Exercise – Serie 5 – Static instability

Exercise 1 - Strut-Mounted Airfoil

A **non-symmetric** airfoil section (chord length c and area S) is placed in a subsonic flow. The airfoil is attached with two springs (having the same stiffness k), at its leading and trailing edges, as shown on the figure below. The airfoil is mounted in such a way that results in an angle of attack of α_r when the springs are both undeformed ($\delta_1 = \delta_2 = 0$). Thus, the incidence angle is $\alpha = \alpha_r + \theta$, where θ is the elastic part of incidence angle. The aerodynamic and gravity centers are located respectively at x_{ac} and x_{cg} from the leading edge. We only consider small angles.

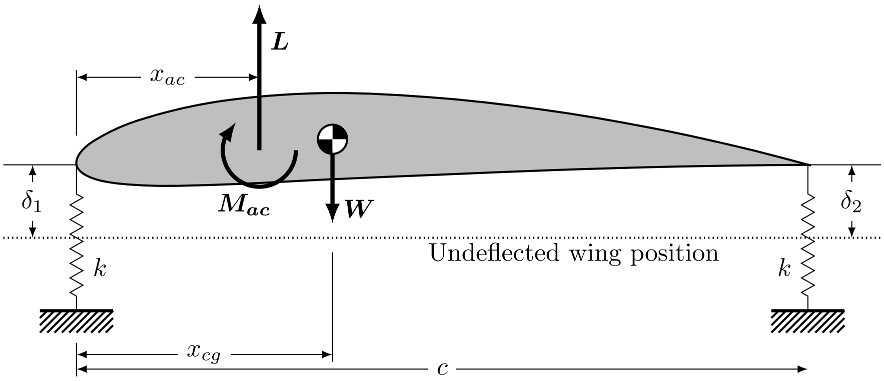


Figure 1 Airfoil mounted on two springs

a) Knowing that the vibration frequency of the first mode is 1 Hz, the upstream velocity is 100 m/s and the chord length is 1 m; Give a definition and compute the reduced velocity. Explain that the problem may be treated within the quasi-static aeroelasticity framework.

b) Express the angle θ as a function of the elongation of the two springs, δ_1 and δ_2.

c) Express the sum of the forces in the vertical direction and the sum of the moments about the trailing edge acting on the wing.

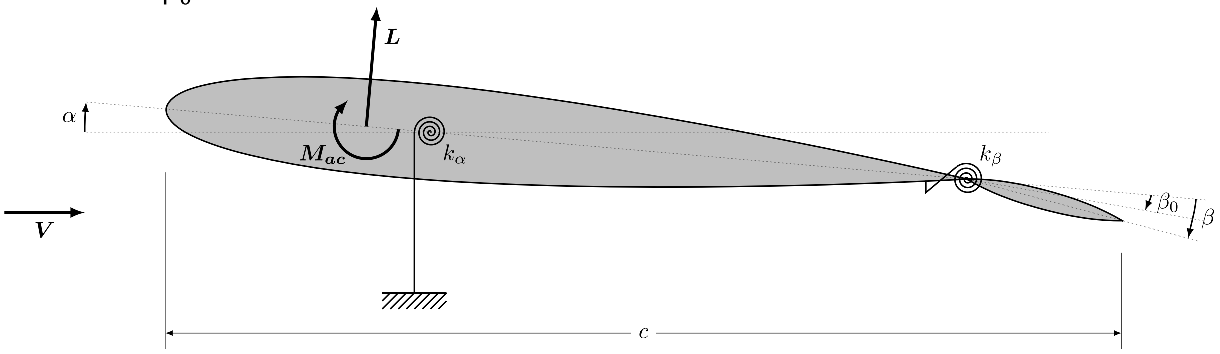
d) Considering $C_{L,\alpha} = \frac{dC_L}{d\alpha}$ and C_{Mac} constant, give the expression of the lift force L and of the moment of aerodynamic forces M_{ac}.

e) Use the equations obtained so far to formulate the equilibrium equation where the only variables are θ and the dynamic pressure, q_∞ (consider the air density as constant). From that equilibrium equations, express θ as a function of q_∞. What kind of instability may develop when the dynamic pressure is increased?

f) Explain why such instability cannot be eliminated in the frame of Thin Airfoil Theory.

Exercise 2 – Aileron Reversal

An airfoil of chord c is mounted in a wind tunnel such that it can only move in pitch against a spring of stiffness k_α. The airfoil is fitted with a flap with a spring restraint of stiffness k_β that creates a restoring moment on the hinge proportional to the incremental deflection from an initial value β_0.



Assume known dynamic pressure q and wing area S. Assume known (non-zero) values of the following aerodynamic derivatives:

$C_{L,\alpha}\left(=\frac{\partial C_L}{\partial \alpha}\right)$, $C_{L,\beta}$, $C_{M_{ac},\beta}$, $C_{H,\alpha}$, $C_{H,\beta}$ Note that $C_{H,\beta}$ is typically negative.

These coefficients are defined as

$$C_L = \frac{L}{qS} \qquad C_{M_{ac}} = \frac{M_{ac}}{qSc} \qquad C_H = \frac{H}{qSc}$$

where L and M_{ac} are the total lift and moment at the aerodynamic center, respectively, and H is the aerodynamic moment of the flap around the hinge. The distance between the aerodynamic center (the point where the lift and moment are applied) and the elastic axis (the point where the torsional spring is attached) is e, with e positive if the aerodynamic axis is ahead of the elastic axis. For simplicity, we ignore the effects of the airfoil weight and assume small angles.

We can approximate the changes in lift and moment coefficients with linear aerodynamics as:

$$C_L = C_{L,\alpha}\alpha + C_{L,\beta}\beta, \qquad C_{M_{ac}} = C_{M_{ac},\beta}\beta, \qquad C_H = C_{H,\alpha}\alpha + C_{H,\beta}\beta$$

a) Show that the static equilibrium equation, in matrix form, will be written as

$$\begin{bmatrix} qSeC_{L,\alpha} - k_\alpha & qSeC_{L,\beta} + qScC_{M_{ac},\beta} \\ qScC_{H,\alpha} & qScC_{H,\beta} - k_\beta \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ -k_\beta \beta_0 \end{bmatrix}$$

b) From the first equation find the control reversal speed. What happens when $k_\beta \to \infty$.

c) Solve the system of question a) to express α and β (Hint: use Cramer's rule).

d) Compute the divergence speed.

e) For $k_\beta \to \infty$, how is the control reversal speed related to the divergence speed? Comment on the results.

Exercise 3 – (REMINDER) Thin airfoil theory, force and moment

The Thin Airfoil Theory (TAT) provides an analytical estimation of the moment and the lift force acting on an airfoil. It is based on the premise that a thin airfoil (thickness $< 12\,\%$ of the chord length) in a uniform inviscid flow V_∞ can be replaced by a vortex sheet, $\gamma(x)$, along its camber line (see figure 1). The disturbance to the uniform flow due to the presence of the airfoil is then modeled as the disturbance caused by the vortex sheet located along the camber line of the airfoil. The strength of the vortex sheet is determined by the fact that this camber line must also be a streamline and therefore satisfying flow tangency on the camber line. (N.B. the theory assumes 2D section of airfoils of infinite length with small angles of attack).

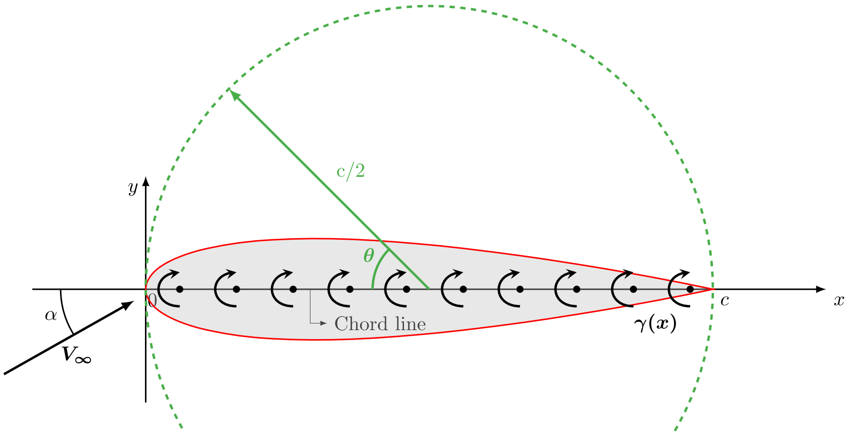


Figure 2 Thin airfoil theory, chord line replaced by a vortex sheet

Consider a thin symmetric airfoil (in that case, the camber line and the chord line are identical). The lift force is obtained using the Kutta-Jukowski equation, $L = \rho V_\infty \Gamma$,where $\Gamma = \int_0^c \gamma(x)dx$. Recall that for a symmetric airfoil, the solution of the fundamental equation of the TAT gives the following vortex sheet strength:

$$\gamma(x) = 2V_\infty \sin(\alpha)\sqrt{\frac{c-x}{x}}$$

a) Derive an expression for x as a function of the angle θ as defined in the figure.

b) Show that for small angles of attack α, the lift coefficient can be approximated by $C_L = 2\pi\alpha$. *Hint*: use the substitution found in the previous question.

c) Compute the moment about the leading edge, given that: $M_{LE} = -\int_0^c x\, dL$, with $dL = \rho V_\infty \gamma(x)dx$

d) Given the lift force and moment coefficients derived above, give an expression for the aerodynamic moment coefficient $C_{M,x}$ about any point along the chord line. Find the location where the aerodynamic moment is independent of the angle of attack.