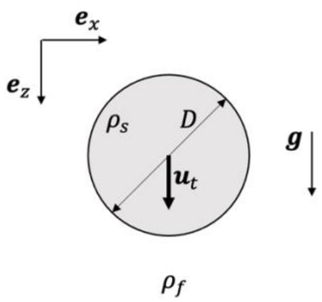
Correction – Serie 4– Added damping

Exercise 1 - Drag of a free-falling sphere in water

Consider a smooth sphere of diameter $D = 2cm$, made of aluminum (density $\rho_s = 2700\frac{kg}{m^3}$) falling vertically into a quiet and unbounded volume of water ($\rho_f = 1000\frac{kg}{m^3}, \mu = 1 \times 10^{-3} Pa.s$). The sphere is assumed to have reached its terminal velocity $\boldsymbol{u_t} = u_t \boldsymbol{e_z}$. The gravitational acceleration is equal to $g = 9.81\ m/s^2$.



Recall that, for a given object, the drag force is given by:

$$\boldsymbol{F}_D = -\frac{1}{2}\rho_f C_D(Re) A \left|\boldsymbol{u}_o - \boldsymbol{u}_f\right|^2 \widehat{\boldsymbol{u}}\,, \qquad Re = \frac{\rho_f \left|\boldsymbol{u}_o - \boldsymbol{u}_f\right| D_e}{\mu}$$

$\boldsymbol{u}_o$ is the velocity of the object and $\boldsymbol{u}_f$ is the far-field fluid velocity.

$\widehat{\boldsymbol{u}} = \frac{(\boldsymbol{u}_o - \boldsymbol{u}_f)}{|\boldsymbol{u}_o - \boldsymbol{u}_f|}$ is a dimensionless unit vector pointing in the direction of $\boldsymbol{u}_o - \boldsymbol{u}_f$.

A is the reference area of the object and D_e is the equivalent diameter of the object.

For a sphere, the drag coefficient C_D in different flow regimes is:

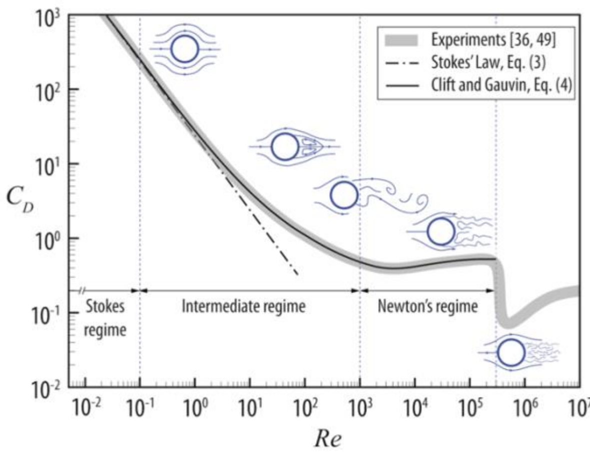


Figure 1: Dependency of C_D on Re for sphere. Streamlines around sphere at various Re are also shown in the plot. From Bagheri et al. 2018.

$$C_D(Re) = \begin{cases} \dfrac{24}{Re} & Re < 0.1 \text{ (Stokes regime, laminar)} \\ f(Re) & 0.1 < Re < 1000 \text{ (Intermediate regime)} \\ \sim 0.45 & 1000 < Re < 3.5 \times 10^5 \text{ (Newton's regime, turbulent)} \end{cases}$$

a) Why should the added mass not be taken into account in the previous hypothesis?

b) Give the expression of the drag force in the Stokes and Newton's regimes.

c) What is the expression of the terminal velocity u_t of the sphere as a function of C_D? Then, deduce its expression in the Stokes and Newton's regimes.

d) Which flow regime is correct in this particular case?

e) In this flow regime, what phenomenon not considered here can make the flow unsteady?

Exercise 2

Structure loaded by a turbulent air flow

Consider a metallic structure dynamically loaded by a turbulent air flow. The density of the structure is assumed to be much larger than that of air.

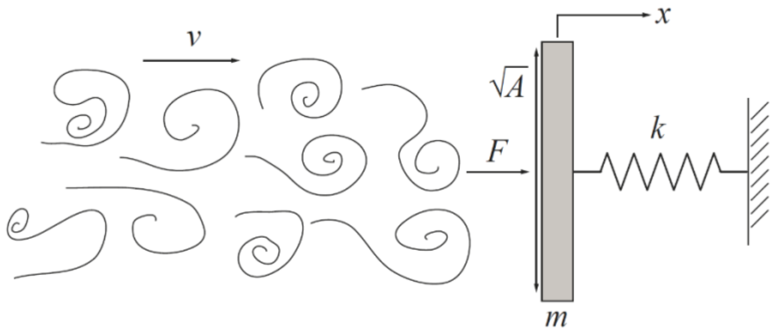


Figure 2: Fluido-dynamic forces acting on a deformable structure.

The known parameters are the drag coefficient C_D, the fluid density ρ, reference area A , the mass m and the stiffness k. v is the fluid velocity and x the displacement of the structure. The objective of this problem to familiarize yourself with deriving linearized equations of motion describing fluctuations about the steady-state values.

a) What type of force is F in the system shown in Figure 2, give its expression.? The drag coefficient given above is considered as a constant and does not depend on relative velocity. Explain the reasoning behind this assumption.

b) Derive the equation of motion for the system shown in figure given the fluid loading F as expressed in point a).

c) The fluid velocity can be expressed as the sum of the mean fluid velocity $\overline{V}$ plus a perturbation v', as $v(t) = \overline{V} + v'(t)$. This leads to the same decomposition for the displacement of the structure: $x = \overline{x} + x'(t)$. Since we intend to use a linear model for the structural response, we consider small perturbations with $v' \ll \overline{V}$, and $x' \ll \bar{x}$. This means that in your linearization, you can neglect second order terms.

d) From your equation of motion, what is the structure's mean displacement $\overline{x}$?

e) What is the equation of motion that governs the fluctuations only?

f) From this linear differential equation, find the natural frequency of the system and compare its value with the no-flow case $F = 0$.