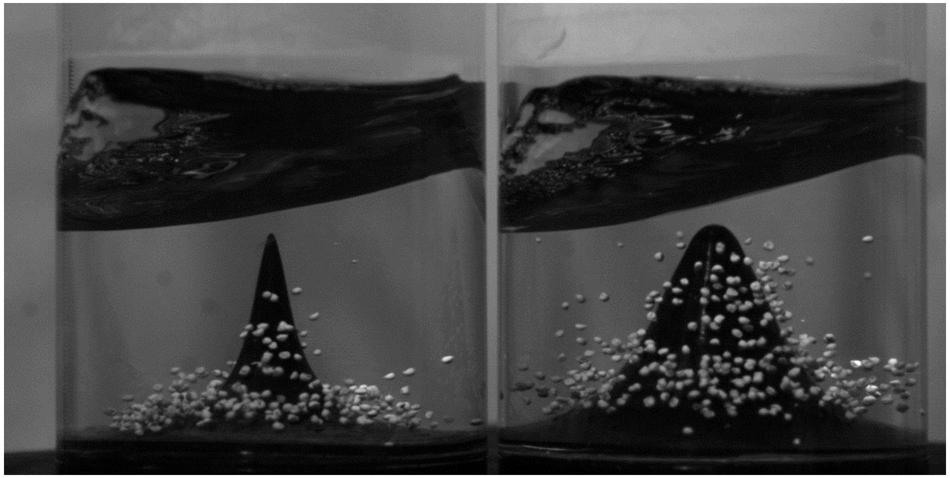
Sloshing Dynamics

- ***Orbital sloshing dynamics in a cylindrical container***
    - ***Effect of container shape ?***
        - ***Semester projects, M. Moosavi 2015, S. Eghbali 2016, Rossi 2018***

Visualisation of mixing improvements with the bump bottom using solid particles in suspension

Ω=135 RPM, d_S=44 mm, H_0=84 mm

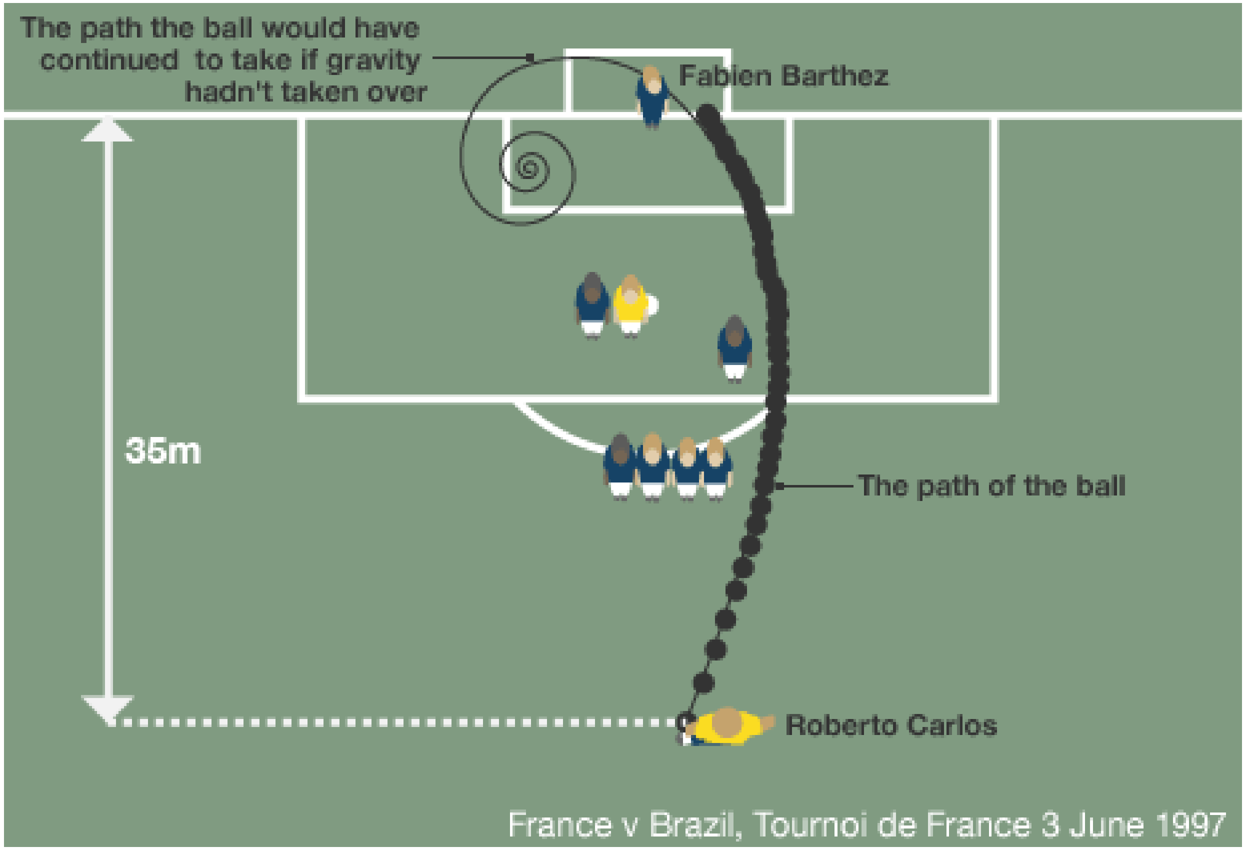
Ω=135 RPM, d_S=44 mm, H_0=84 mm



Examples of Fluid-Structure Interaction

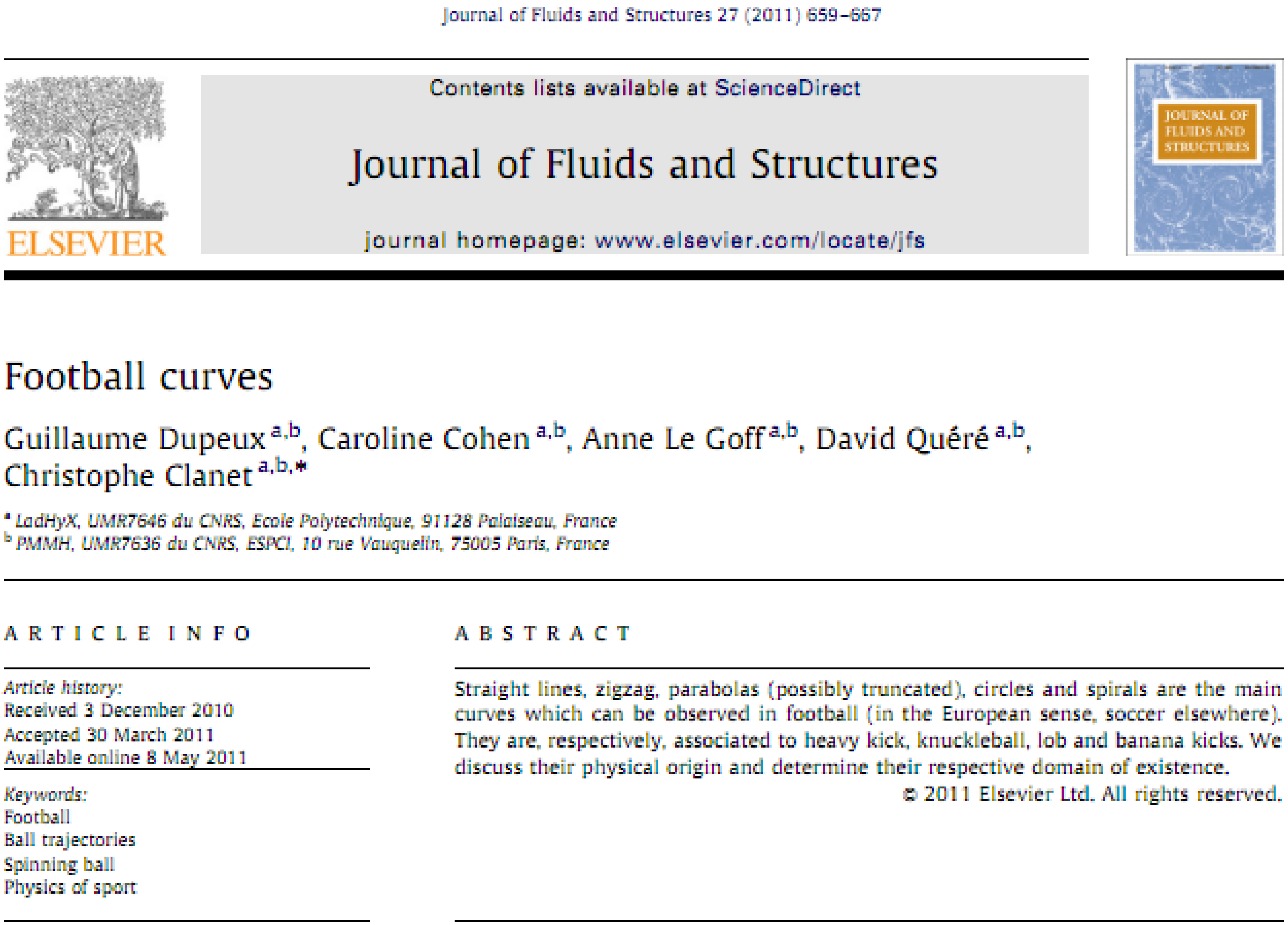
Roberto Carlos Spiral: Fluid-Solid interaction in football

How Carlos scored an 'impossible' goal



Examples of Fluid-Structure Interaction

The Roberto Carlos Spiral (Aerodynamics related to soccer ball)



Journal of Fluids and Structures 27 (2011) 659–667

Contents lists available at ScienceDirect

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Football curves

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ARTICLE INFO

Article history:
Received 3 December 2010
Accepted 30 March 2011
Available online 8 May 2011

Keywords:
Football
Ball trajectories
Spinning ball
Physics of sport

ABSTRACT

Straight lines, zigzag, parabolas (possibly truncated), circles and spirals are the main curves which can be observed in football (in the European sense, soccer elsewhere). They are, respectively, associated to heavy kick, knuckleball, lob and banana kicks. We discuss their physical origin and determine their respective domain of existence.

Examples of Fluid-Structure Interaction

The Roberto Carlos Spiral