Exercise – Serie 6 – Dynamic Instability

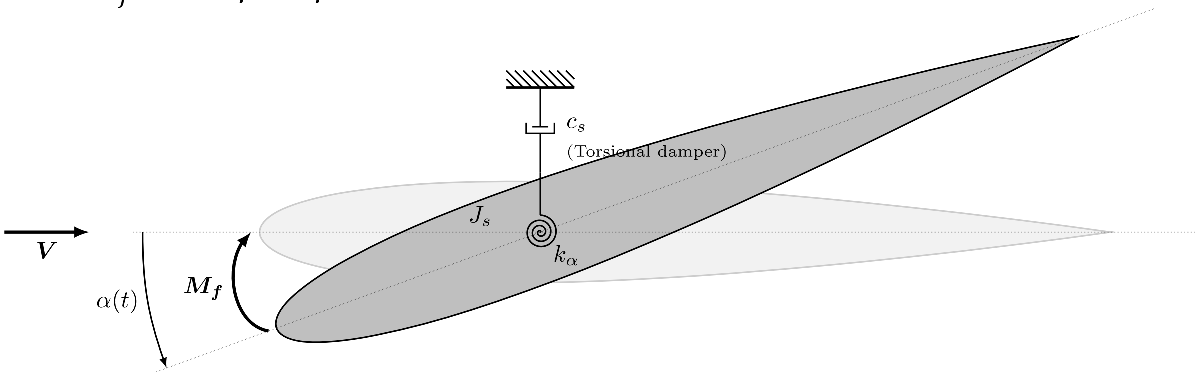
For the questions requiring numerical applications, use Matlab or Python.

Exercise 1 - Fluid-structure coupling for an oscillating hydrofoil.

The motion of a freely oscillating **symmetric** hydrofoil, as depicted on the following figure (pitching motion in α only), can be described by the following equation:

$$M_f = J_s\ddot{\alpha} + c_s\dot{\alpha} + k_s\alpha \tag{1}$$

where M_f is the hydrodynamic moment due to the flow.



For the following questions, note that the hydrofoil is placed in water.
It is at rest at $\alpha = 0$ and we consider the initial incidence angle to be $\alpha_0 = \alpha(t=0) = 2°$, with no initial velocity.

a) We will model the oscillation of the foil using the quasi-static approach as seen in the lectures. Find the hydrodynamic moment M_f using the quasi-static aerodynamic forces and write the equation of motion of the hydrofoil. The chord length is $c = 0.1$ m, the hydrofoil span is $b = 1$ mm, the aerodynamic center is located at $c/4$, the pitch axis at $c/2$ and $C_{L,\alpha} = 2\pi$.

b) The added mass of an accelerating thin hydrofoil can be approximated as the added mass of a flat plate. Compute this added mass using the expression below and inject it in the equation of motion derived in point 2.

$$J_{f,plate} = \frac{1}{128}\rho\pi c^4 b$$

c) To obtain more realistic results, a research conducted by Münch[1] showed that the flow response to the hydrofoil motion can be modelled by a linear combination of the angular position α and its derivatives (assumption valid in the case of small angles of attack), the hydrodynamic moment can be evaluated as:

$$M_f = -\left(J_f\ddot{\alpha} + c_f\dot{\alpha} + k_f\alpha\right) \tag{2}$$

where J_f, c_f, and k_f represent the fluid's added mass, fluid's damping, and fluid's stiffness, respectively.

In his research, he also showed that CFD simulations of the hydrofoil forced motion enabled identifying the values of J_f, c_f, and k_f and this linear model showed an excellent agreement with coupled fluid-structure simulations of a freely oscillating hydrofoil. Münch *et al.*

results thus showed that the accurate modeling of a freely oscillating hydrofoil could be obtained at a much lesser computational cost using hydrofoil forced motion simulation.
The values found for J_f, c_f, and k_f are summarized in table 1 below for two different reduced frequencies, $k = {}^{\omega c}/_{2\,C_{ref}}$ (c being the chord length, ω the oscillating frequency and $C_{ref} =$ 5 m/s the freestream velocity).

Considering equations (1) and (2) and the six cases provided in Table 1 compute the motion of the hydrofoil $\alpha(t)$. Solve the equation analytically or numerically and plot the solution from 0 to 60 ms.

d) Solve the equations of question a) and b) analytically or numerically and plot the solution from 0 to 100 ms. Compare the results with the ones obtained before.

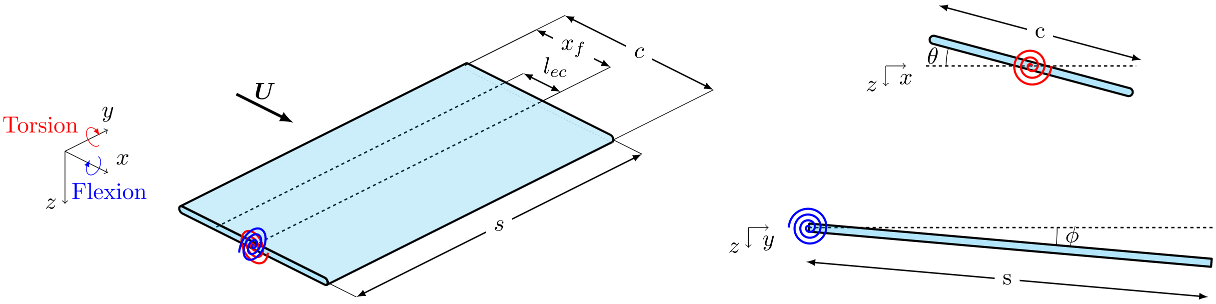
Table 1: Hydrofoil properties, added mass, fluid stiffness, and fluid damping.

	k	J_s (kg m^2)	k_s (Nm)	c_s(kg m^2 s^{-1})	J_f (kg m^2)	k_f (Nm)	c_f(kg m^2 s^{-1})
Case 1a	2.62	$1 \cdot 10^{-5}$	1	$4 \cdot 10^{-4}$	$2.45 \cdot 10^{-6}$	-0.14	$3.82 \cdot 10^{-4}$
Case 1b	2.62	$1 \cdot 10^{-5}$	1	$6.2 \cdot 10^{-3}$	$2.45 \cdot 10^{-6}$	-0.14	$3.82 \cdot 10^{-4}$
Case 1c	2.62	$1 \cdot 10^{-5}$	1	$2 \cdot 10^{-2}$	$2.45 \cdot 10^{-6}$	-0.14	$3.82 \cdot 10^{-4}$
Case 2a	15.5	$1 \cdot 10^{-5}$	30	$5 \cdot 10^{-3}$	$2.45 \cdot 10^{-6}$	-0.14	$2.59 \cdot 10^{-4}$
Case 2b	15.5	$1 \cdot 10^{-5}$	30	$3.9 \cdot 10^{-2}$	$2.45 \cdot 10^{-6}$	-0.14	$2.59 \cdot 10^{-4}$
Case 2c	15.5	$1 \cdot 10^{-5}$	30	$1.2 \cdot 10^{-1}$	$2.45 \cdot 10^{-6}$	-0.14	$2.59 \cdot 10^{-4}$

1. Münch, C., Ausoni, P., Braun, O., Farhat, M. & Avellan, F. Fluid–structure coupling for an oscillating hydrofoil. *Journal of Fluids and Structures* **26**, 1018–1033 (2010).

Exercise 2 - Loads on a rectangular wing model.

Consider the simple rectangular and cantilevered wing model as shown on the figure below[2]. The wing of span s and chord c is considered rigid. It however possesses two springs at its root which provide torsion (θ) and flexion (φ) degrees of freedom. The incidence angle is $\alpha = \alpha_r + \theta$, where $\alpha_r = 0$ is its rigid part and θ is its elastic part. The springs are attached at a distance l_{ec} behind the aerodynamic center (itself located at the quarter chord length). The mass distribution of the wing is considered uniform.



The displacement of a point on the wing in the z-direction is given by:

$$z(x, y, t) = y\varphi(t) + (x - x_f)\theta(t)$$

And the quasi-static lift and pitching moment for each elemental strip dy are given by:

$$dL = \frac{1}{2}\rho U^2 c C_{L,\alpha}\theta dy$$

$$dM = \frac{1}{2}\rho U^2 c^2 e C_{L,\alpha}\theta dy$$

where e is the eccentricity between flexural axis and aerodynamic center, $e = x_f/c - 0.25$.

a) Using Lagrange's equation, find the equations of motion of the system without considering aerodynamic forces and express them in a matrix form.
Hint: To compute the kinetic energy, use the mass per unit area of the wing σ (infinitely thin wing assumption).

b) Considering now quasi-static aerodynamic forces, solve the eigenvalue problem to determine the aeroelastic system frequencies and damping ratios at different flight conditions (velocities from 1 to 270 m/s). From those results, determine the flutter speed U_f.
The generalized aerodynamic forces Q_φ and Q_θ associated with the generalized coordinates φ and θ are given by:

$$Q_\varphi = -\int_0^s y dL\,, \qquad Q_\theta = \int_0^s dM$$

The considered flight conditions are summarized in Table 2 below.

Table 2: Flight parameters.

Span s	7.5 m
Chord c	2 m
Flexural axis x_f	$0.48c$
Mass axis	$0.5c$
Mass per unit area σ	100 kg/m^2
Flap stiffness k_φ	$2812500\pi^2$ Nm
Pitch stiffness k_θ	$200960\pi^2$ Nm
Lift slope $\partial C_l/\partial\alpha$	2π
Air density ρ	1.225 kg/m^3
Air velocities U	1.0, 1.1, 1.2, ..., 270 m/s

Hints: Assume a solution of the form $\theta = \bar{\theta}e^{\lambda t}$ and $\varphi = \bar{\varphi}e^{\lambda t}$, where λ is a complex value. The system frequencies ω_j and damping ratios ξ_j can be evaluated from the eigenvalues λ_j as such:

$$\lambda_j = -\xi_j\omega_j \pm i\omega_j\sqrt{1-\xi_j^2}$$

And consequently:

$$f_j = \frac{\omega_j}{2\pi} = \frac{\sqrt{Re(\lambda_j)^2 + Im(\lambda_j)^2}}{2\pi}$$

$$\xi_j = -\frac{Re(\lambda_j)}{\omega_j}$$

c) Compute the motion $\varphi(t)$ and $\theta(t)$ of the system at three different speeds: $U = 0.75U_f, U = U_f$ and $U = 1.1U_f$. Consider an initial disturbance $\theta(t=0) = 2°$ (the other initial conditions are zero).

2. Hancock, G. J., Wright, J. R. & Simpson, A. On the teaching of the principles of wing flexure-torsion flutter. *Aeronaut. j.* **89**, 285–305 (1985)