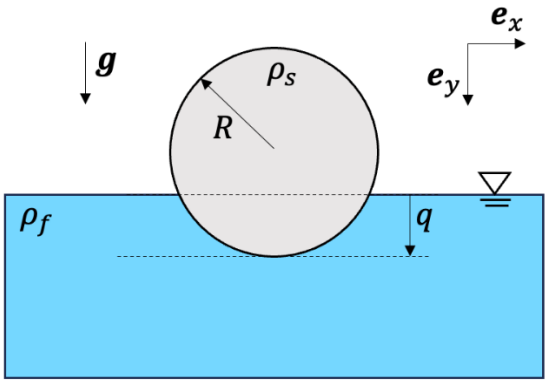
Exercises – Serie 2 – Oscillations of a floating sphere

We want to study the oscillatory motion of a solid sphere of density ρ_s and radius R which is partially immersed under water (density $\rho_f = 1000 kg/m^3$).
We suppose that the sphere only moves in the vertical direction and we define the parameter $q \in [0, 2R]$ as the distance between the free surface and the bottom of the sphere. $\boldsymbol{g} = g\boldsymbol{e}_y$ is the gravitational acceleration constant equal to 9.81 m/s^2. The density of the sphere is considered smaller than the one of water and we neglect viscous and added mass effects.



1) Find the expression of the immersed volume $V_f(q)$.

2) Using the Newton's second law of motion, write the ordinary differential equation (ODE) characterizing the motion of the sphere $\ddot{q} = f(q)$. Why does it differ from the case of a rectangular floating object?

3) Express the algebraic equation at the equilibrium point $0 = f(\bar{q})$.

4) Solve this ODE numerically using *Matlab's* 'ode45' function for various values of R and ρ_s and different initial conditions $[q(0), \dot{q}(0)]$. Can you find a set of parameters for which the solution differs from a simple harmonic motion?

5) Linearize the ODE around the equilibrium point $q = \bar{q} + q' \Longrightarrow \ddot{q}' = f(q', \bar{q})$ (with $q' \ll 1$) and find the expression of the natural frequency of the sphere $\omega_0(\bar{q})$ around this point.

6) For a given set of parameters, compare the oscillation frequency found from the numerical solution with the natural frequency obtained in question 5. You can use *Matlab's 'fsolve'* function to find the equilibrium point $\bar{q}$.

7) At home: Set up a basic experiment using any common floating object (e.g. an ice cube, an apple, a half-filled water bottle, or any floating object you can think of). Find a simple way to measure the oscillation frequency of the object and compare it with the frequencies predicted by your numerical findings.