

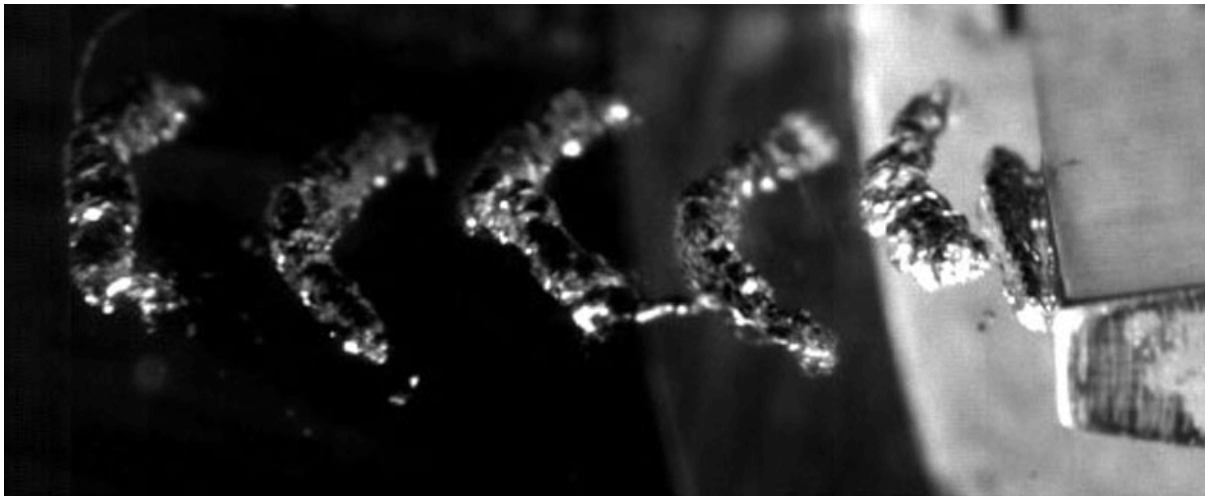
Common Chapter:

Cavitation and Interface Phenomena

Chapter 5.1: Vortex Cavitation - Introduction

Aeroelasticity & Fluid-Structure Interaction

Chapter 7.1: Vortex Induced Vibration - Introduction

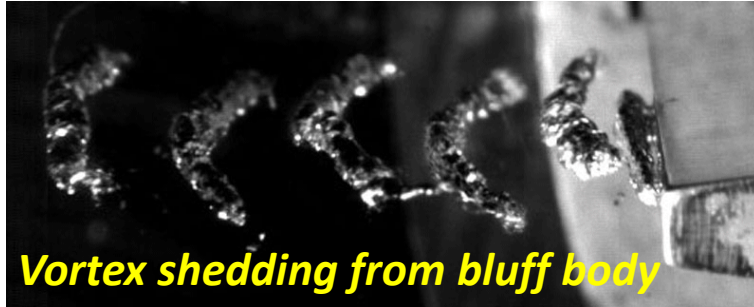


Dr Mohamed FARHAT Assistants: A. Sache, T. Berger

EPFL – Cavitation Research Group, Avenue de Cour 33 bis, 1007 Lausanne

Vortices – Source of cavitation and vibration

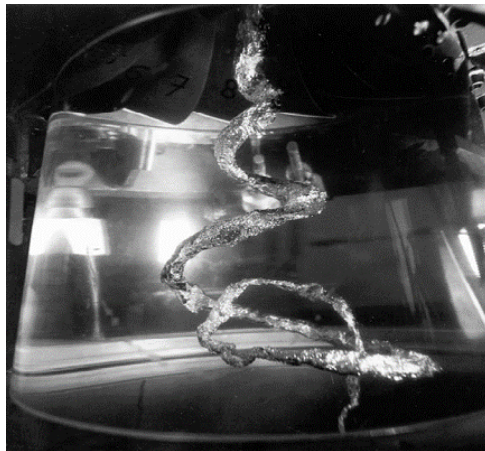
Vortices are highly relevant for both Cavitation and Flow-induced Vibration
They are present in a variety of applications:



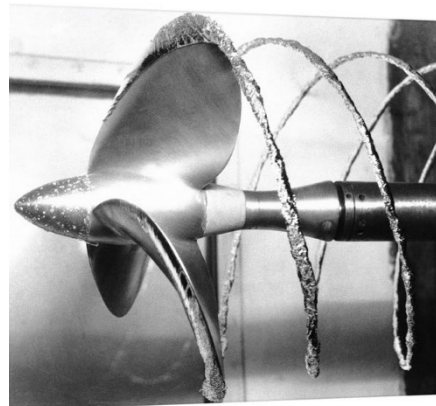
Vortex shedding from bluff body



Hurricanes



Part-load rope in a Francis Turbine



Tip vortex cavitation in a marine propeller



Tip vortex from a plane wing



Vortex-induced vibration of soft coral → improve nutrition

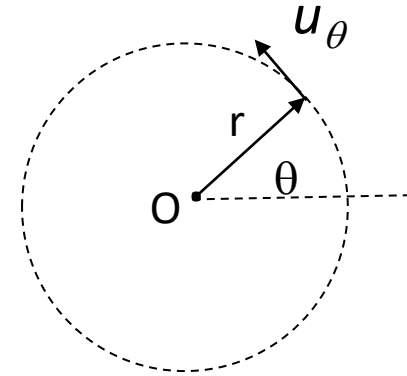
Vortices – Source of cavitation and vibration

What is a vortex ?

- A vortex may be seen as region of a flow in which the fluid rotates around a straight or curved axis line
- Free vortex (potential flow, incompressible):

- Velocity field: $\vec{u} = (u_r, u_\theta, u_z) = \left(0, \frac{\Gamma}{2\pi r}, 0\right)$

$$\Gamma = \oint_C u_\theta(r) dl = \text{cste} \quad \forall C, \text{ a closed contour around } O$$



We may verify that the flow is incompressible ($\vec{\nabla} \cdot \vec{u} = 0$) and irrotational ($\vec{\nabla} \times \vec{u} = 0$)

$$\vec{\nabla} \cdot \vec{u} = \frac{1}{r} \frac{\partial(ru_r)}{\partial r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

$$\vec{\nabla} \times \vec{u} = \left(\frac{1}{r} \frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}, \frac{\partial u_r}{\partial r} - \frac{\partial u_z}{\partial r}, \frac{1}{r} \left(\frac{\partial(ru_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta} \right) \right) = (0, 0, 0) \quad \forall r > 0$$

Vortices – Source of cavitation and vibration

- **Free vortex (potential flow):**

- **Pressure field :**

- **Navier-Stokes in cylindrical coordinates (radial equilibrium):**

$$\frac{\partial p}{\partial r} = \rho \frac{u_{\theta}^2}{r} = \rho \frac{\Gamma^2}{4\pi^2 r^3}$$

$$\text{Integration} \rightarrow p(r) = p_{\infty} - \frac{\rho \Gamma^2}{8\pi^2 r^2}$$

- **Alternate method - Bernoulli equation:**

Since the flow is **irrotational** and steady, Bernoulli equation reads:

$$\forall r > 0, \quad p(r) + \rho \frac{u_{\theta}^2}{2} = \text{constant}$$

$$\Rightarrow \forall r > 0, \quad p(r) = p_{\infty} - \rho \frac{u_{\theta}^2}{2} = p_{\infty} - \frac{\rho \Gamma^2}{8\pi^2 r^2}$$

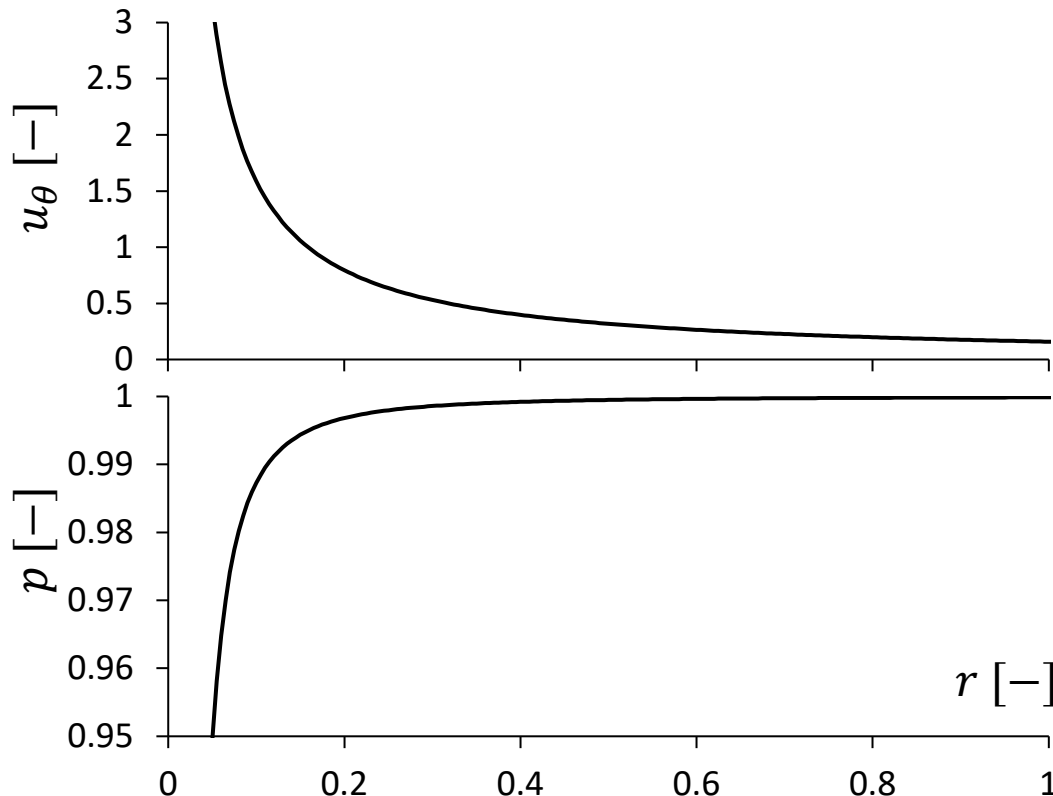
Vortices – Source of cavitation and vibration

- **Free vortex (potential flow):**

- **Pressure field :** $\forall r > 0, \quad p(r) = p_\infty - \rho \frac{u_\theta^2}{2} = p_\infty - \frac{\rho \Gamma^2}{8\pi^2 r^2}$

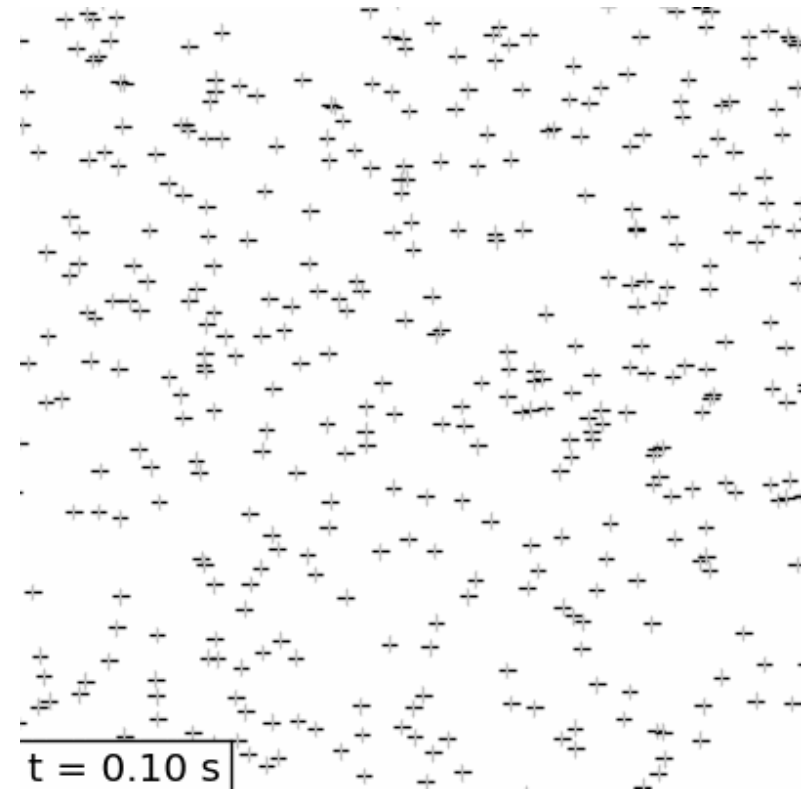
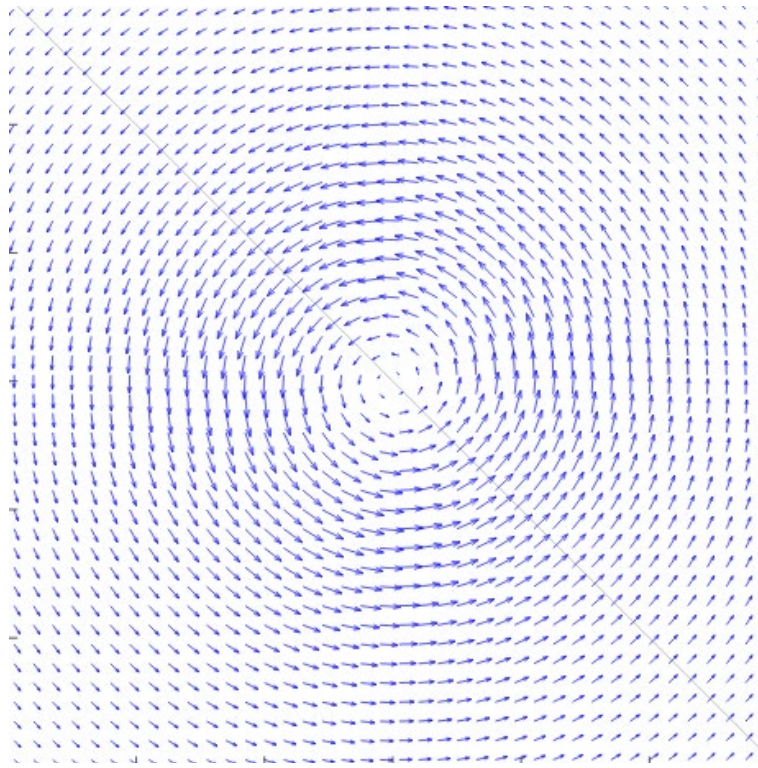
Limitations: $\lim_{r \rightarrow 0} u_\theta(r) = +\infty$ and $\lim_{r \rightarrow 0} p(r) = -\infty$

Not realistic !



Vortices – Source of cavitation and vibration

- **Observation of vortices in real life leads to the following:**
 - **Far from the vortex axis, the flow is irrotational**
 - **As we approach the axis, the flow becomes rotational and its velocity decreases gradually to vanish on the axis.**
 - **The maximum velocity is reached at the border between rotational and irrotational domains**



Vortices – Source of cavitation and vibration

- **Rankine model:**

- Close to the axis, the viscous forces limits the velocity

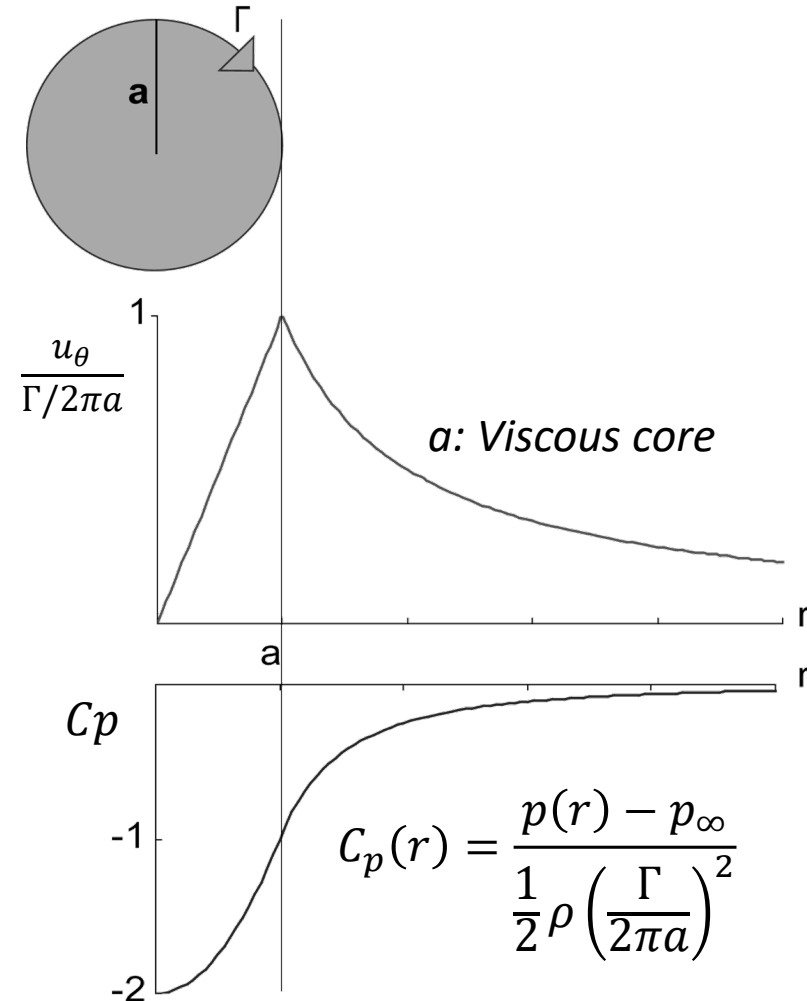
- Velocity field:

$$r \geq a, \quad u_\theta = \frac{\Gamma}{2\pi r}$$

$$r \leq a, \quad u_\theta = \omega r$$

where Γ is the circulation:

$$\Gamma = \int_0^{2\pi} u_\theta r d\theta$$



- Continuity of the velocity at $r = a$: $\omega a = \frac{\Gamma}{2\pi a} \Rightarrow \Gamma = 2\pi\omega a^2$

Vortices – Source of cavitation and vibration

- **Rankine model:**

- **Pressure field (Navier Stokes)**

$$\frac{\partial p}{\partial r} = \rho \frac{u_\theta^2}{r}$$

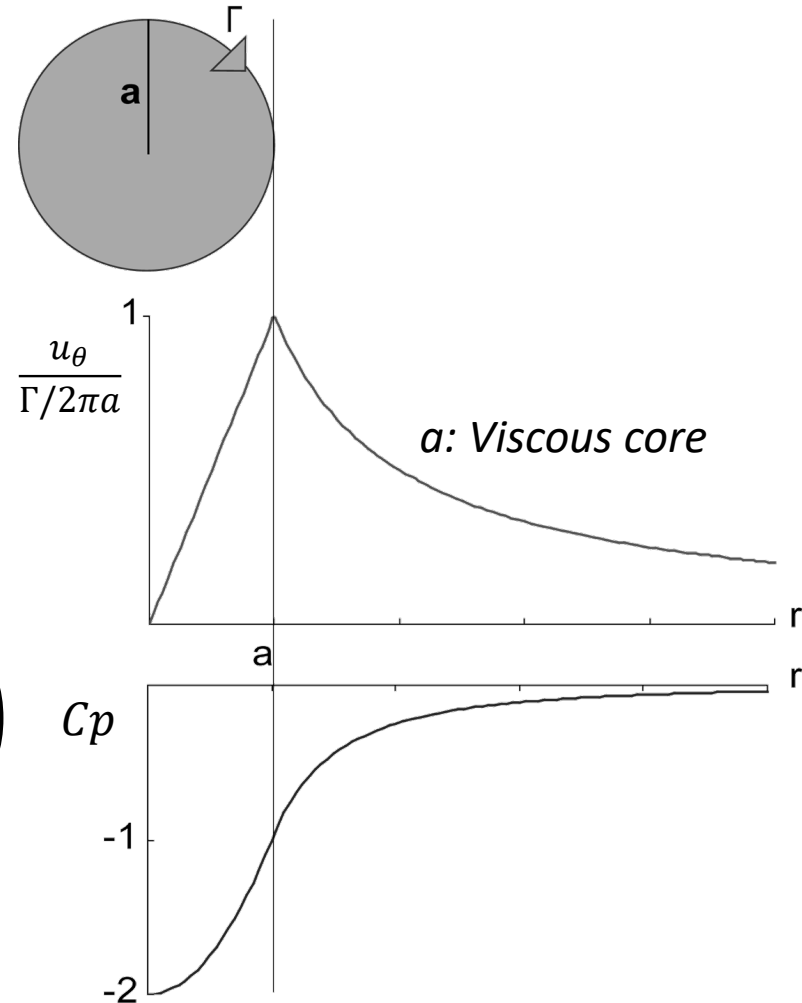
- **Integration:**

$$r \geq a, \quad p(r) = p_\infty - \frac{\rho \Gamma^2}{8\pi^2 r^2}$$

$$r \leq a, \quad p(r) = p_\infty - \frac{\rho \Gamma^2}{8\pi^2 a^2} \left(2 - \frac{r^2}{a^2} \right)$$

$$C_p(r) = \frac{p(r) - p_\infty}{\frac{1}{2} \rho \left(\frac{\Gamma}{2\pi a} \right)^2}$$

$$C_{p,min} = -2$$



- **Limitation: Discontinuity of the velocity derivative at $r = a$**

- **About Bernoulli equation: In the viscous core, the flow is rotational and Bernoulli equation is only valid along a streamline (useless)**

Vortices – Source of cavitation and vibration

More realistic models for vortex flows

- **Lamb-Oseen model:**

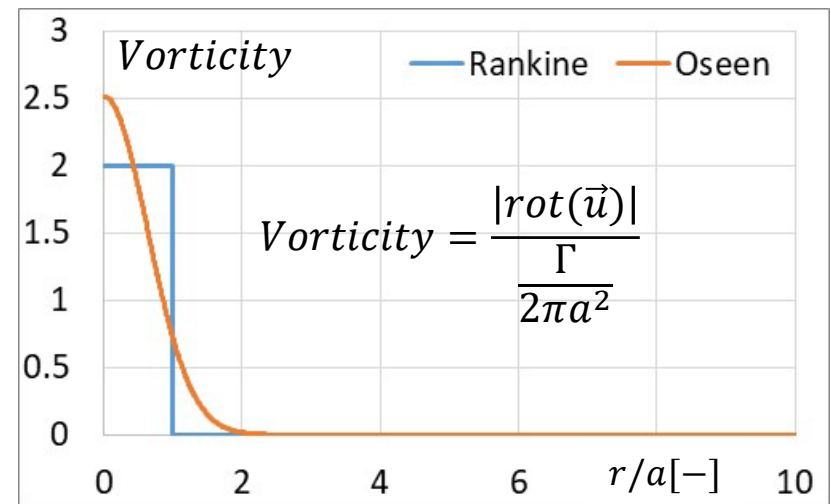
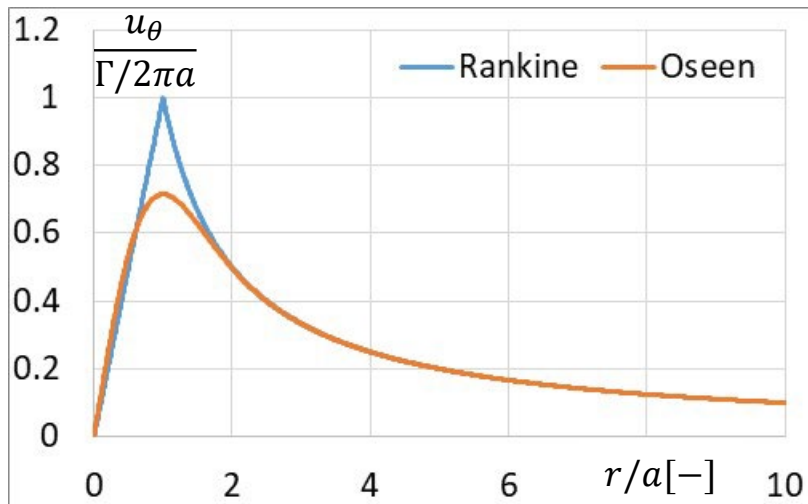
- **Velocity field:**

$$\forall r > 0, \quad u_\theta = \frac{\Gamma}{2\pi r} \left(1 - e^{-\lambda r^2/a^2}\right)$$

Where λ is a constant ($\lambda \approx 1.256$)

$$u_{\theta,max} \approx 0.715 \frac{\Gamma}{2\pi a}$$

→ **Smooth transition from rotational to irrotational flow**



Vortices – Source of cavitation and vibration

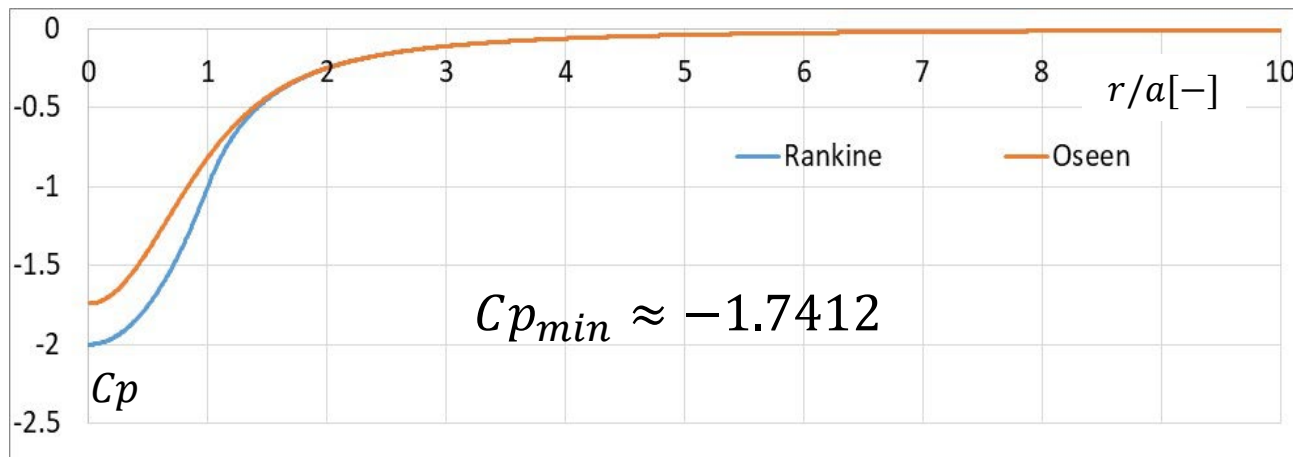
- **Lamb-Oseen model:**

- **Pressure field: integration of** $\frac{\partial p}{\partial r} = \rho \frac{u_\theta^2}{r}$

$$p(r) = p_\infty - \int_r^{+\infty} \rho \frac{u_\theta^2}{r} dr = p_\infty - \rho \left(\frac{\Gamma}{2\pi a} \right)^2 \int_{r/a}^{+\infty} \frac{(1 - e^{-\lambda s^2/a^2})^2}{s^3/a^3} ds$$

$$\dots C_p(r^*) = \frac{p(r^*) - p_\infty}{\frac{1}{2}\rho \left(\frac{\Gamma}{2\pi a} \right)^2} = 2\lambda Ei(-\lambda r^{*2}) - 2\lambda Ei(-2\lambda r^{*2}) + \frac{2e^{-\lambda r^{*2}} - e^{-2\lambda r^{*2}} - 1}{r^{*2}}$$

Where Ei stands for the Exponential Integral, defined by: $Ei(x) = \int_{-\infty}^x \frac{e^s}{s} ds$ and $r^* = r/a$



Vortices – Source of cavitation and vibration

- **Vatistas Model**

- **Velocity field:**

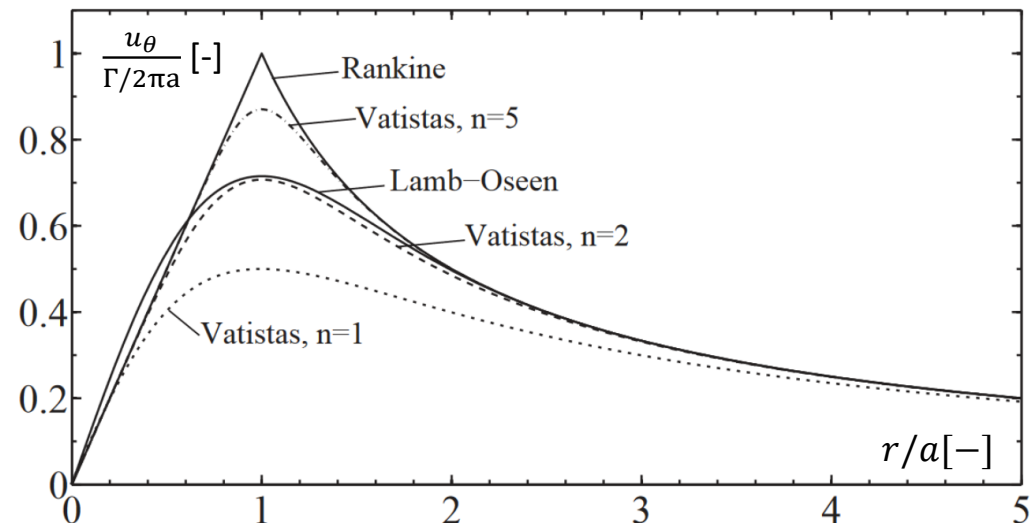
$$\forall r \geq 0, \quad u_\theta = \frac{\Gamma}{2\pi} \frac{r}{(a^{2n} + r^{2n})^{1/n}}$$

Where n is a shape parameter

- **Key parameters of the vortex**

- a : **Viscous core radius**

- Γ : **Circulation**



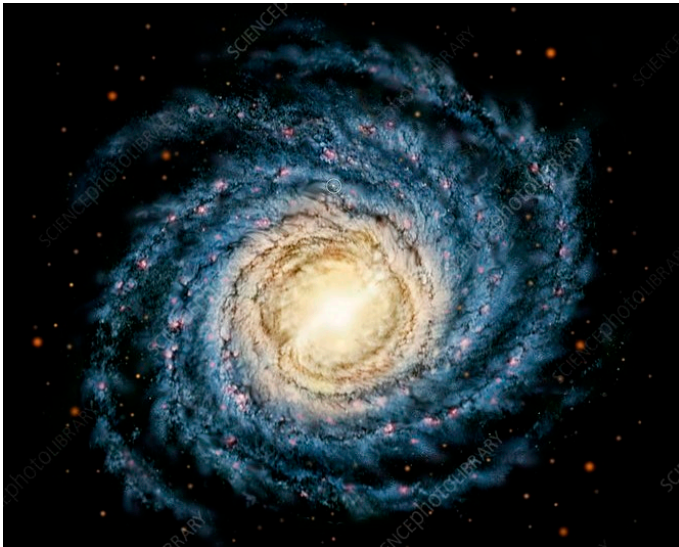
- **Other models are available (e.g. Batchelor, VM2, ...)**

Vortices – Source of cavitation and vibration

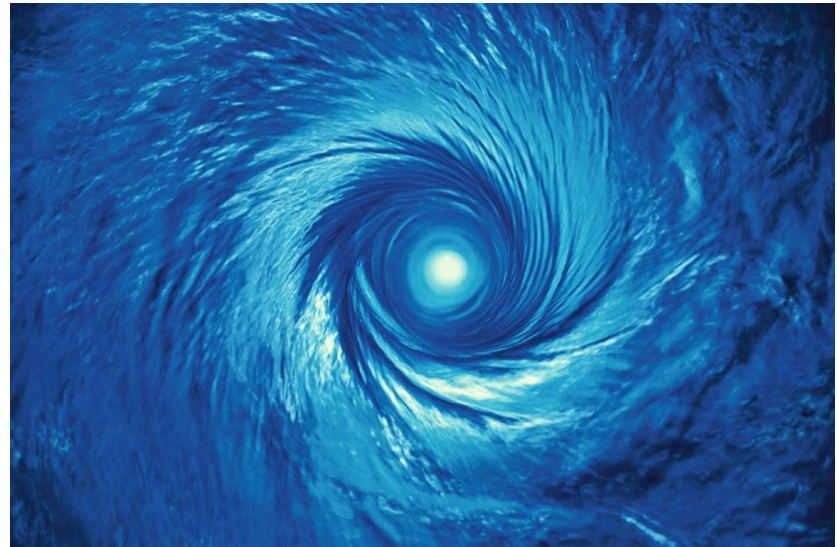
Similarity with astrophysics

- ***The motion of stars in a galaxy looks like a vortex***
 - ***It is possible to fit a vortex model to describe the motion of stars ?***

Milky way



Sink (Lavabo)



Vortices – Source of cavitation and vibration

Example: Tip Vortex Cavitation

Tip vortex cavitation (TVC) on an elliptic hydrofoil

Upstream velocity (U_∞): 13.5 m/s, Incidence: 10°

$\sigma = 2$

$\sigma = 1.5$

$\sigma = 1.3$

$\sigma = 1$

$\sigma = 0.8$



Where σ is the cavitation number, defined as:

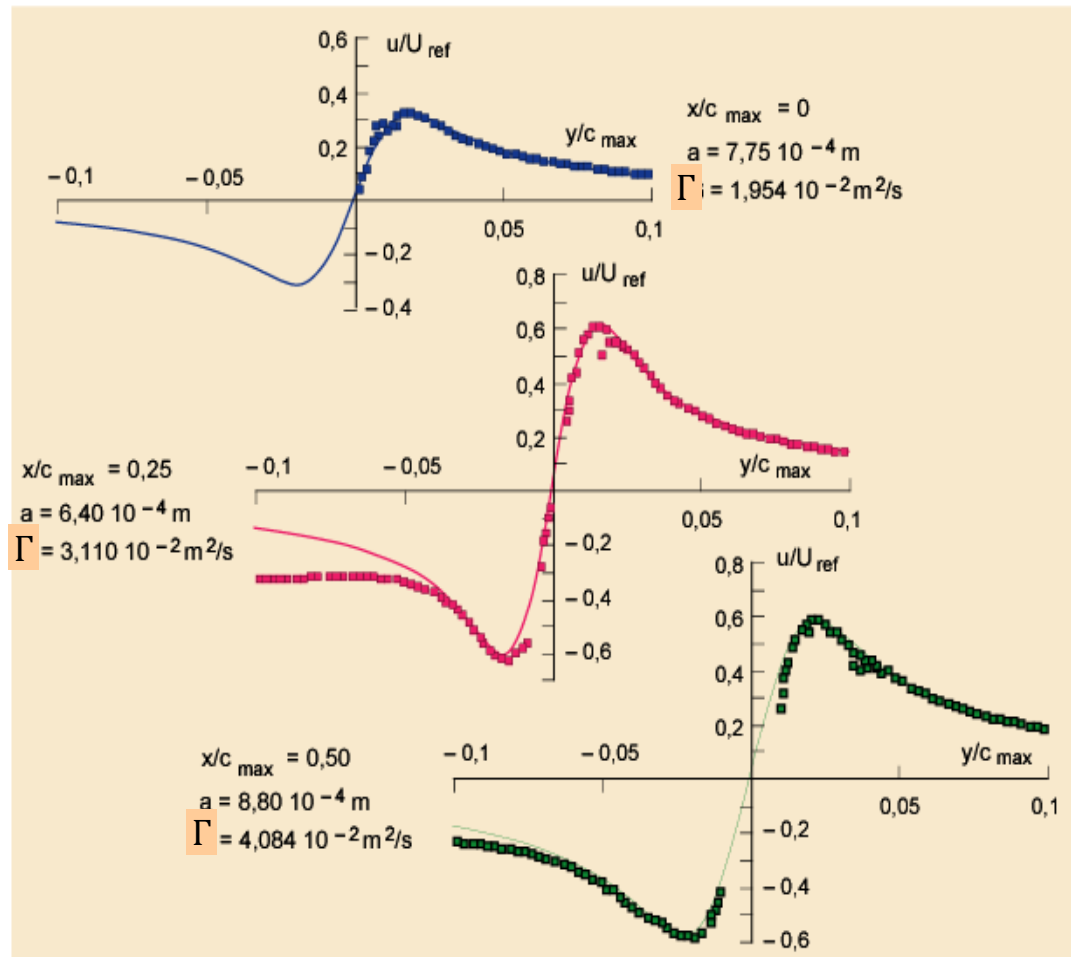
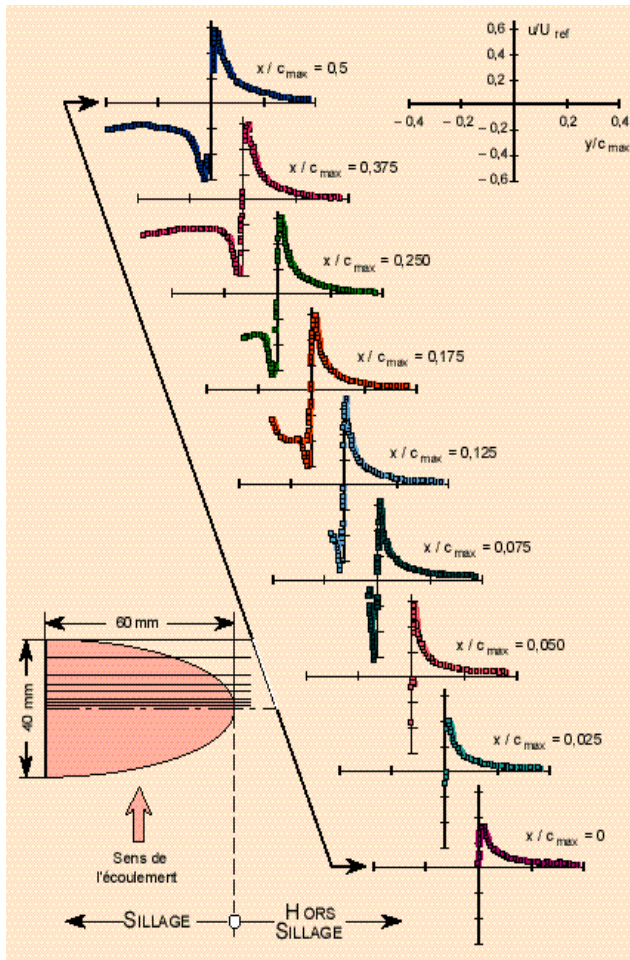
With p_v is the vapor pressure

$$\sigma = \frac{p_\infty - p_v}{\frac{1}{2} \rho U_\infty^2}$$

Vortices – Source of cavitation and vibration

Example: Tip Vortex Cavitation

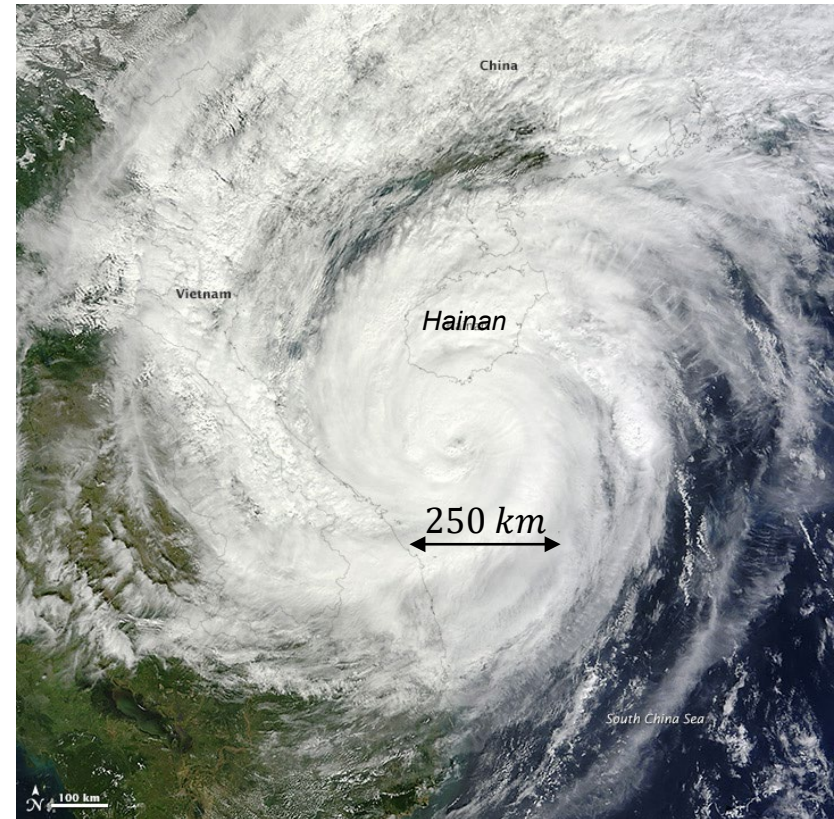
Velocity profiles downstream an elliptic foil, measured with LDV
Lamb-Oseen model is used to fit the experimental data



Vortices – Source of cavitation and vibration

Example: Super Typhoon Haiyan, Philippines, November 2013

- Maximum wind speed ~ 360 km/h
 - Pressure in the vortex axis ?
- Assumptions:
 - Air density $\rho = 1$ kg/m³
 - Atm. pressure = 1000 hPa
 - 2D flow
- Rankine model:



$$p(r = 0) = p_{\infty} - \frac{\rho \Gamma^2}{4\pi^2 a^2} = p_{\infty} - \rho u_{\theta, \max}^2 \approx 900 \text{ hPa}$$

Vortices – Source of cavitation and vibration

Example: Super Typhoon Haiyan, Philippines, November 2013

Le Nouvelliste

VALAIS SUISSE SPORTS ECONOMIE MONDE SORTIR LIFESTYLE DOSSIERS **PRFMIIM**

DÉFINITION Typhons, ouragans et cyclones sont autant de déclinaisons pour désigner le même phénomène météorologique: une dépression tropicale capable de dégager une puissance équivalente à dix bombes nucléaires.

Typhons, ouragans et cyclones sont un même phénomène **Météo** désignant la plus violente manifestation des dépressions tropicales. Elles sont capables de dégager une puissance équivalente à dix fois la bombe d'Hiroshima.

Le typhon qui s'est abattu sur les Philippines, où les autorités évoquent plus de 10'000 morts et 2000 disparus, est le mot asiatique pour un phénomène qualifié d'ouragan dans les Caraïbes et de cyclone dans les régions tropicales.

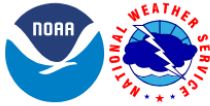
Echelle de Saffir-Simpson

Les cyclones, terme générique pour ces phénomènes tourbillonnaires, sont mesurés par l'échelle de Saffir-Simpson en cinq catégories selon la force des vents maximum et l'ampleur des dégâts potentiels.

Le super typhon Haiyan est un typhon de catégorie 5, la plus élevée, avec des vents maximum estimés à 315 km/h et des rafales atteignant 380 km/h selon l'agence météo japonaise et le centre américain inter-armes de prévision des cyclones tropicaux.

Au plus fort de son intensité, la pression en son centre était comprise, selon les estimations, entre 870 et 895 hectoPascal. Il pourrait s'avérer le plus violent mesuré à ce jour. Le précédent record est détenu par Tip, dans l'océan Pacifique, avec 870 hPa mesurés le 12 octobre 1979 et des vents maximum alors estimés à 305 km/h, selon le site de Météo France.

Vortices – Source of cavitation and vibration

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Hurricane MELISSA



[US Dept of Commerce](#)
[National Oceanic and Atmospheric Administration](#)
National Hurricane Center
11691 SW 17th Street
Miami, FL, 33165
nhcwebmaster@noaa.gov

ZCZC MIATCPAT3 ALL
TTAA00 KNHC DDHMM

BULLETIN

Hurricane Melissa Intermediate Advisory Number 28A
NWS National Hurricane Center Miami FL AL132025
800 AM EDT Tue Oct 28 2025

...EYE OF EXTREMELY DANGEROUS CATEGORY 5 MELISSA APPROACHING
WESTERN JAMAICA...
...CATASTROPHIC WINDS, FLASH FLOODING, AND STORM SURGE EXPECTED ON
THE ISLAND TODAY...

SUMMARY OF 800 AM EDT...1200 UTC...INFORMATION

LOCATION...17.5N 78.1W
ABOUT 55 MI...90 KM SSE OF NEGRIL JAMAICA
ABOUT 265 MI...430 KM SW OF GUANTANAMO CUBA
MAXIMUM SUSTAINED WINDS...175 MPH...280 KM/H
PRESENT MOVEMENT...NNE OR 20 DEGREES AT 7 MPH...11 KM/H
MINIMUM CENTRAL PRESSURE...901 MB...26.61 INCHES

WATCHES AND WARNINGS

CHANGES WITH THIS ADVISORY:

None.

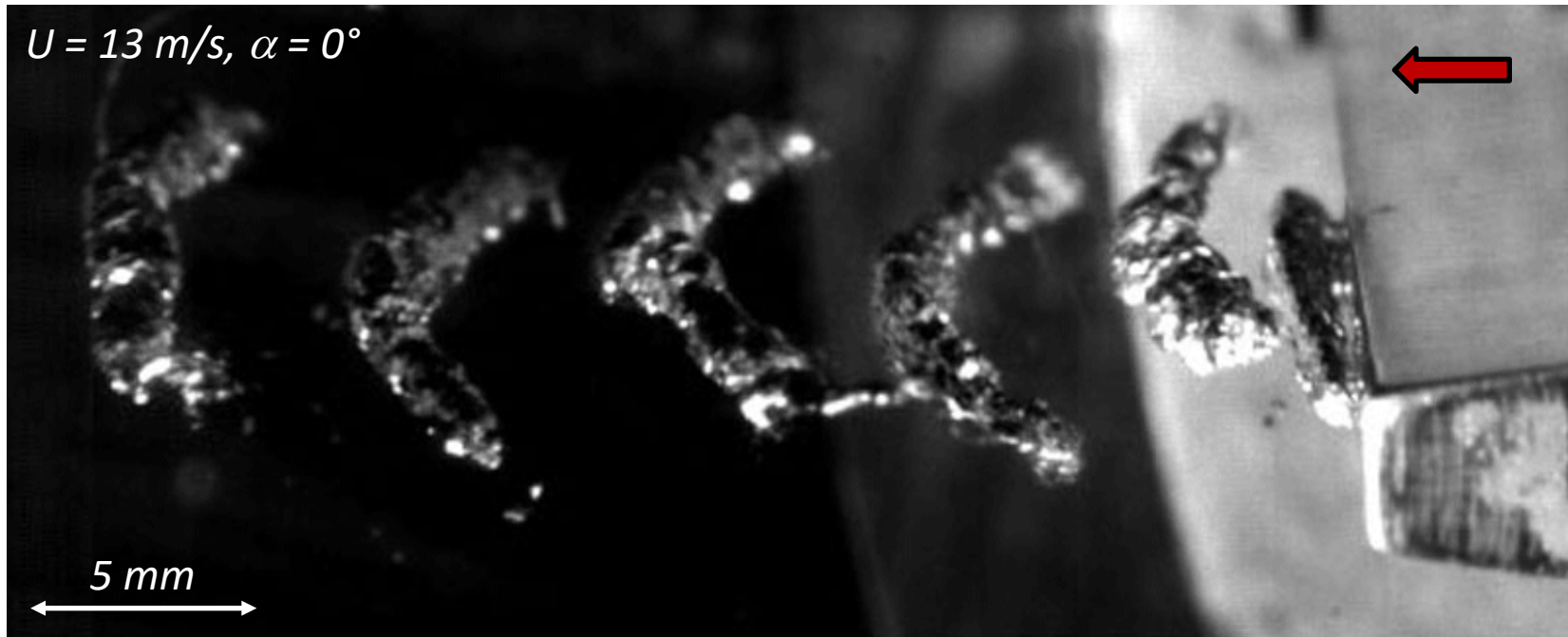
SUMMARY OF WATCHES AND WARNINGS IN EFFECT:

A Hurricane Warning is in effect for...
* Jamaica
* Cuban provinces of Granma, Santiago de Cuba, Guantanamo, and
Holguin

Vortices – Source of cavitation and vibration

Wake dynamics:

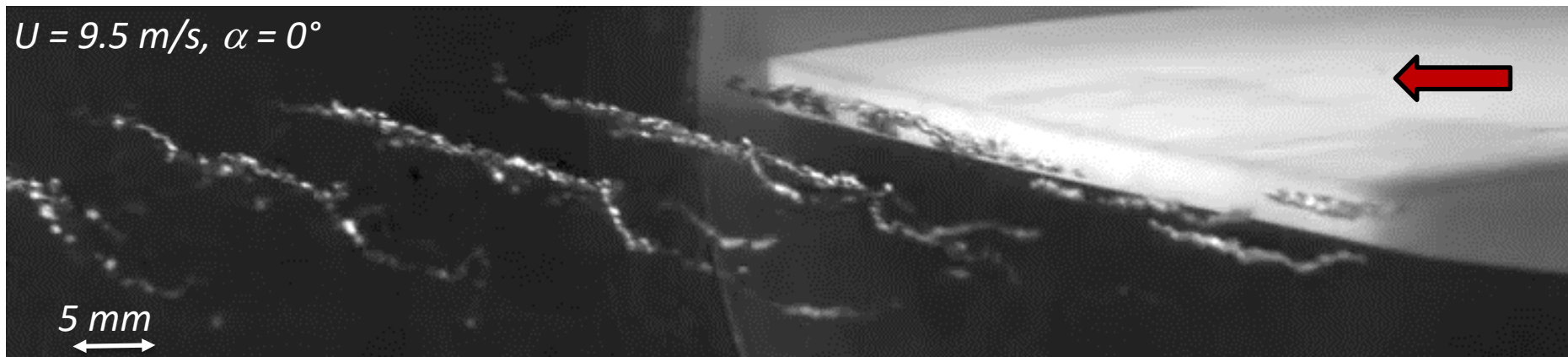
- *Placed in a fluid stream, bodies generate separated flow that extends to their wake with a possible formation of alternate vortices (Kàrman vortices)
→ Fluctuating lift → Vibration (risk of resonance)*
- *Example of Karman vortices in the wake of a **stainless steel** Naca0009 hydrofoil with a blunt trailing edge (Chord length=100 mm, max thickness=1 cm)
Velocity: 13 m/s; 0° incidence angle; resonance (**first torsional mode**)*



Vortices – Source of cavitation and vibration

Wake dynamics:

- *Placed in a fluid stream, bodies generate separated flow that extends to their wake with a possible formation of alternate vortices (Kàrman vortices)
→ Fluctuating lift → Vibration*
- *Example of Karman vortices in the wake of a **POM** Naca0009 hydrofoil with a blunt trailing edge (Chord length=100 mm, max thickness=1 cm)
Velocity: 9.5 m/s; 0° incidence angle; resonance (**second torsional mode**)*



Vortices – Source of cavitation and vibration

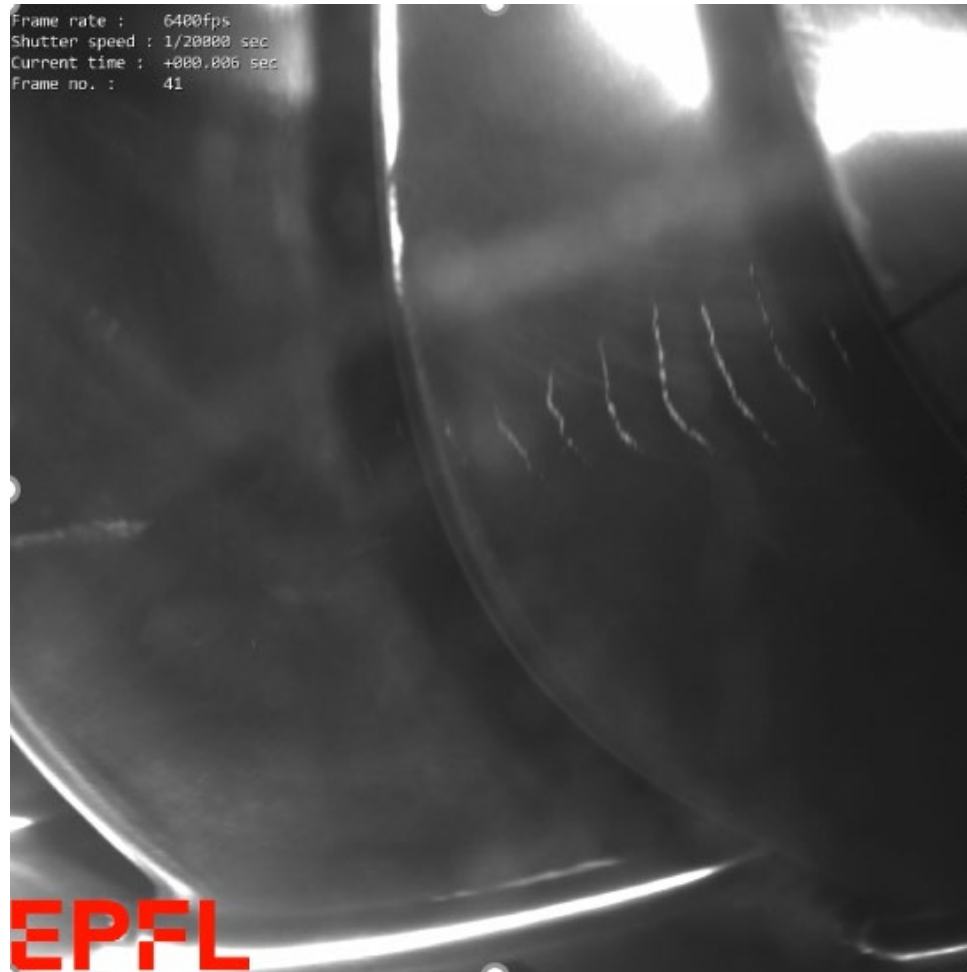
Wake dynamics:

- ***Placed in a fluid stream, bodies generate separated flow that extends to their wake with a possible formation of alternate vortices (Kàrman vortices)***
 → Fluctuating lift → Vibration
- ***Example: Karman vortices in the wake of a Kaplan turbine blades***
 The vortices are visible because of cavitation occurrence



Vortices – Source of cavitation and vibration

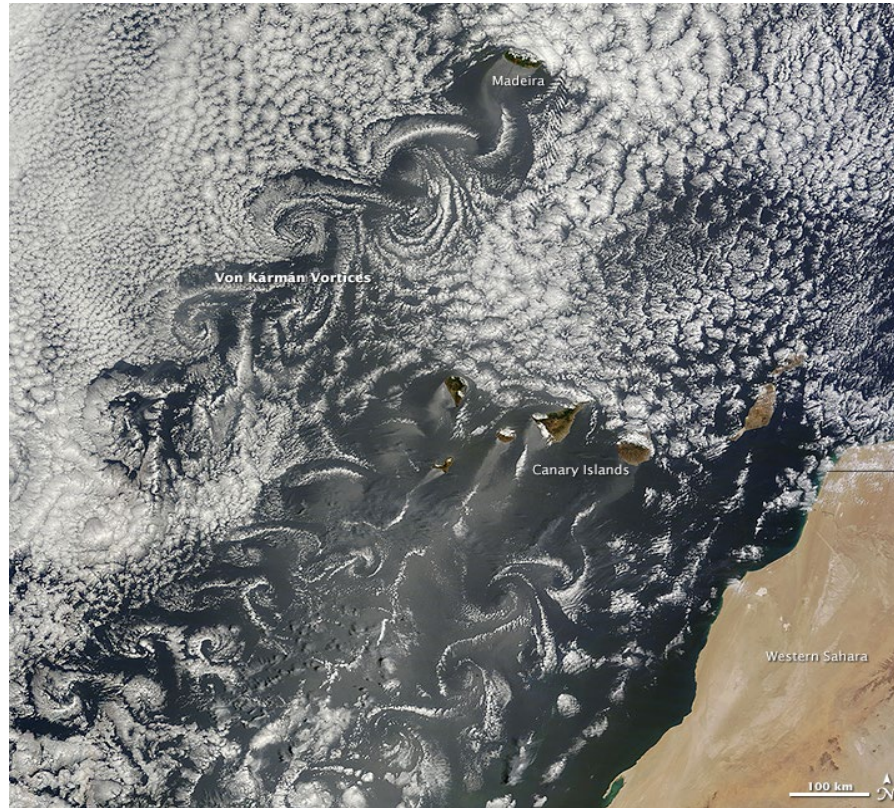
*Karman vortices in the wake of **Francis** turbine blades (reduced scale model)*



Vortices – Source of cavitation and vibration

Wake dynamics:

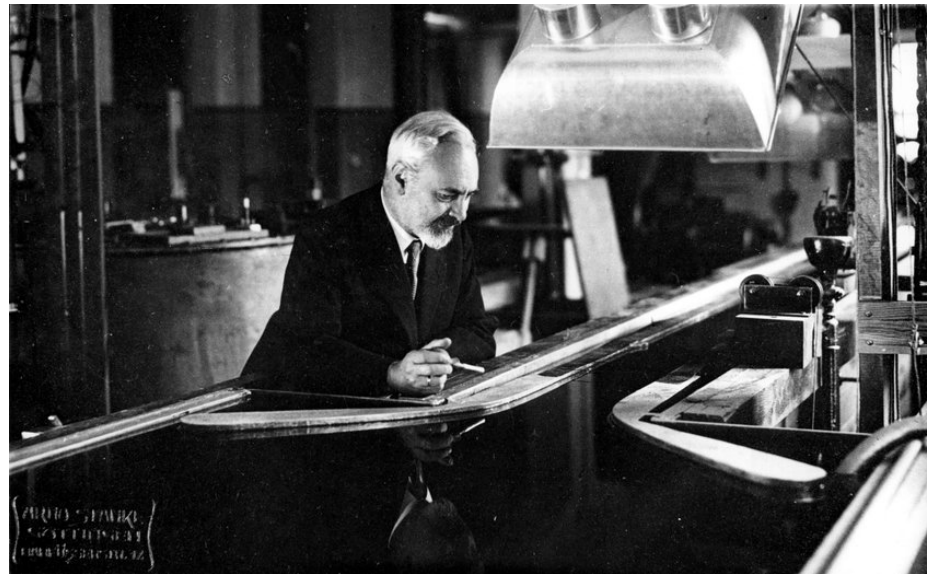
- *Placed in a fluid stream, bodies generate separated flow that extends to their wake with a possible formation of alternate vortices (Kàrman vortices)
→ Fluctuating lift → Vibration*
- *Example: Karman vortices in the wake of Madeira and Canary islands*



Vortices – Source of cavitation and vibration

How It All Started: The Origins of Vortex-Induced Vibrations

- *Göttingen, 1911: Prof. Ludwig Prandtl – the father of modern fluid mechanics*
- *Mission: Exp. validation of his BL theory through a flow around a cylinder!*
 - *Location of flow separation ?*
- *Hiemenz, a devoted PhD student – perfectionist, hard-working, obedient*
 - *Unexpectedly, the flow was oscillating violently !*
 - *Prandtl: “Obviously your cylinder is not circular !”*
 - *He polishes, adjusts, realigns... again and again...*
 - *But the flow keeps oscillating violently !*



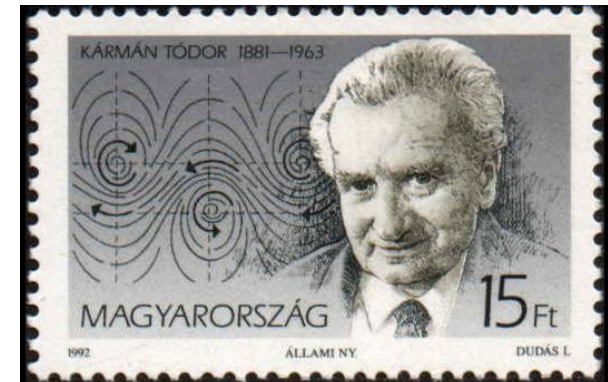
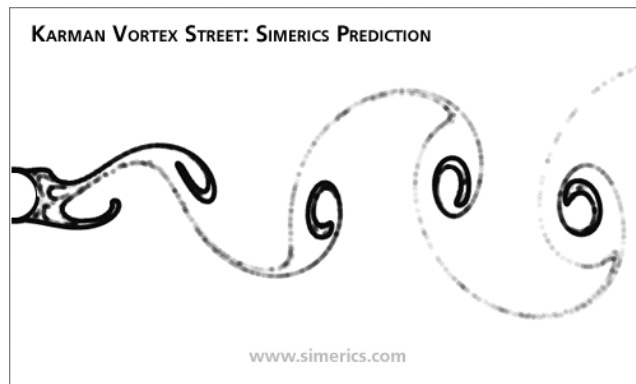
Vortices – Source of cavitation and vibration

How It All Started: The Origins of Vortex-Induced Vibrations

Enter Theodore von Kármán (the witness):
newly hired graduate assistant – brilliant and curious

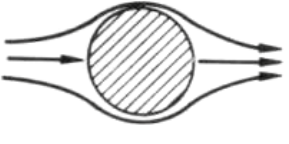
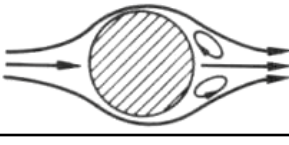

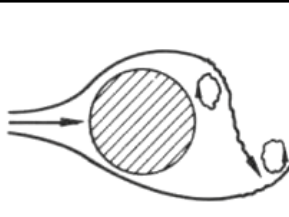
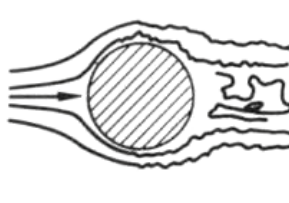

Kármán spent one weekend working alone and came out with the 1st theory,
which demonstrates that:

- The flow oscillates because of alternate & periodic vortices formed in the wake and named after him. The oscillation is intrinsic to the flow, not due to experimental imperfections
- Only a staggered (asymmetric) arrangement of vortices is stable.
- $\frac{B}{A} = \frac{1}{\pi} \cosh^{-1}(\sqrt{2}) \approx 0.281$



Vortices – Source of cavitation and vibration

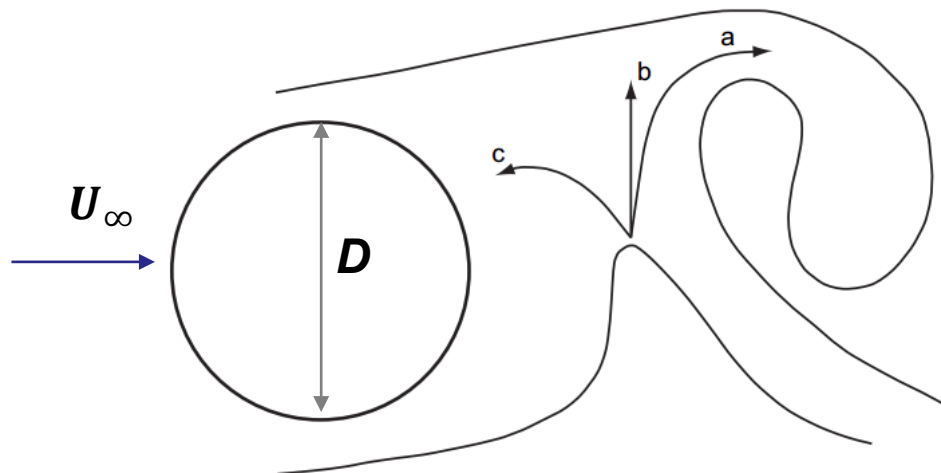
Wake dynamics: The case of a flow over a cylinder

	$R_e < 5$: Laminar flow with 2 stagnation points. The flow moves smoothly around the obstacle without separation.
	$5 \text{ to } 15 < R_e < 40$: Laminar flow with the formation of fixed pair of vortices in the wake
	$40 < R_e < 90$ AND $90 < R_e < 150$: Two regimes in which vortex street is laminar
	$150 < R_e < 300$: Transition range to turbulent shedding $300 < R_e < 3 \cdot 10^5$: Vortex street is fully turbulent
	$3 \cdot 10^5 < R_e < 3.5 \cdot 10^6$: Laminar boundary layer has undergone turbulent transition and wake is narrower and disorganized. This coincides with the so-called drag crisis
	$R_e > 3.5 \cdot 10^6$: Re-establishment of turbulent vortex street

Vortices – Source of cavitation and vibration

Wake dynamics: The case of a flow over a cylinder

- *The mutual interaction between the two separating shear layers leads to the formation of a vortex, which continues to grow, fed by circulation from its connected shear layer, until it is strong enough to draw the opposing shear layer across the wake.*



Vortex-formation model, Gerrard (1966)

- *The alternate shedding of vortices is the result of a complex interaction between 3 shear layers: (i) The boundary layer, (ii) the formation region and (iii) the wake.*

Vortices – Source of cavitation and vibration

Wake dynamics: The case of a flow over a cylinder

- Shedding Frequency f_s ?

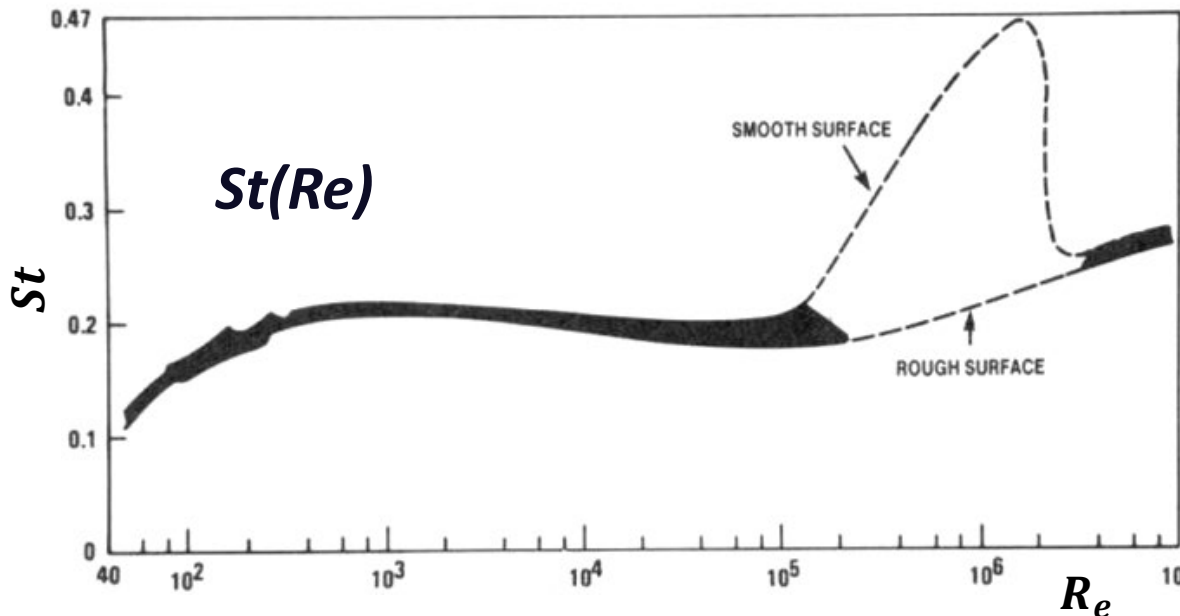
- Dimensional analysis \rightarrow

$$F \left(\frac{f_s D}{U_\infty}, \frac{\rho U_\infty D}{\mu} \right) = 0$$

St (Strouhal Nb)

Re Reynolds Nb

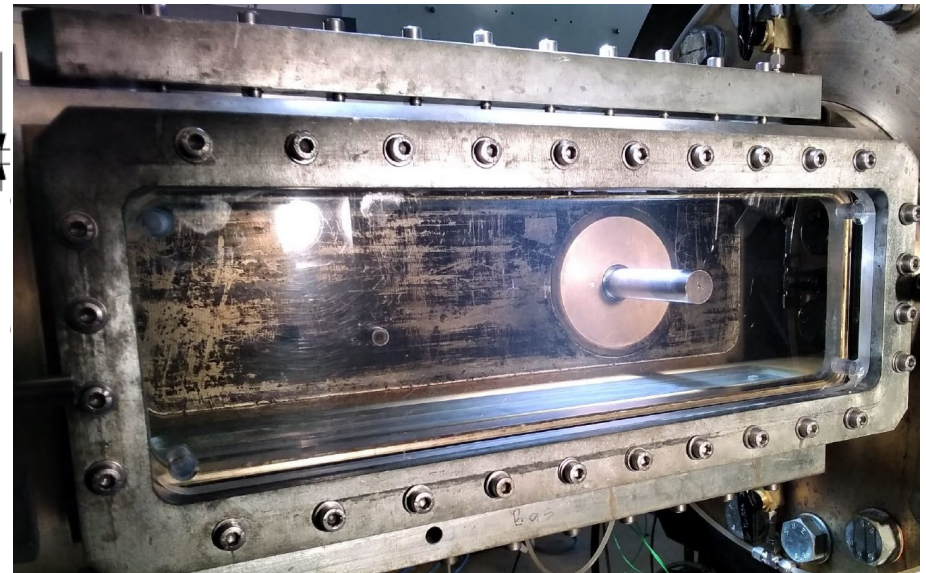
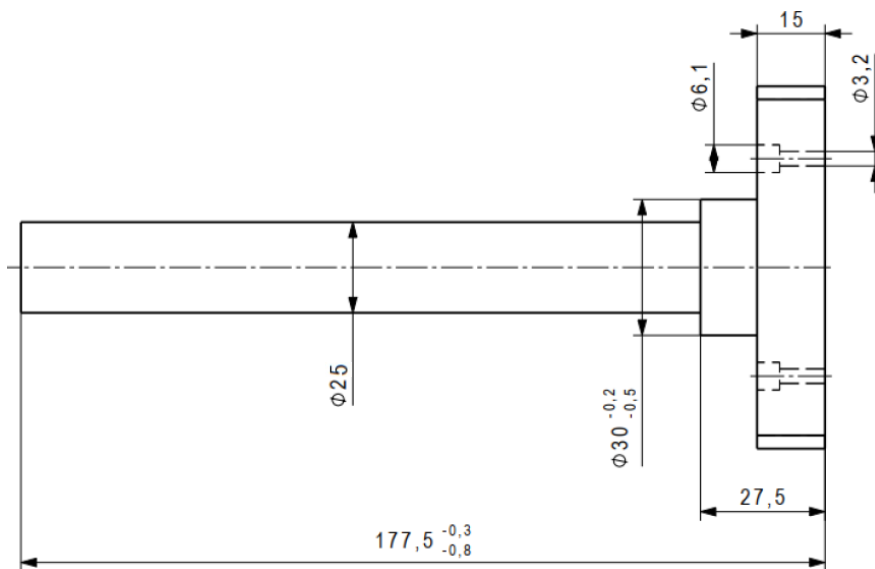
- Empirical relation:



Vortices – Source of cavitation and vibration

Vortex Induced Vibration

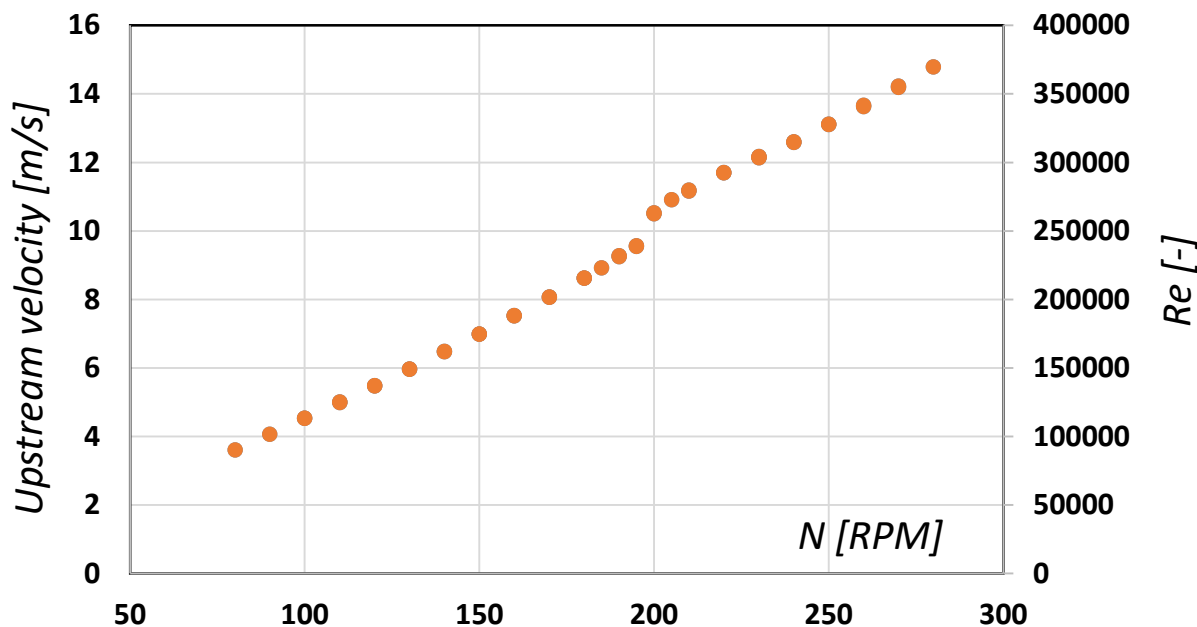
- *Case of a cylinder in EPFL Cavitation Tunnel (semester project 2021)*
 - *Stainless steel, made from one bloc of metal*
 - *Several cylinders made of assembly of 2 parts were destroyed because of too much vibration due to cavitating Karman vortices !!*
 - *25 mm diameter, 150 mm span*
 - *Measurement of vibration, Lift&Drag, High-speed visualization*



Vortices – Source of cavitation and vibration

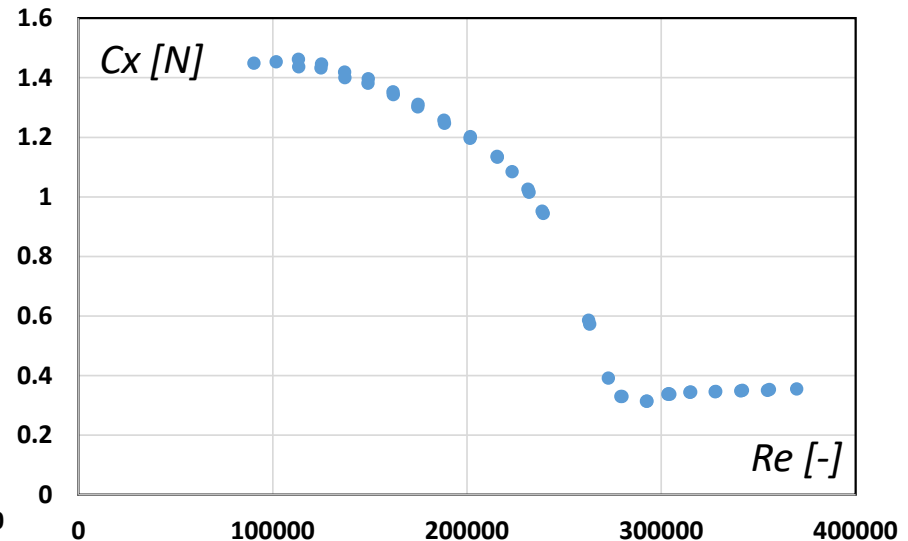
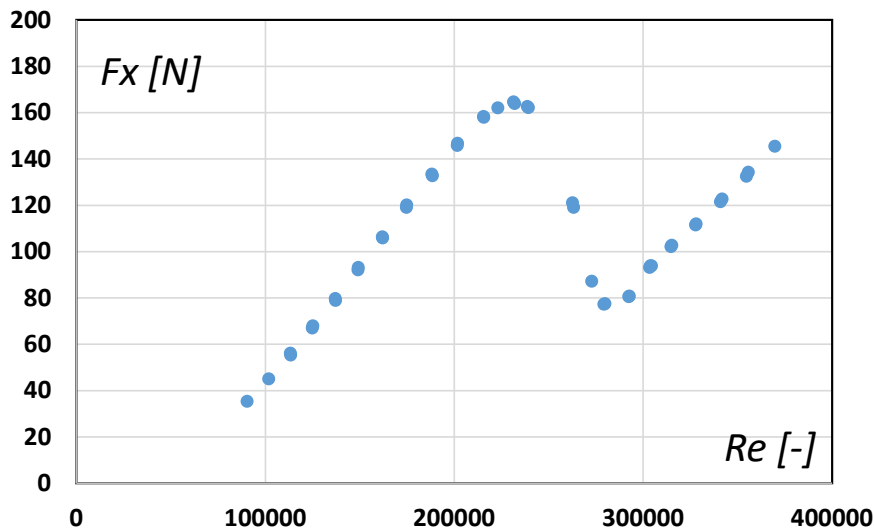
Vortex Induced Vibration

- **Case of a cylinder in EPFL Cavitation Tunnel (semester project 2021)**
 - **Evidence of drag crisis (Cavitation free)**
 - **Upstream velocity vs Rotation speed of the pump**
 - **Constant acceleration of the pump during 3 minutes**
 - **Sudden increase around $Re=250'000$**
 - **Due to a sudden drop of the cylinder drag**



Vortex Induced Vibration

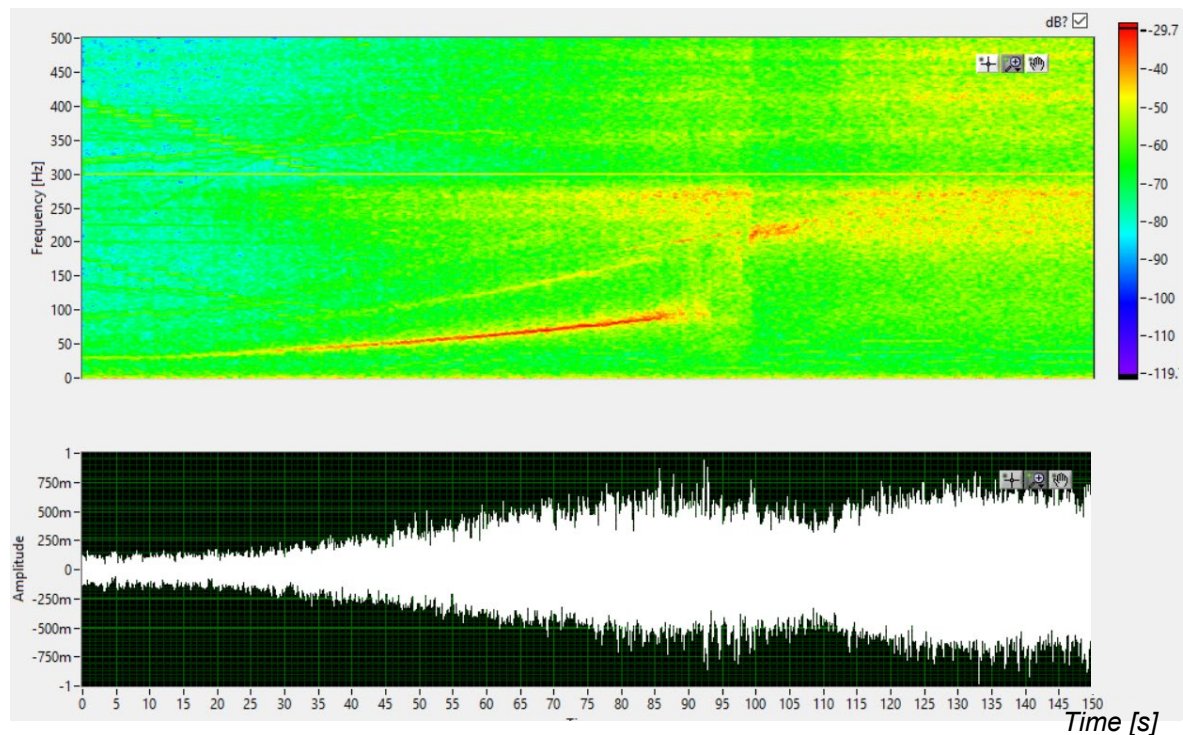
- *Case of a cylinder in EPFL Cavitation Tunnel (semester project 2021)*
 - *Evidence of drag crisis*
 - *Drag force and drag coefficient vs Reynolds number*
 - *Significant decrease of the drag force around $Re=250'000$*
 - *Due to a delay in boundary layer separation*



Vortices – Source of cavitation and vibration

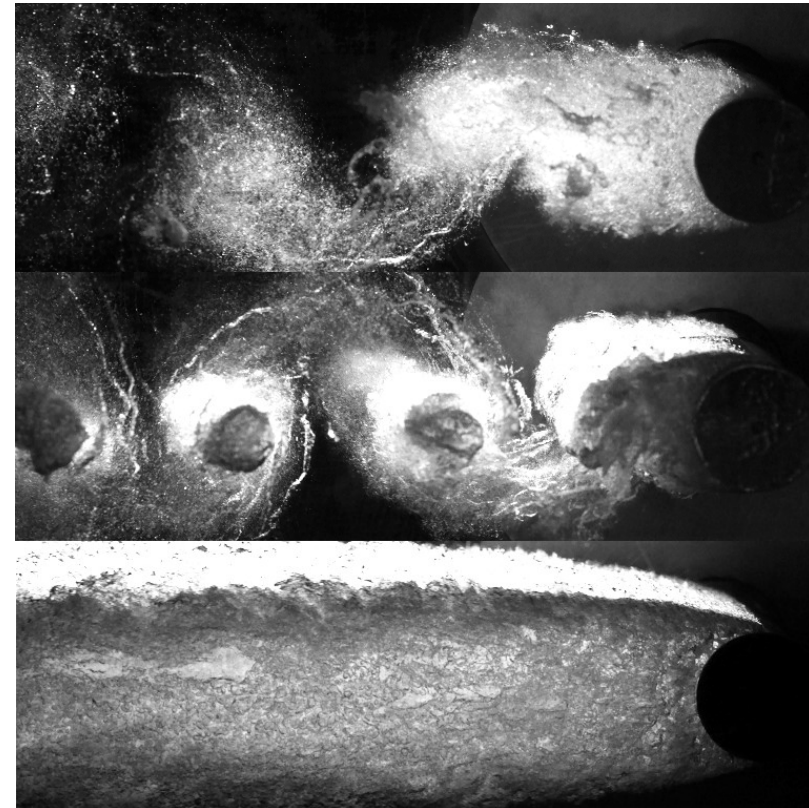
Vortex Induced Vibration

- **Case of a cylinder in EPFL Cavitation Tunnel (semester project 2021)**
 - **Flow induced vibration (upstream velocity 3.6 → 13 m/s)**
 - **No significant increase of vibration at onset or beyond drag crisis**
 - **Strouhal frequency dominant before drag crisis ($St \sim 0.2$)**
 - **Wake less organized beyond drag crisis**



Vortex Induced Vibration

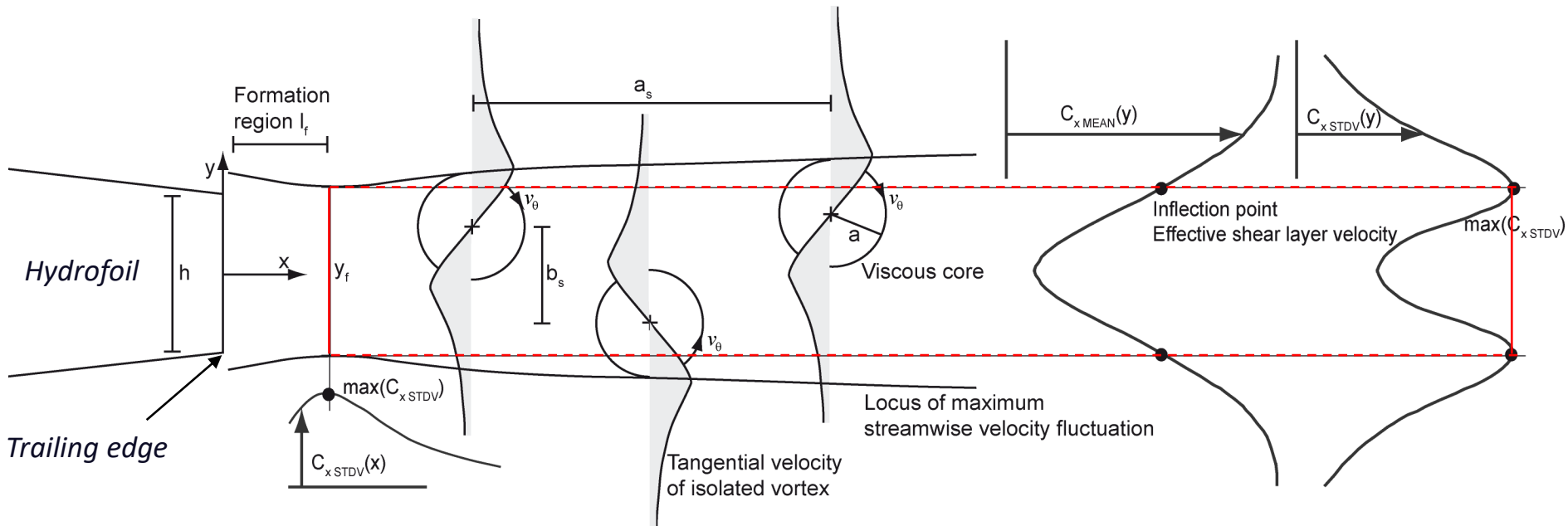
- *Case of a cylinder in EPFL Cavitation Tunnel (semester project 2021)*
 - *Effect of cavitation on fluid-structure vibration:*
 - *Cavitation occurrence in the wake*
 - *the vortices are more coherent*
 - *A tremendous increase of vibration*
 - *Supercavitation:*
 - *No vortex shedding*
 - *Minimum vibration*
 - *Minimum drag*
 - *Further research is underway to understand these peculiar effects*



Vortices – Source of cavitation and vibration

The case of a flow over a hydrofoil

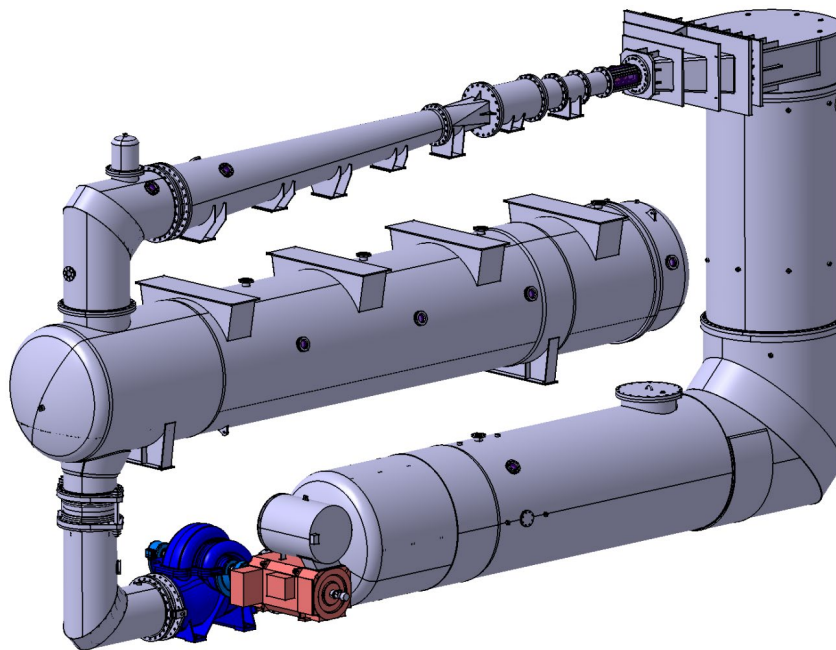
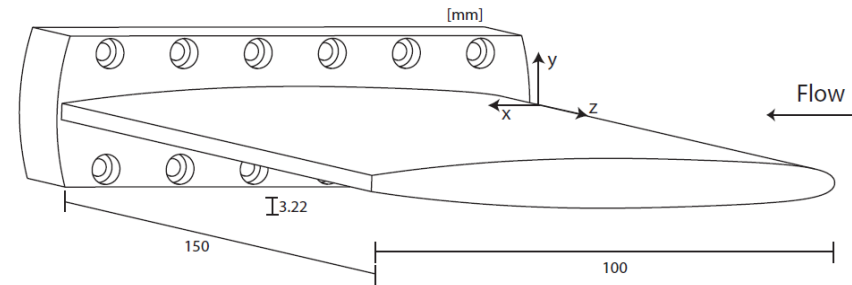
- **Wake structure (similar to the Cylinder case)**
 - **Vortex formation region (l_f):**
 - **bound by X-Position of maximum streamwise velocity fluctuation**
 - **Wake width (y_f):**
 - **Distance between the streamwise velocity fluctuations maxima**
 - **$y_f = 4a$, where a is the viscous core of karman vortices**



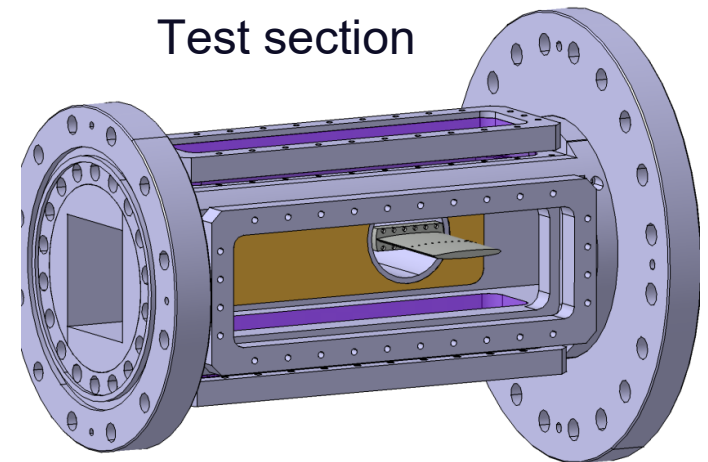
Vortices – Source of cavitation and vibration

The case of a flow over a hydrofoil

- **Experimental investigations**
 - **2D NACA0009 hydrofoil in the EPFL high speed cavitation tunnel**
 - **Blunt trailing edge**
 - **Smooth and rough leading edge**



Test section



Vortices – Source of cavitation and vibration

The case of a flow over a hydrofoil

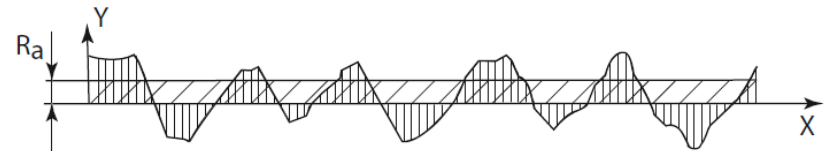
- **Boundary layer tripping through leading edge roughness**

- **Surface roughness – Definition**

- **R_a: Arithmetic Mean Roughness**

R_a is the average value of all absolute distances of the roughness profile from the center line within the measuring length

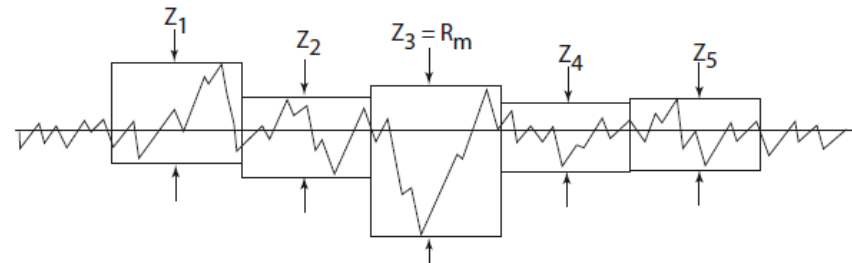
$$R_a = \frac{1}{l_r} \int_{x=0}^{x=l_r} |y| dx$$



- **R_z: Average Maximum Peak to Valley**

R_z is the average maximum peak-to-valley of five consecutive sampling lengths within the measuring length

$$R_z = 1/5(Z_1 + Z_2 + Z_3 + Z_4 + Z_5) = 1.5 \mu\text{m}$$

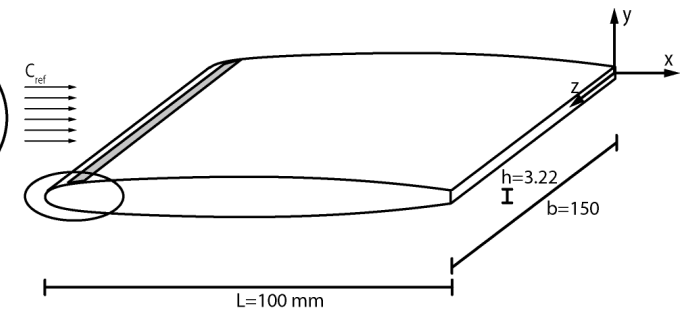
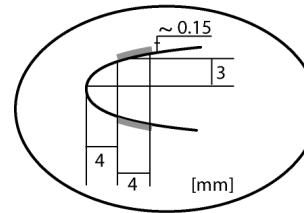


Vortices – Source of cavitation and vibration

The case of a flow over a hydrofoil

- **Boundary layer tripping through leading edge roughness**
- **Natural transition to turbulence**
 - ✓ Polished surface ($R_z \sim 2 \mu\text{m}$)
 - ✓ Foil hydraulically smooth (H. Schlichting)

$$\frac{C_{ref} R_z}{\nu} < 100$$
$$\Rightarrow C_{ref} < 50 \text{ m/s}$$



- **Tripped transition to turbulence**
 - ✓ Glue + $125 \mu\text{m}$ diameter sand
 - ✓ Transition to turbulence (H.L. Dryden:) :

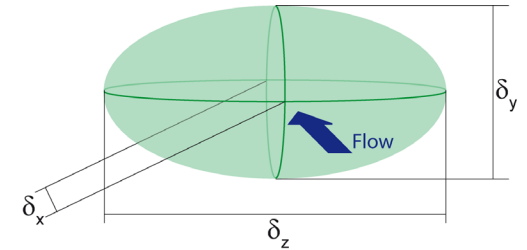
$$\frac{C_{ref} k}{\nu} > 900 \quad k \text{ is the mean of roughness height } \sim 150 \mu\text{m}$$

→ transition to turbulence occurs for upstream velocity above 6 m/s

Vortices – Source of cavitation and vibration

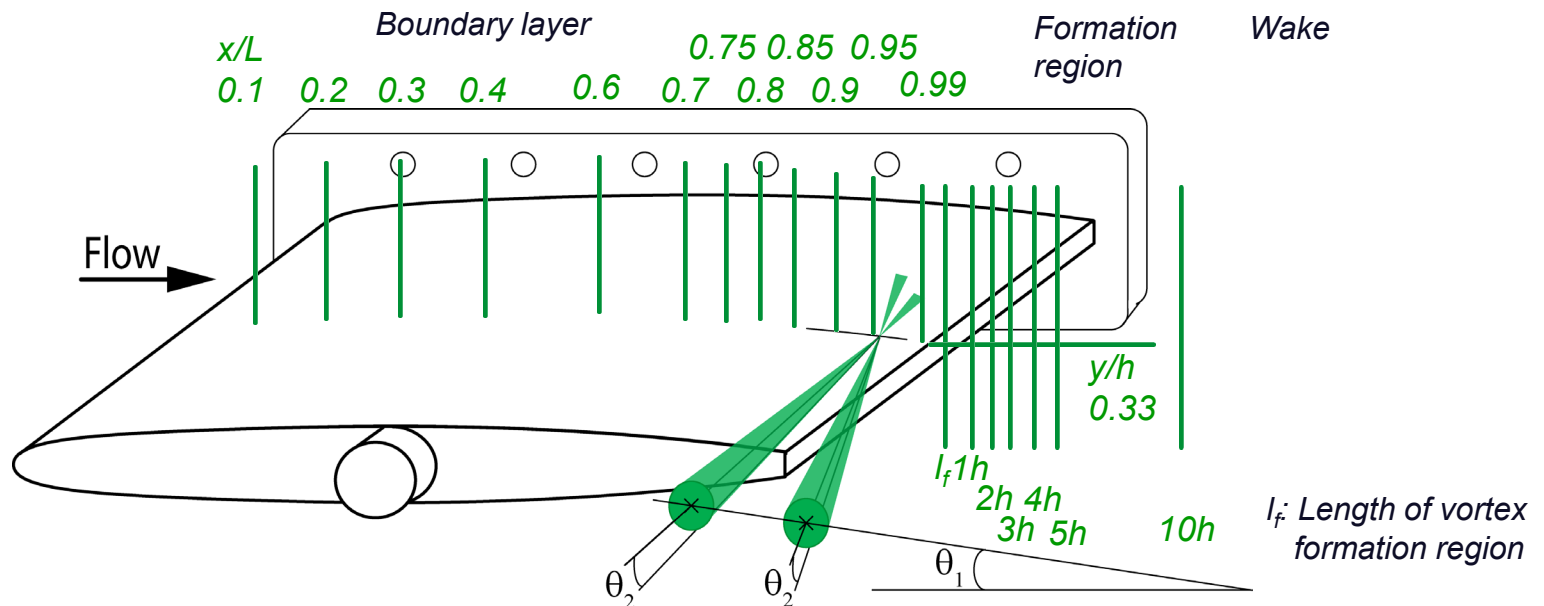
The case of a flow over a hydrofoil

- **Laser Doppler Velocimetry (LDV)**
 - **Seeding:** 10 μm diameter hollow glass spheres
 - **Laser light:** 10 W argon-ions source
 - **Ellipsoidal measurement volume:** diameter: 0.074 mm; length: 1.3 mm
 - **Velocity profiles:**



$$\delta_x = \delta_y = 0.074 \text{ mm}$$

$$\delta_z = 1.307 \text{ mm}$$

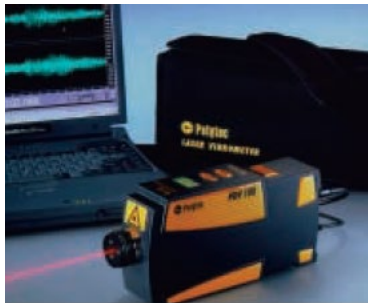


Vortices – Source of cavitation and vibration

The case of a flow over a hydrofoil

- **Vibration measurements:**

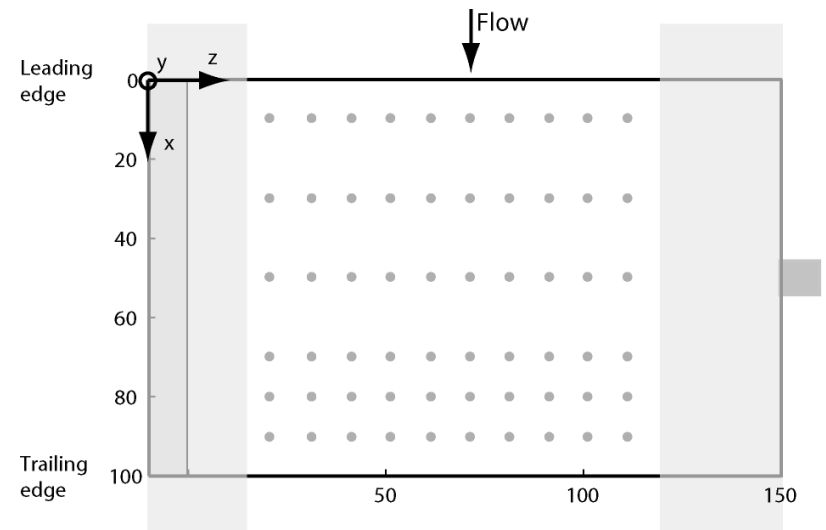
- Piezoelectric accelerometer fitted on the hydrofoil support
- Non intrusive Laser Doppler Vibrometer (Polytec PDV100)



Specification	
Laser type and class	HeNe, class II
Velocity range	20, 100, 500 mm/s
Frequency range	22 kHz
Low pass filter	1, 5, 22 kHz digital FIR type
High pass filter	100 Hz analog, 3 rd order Butterworth
Working distance	> 0.1 m

- **Hydrofoil eigen mode identification:**

- **Accelerometer: Fixed position**
 - Reference signal
- **Laser vibrometer:**
 - Multiple measurement points



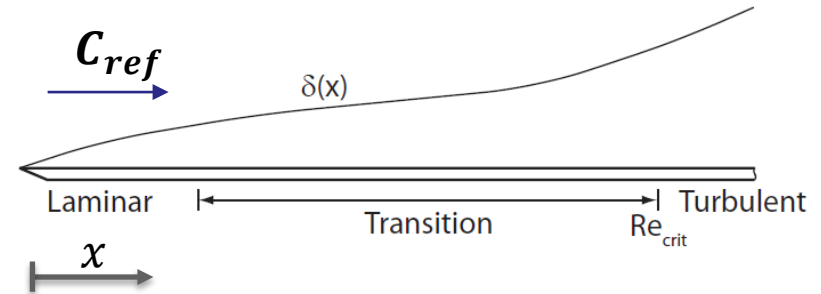
Vortices – Source of cavitation and vibration

Boundary layer properties

Definition: “A thin layer of the fluid close to a boundary surface where the viscous forces are dominant”

- **99% boundary layer thickness:**

$$\delta = y|_{C_x=0.99C_{ref}}$$



- **Displacement thickness:** $\delta_1 = \int_0^{\delta} \left(1 - \frac{C_x}{C_{ref}}\right) dy$

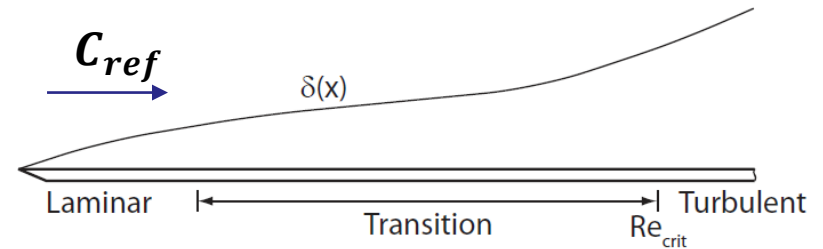
- **Momentum thickness:** $\delta_2 = \int_0^{\delta} \left(1 - \frac{C_x}{C_{ref}}\right) \frac{C_x}{C_{ref}} dy$

- **Form (or Shape) factor:** $H_{12} = \frac{\delta_1}{\delta_2}$

Provides an indication of the boundary layer state (laminar or turbulent)

Vortices – Source of cavitation and vibration

Boundary layer properties



Boundary layer on a flat plate

	Symbol	Laminar flow ¹	Turbulent flow ²
Boundary layer thickness	$\delta(x)$	$5.0 x Re_x^{-1/2}$	$0.37 x Re_x^{-1/5}$
Boundary layer displacement thickness	$\delta_1(x)$	$0.344 \delta(x)$	$0.125 \delta(x)$
Boundary layer momentum thickness	$\delta_2(x)$	$0.133 \delta(x)$	$0.097 \delta(x)$
Form factor	$H_{1,2}$	2.59	1.3 – 1.4
Skin friction coefficient	c_f	$0.664 Re_x^{-1/2}$	$0.0576 Re_x^{-1/5}$

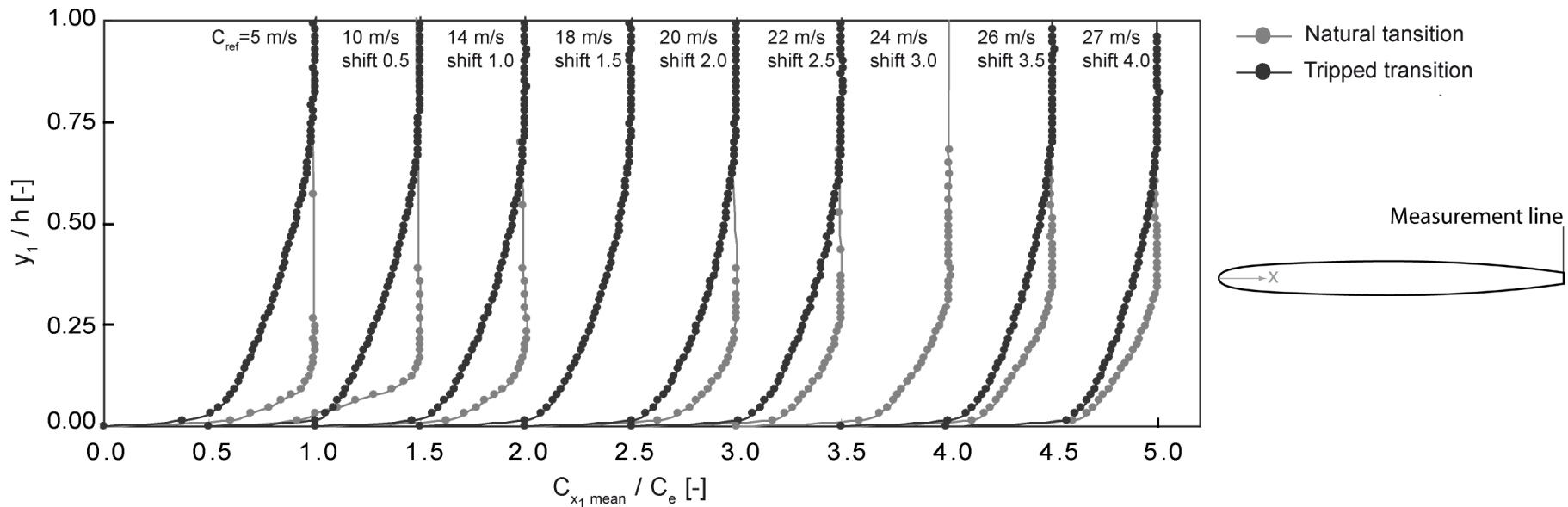
Where Re_x is defined by: $Re_x = \frac{\rho C_{ref} x}{\mu}$

¹ Blasius Theory; ² Empirical

Vortices – Source of cavitation and vibration

Effect of the boundary layer on vortex shedding

- **Boundary layer vs Reynolds number**
 - **Time-averaged velocity profiles: Trailing edge**
 - ✓ **Velocity profiles: shifted on the horizontal axis for clarity**

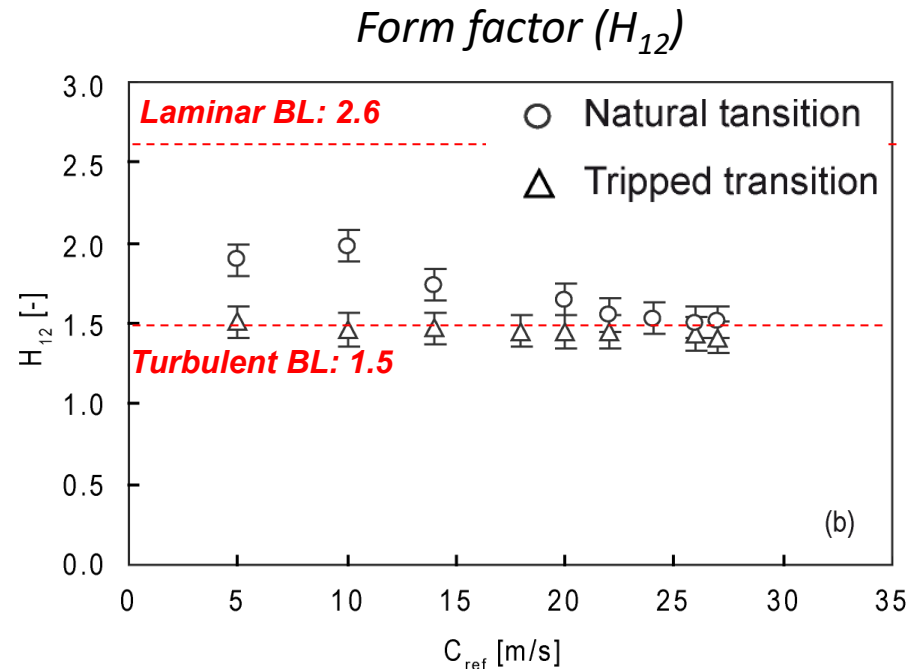
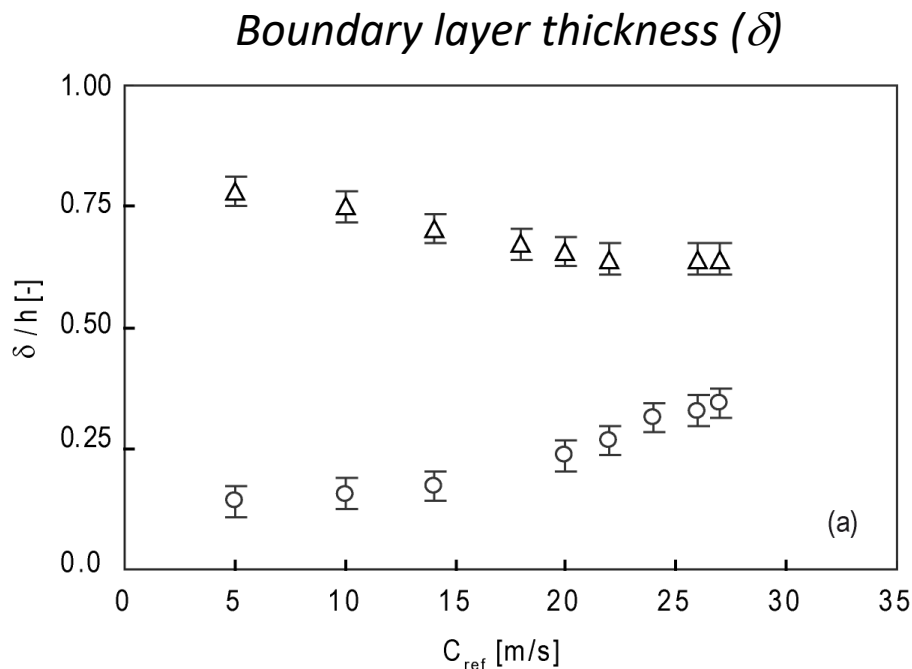
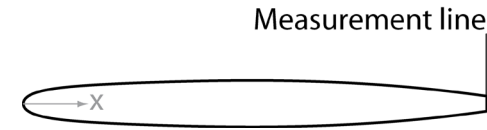


- ✓ **Tripped transition: δ significantly increases**
- ✓ **Reynolds effects: For increasing C_{ref}**
 - **Natural transition: δ increases (transition point moves upstream)**
 - **Tripped transition: δ decreases**

Vortices – Source of cavitation and vibration

Effect of the boundary layer on vortex shedding

- **Boundary layer flow**
 - **Time-averaged velocity profiles: Trailing edge**



- ✓ **Natural transition: up to 14 m/s , H_{12} transitional boundary layer then, H_{12} tends to turbulent value \rightarrow A natural transition occurs along the chord**
- ✓ **Tripped transition: Turbulent BL at the leading edge for all tested velocities**

Vortices – Source of cavitation and vibration

Effect of the boundary layer on vortex shedding

Boundary layer flow

- Natural transition (Location of the transition point ?)

Flat plate boundary layer, Schlichting (1979)

	Laminar flow	Turbulent flow
$\delta(x)$	$5.0 x Re_x^{-1/2}$	$0.37 x Re_x^{-1/5}$

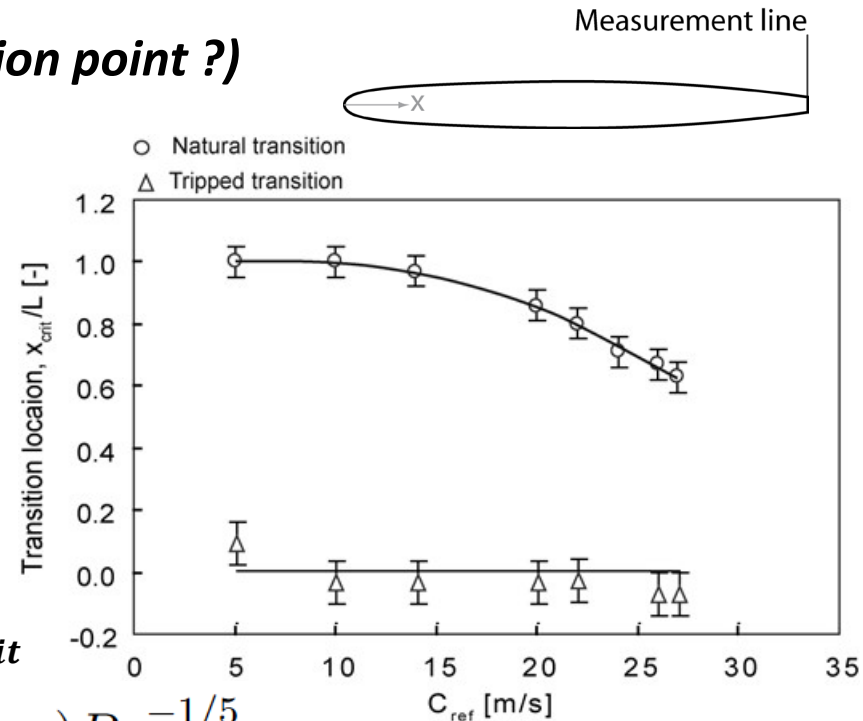
x_{crit} : Location of BL transition to turbulence

$\delta_{T.E.}$: Boundary layer thickness at the trailing edge

- Measurement of $\delta_{T.E.}$ → Evaluation of x_{crit}

$$\delta_{T.E.} = 5.0 x_{crit} Re_{x_{crit}}^{-1/2} + 0.37(L - x_{crit}) Re_{L-x_{crit}}^{-1/5}$$

- Benchmark condition: $C_{ref} = 20 \text{ m/s}$, transition at $0.85L$
- Extrapolation → Natural transition at leading edge for $C_{ref} = 40 \text{ m/s}$
- Tripped transition: Fully turbulent flow at leading edge



L : Chord length

$$Re_x = \frac{\rho C_{ref} x}{\mu}$$

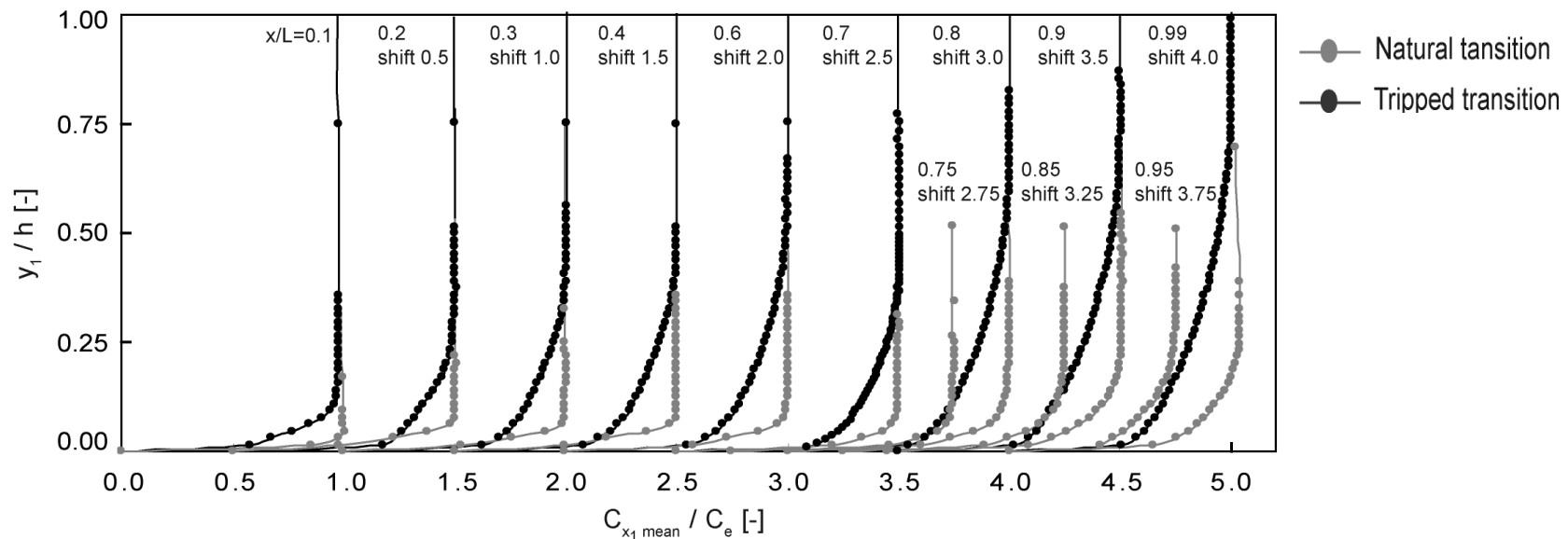
Vortices – Source of cavitation and vibration

Effect of the boundary layer on vortex shedding

- **Boundary layer flow**

- **Development along the hydrofoil chord**
- **Time-averaged velocity profiles ($C_{ref}=20$ m/s)**

✓ **Velocity profiles: shifted on the horizontal axis for clarity**



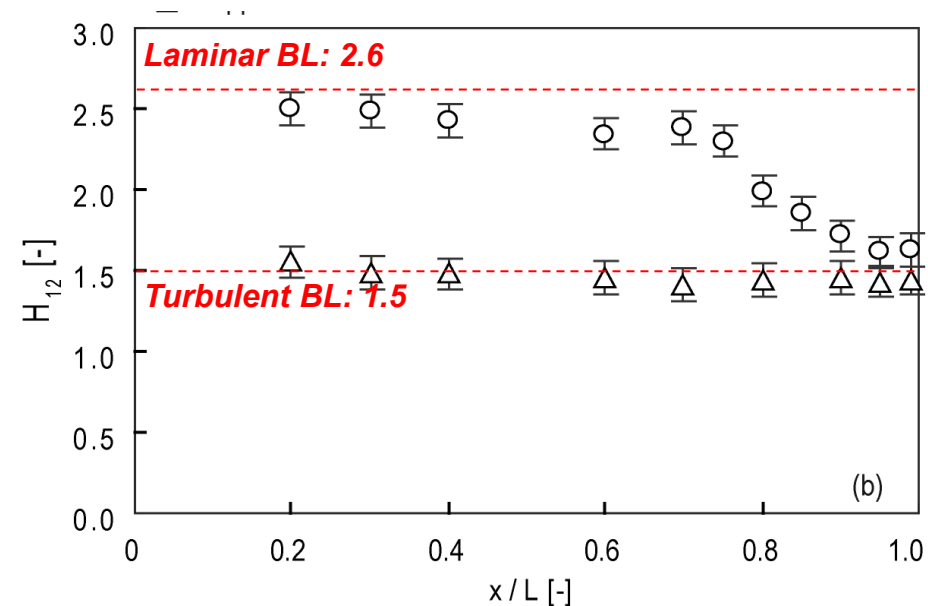
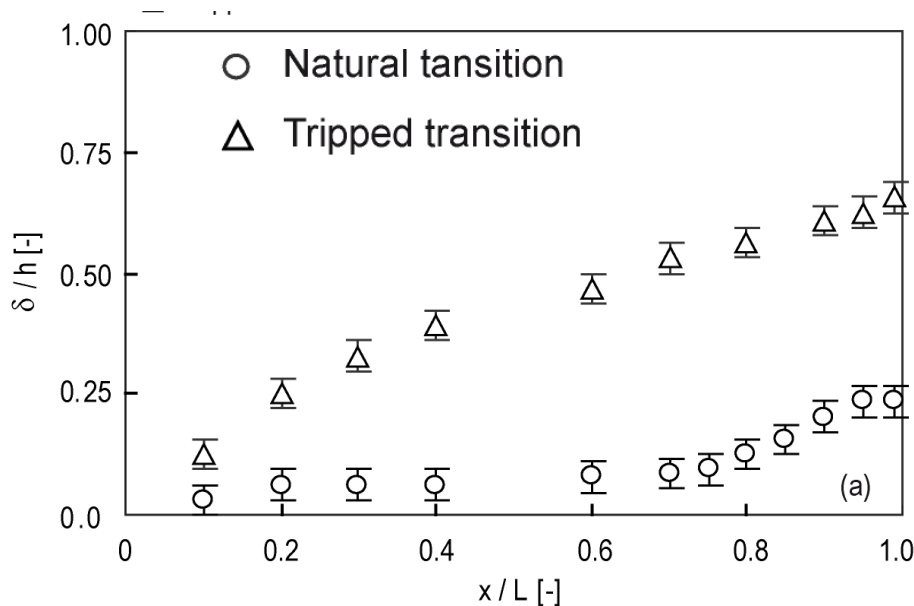
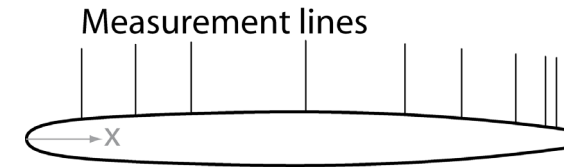
✓ **Natural transition: δ increases with increasing distance from leading edge**

✓ **Tripped transition: Larger growth of the BL thickness compared to natural case**

Vortices – Source of cavitation and vibration

Effect of the boundary layer on vortex shedding

- **Boundary layer flow**
 - **Development along the hydrofoil chord**
 - **Natural and tripped transitions ($C_{ref}=20$ m/s)**



✓ **Natural transition:**

✓ **Form factor decreases from 2.5 (Laminar BL) to 1.6 (Turbulent BL)**

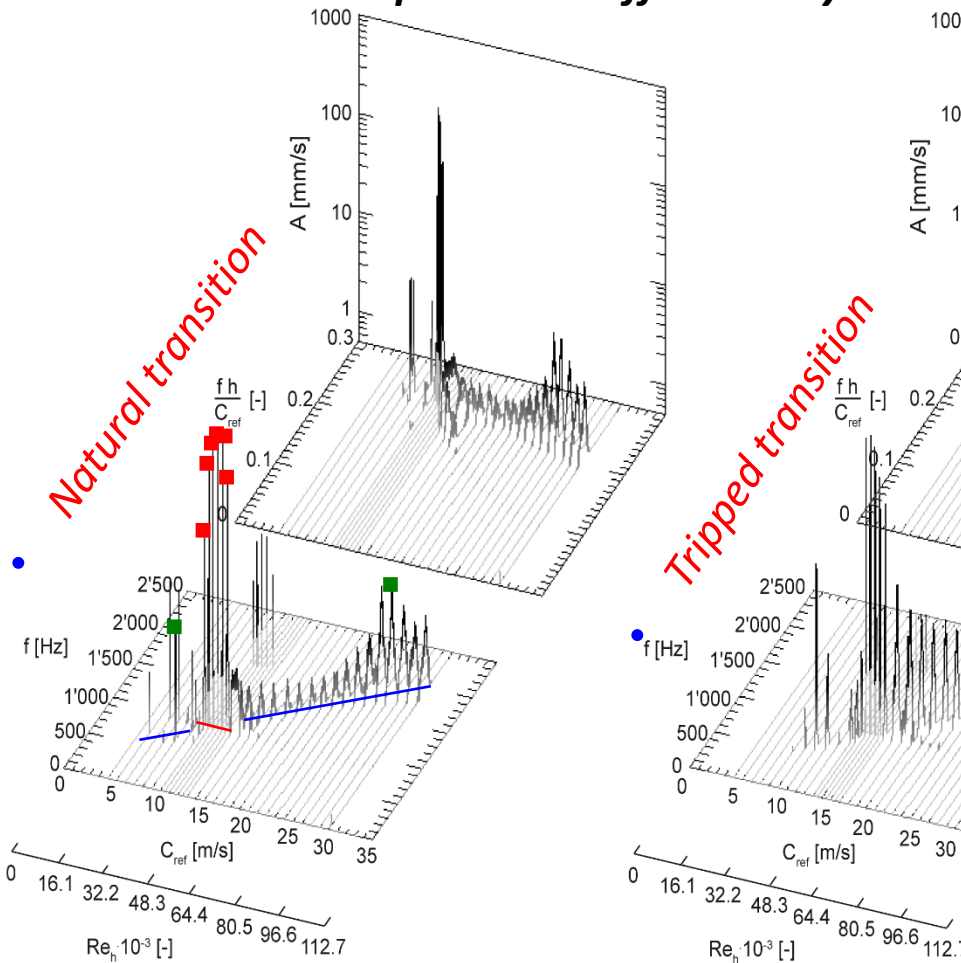
Natural turbulent boundary layer transition @ 0.8 L

✓ **Tripped transition: Mean $H_{12} \sim 1.46$: Turbulent BL develops along the entire chord**

Vortices – Source of cavitation and vibration

Effect of the boundary layer on vortex shedding

- **Vortex-induced vibration and vortex shedding frequency**
 - **Vibration spectra at different Reynolds numbers (natural and tripped transition)**



- ✓ **Strouhal behavior**
 f_s evolves linearly with C_{ref}
- ✓ **Resonance**
- ✓ **Lock-in**

Tripped vs. Natural transition

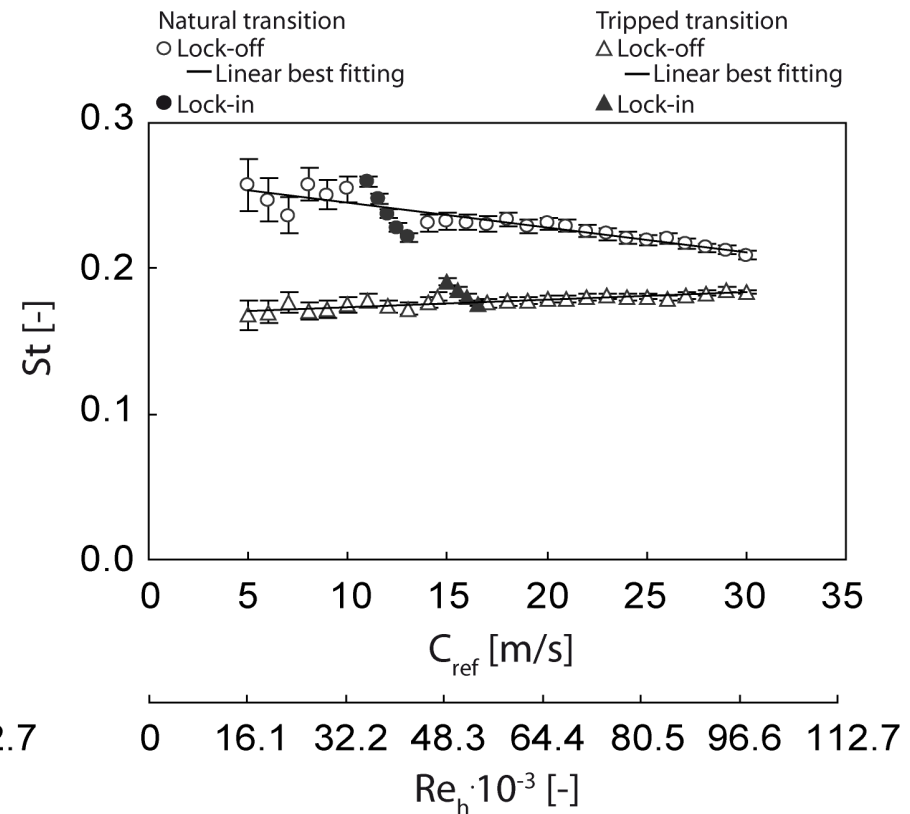
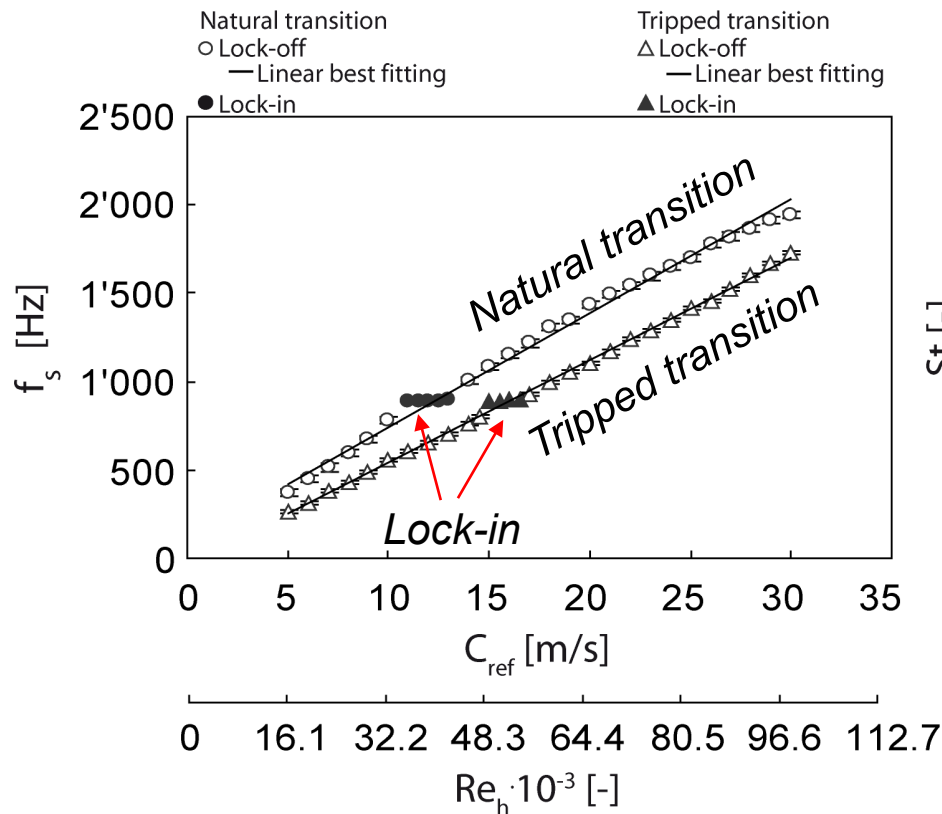
- **Decrease of shedding frequency (22%)**
- **Increase of the vortex-induced vibration**

Vortices – Source of cavitation and vibration

Effect of the boundary layer on vortex shedding

- Vortex shedding frequency and Strouhal number

$$St_h = \frac{f_s h}{C_{ref}}$$



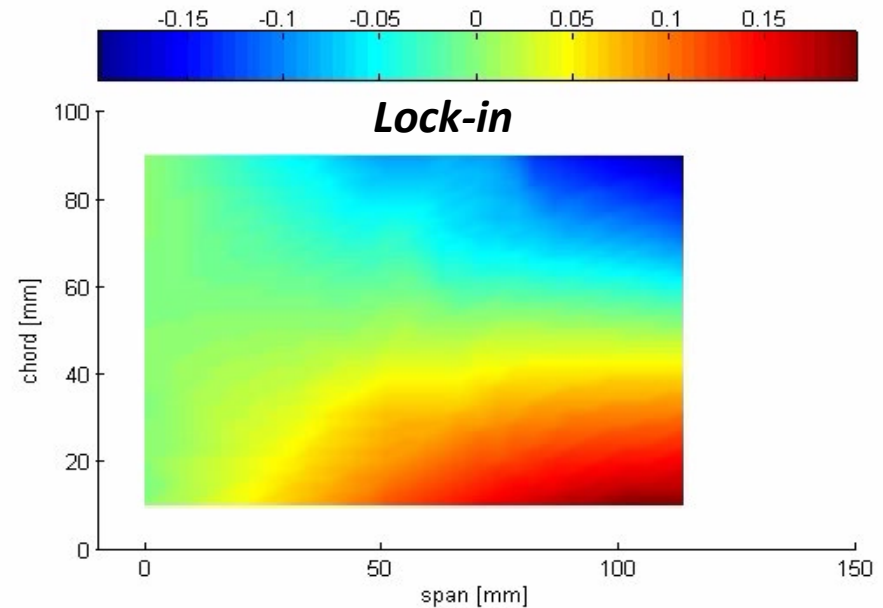
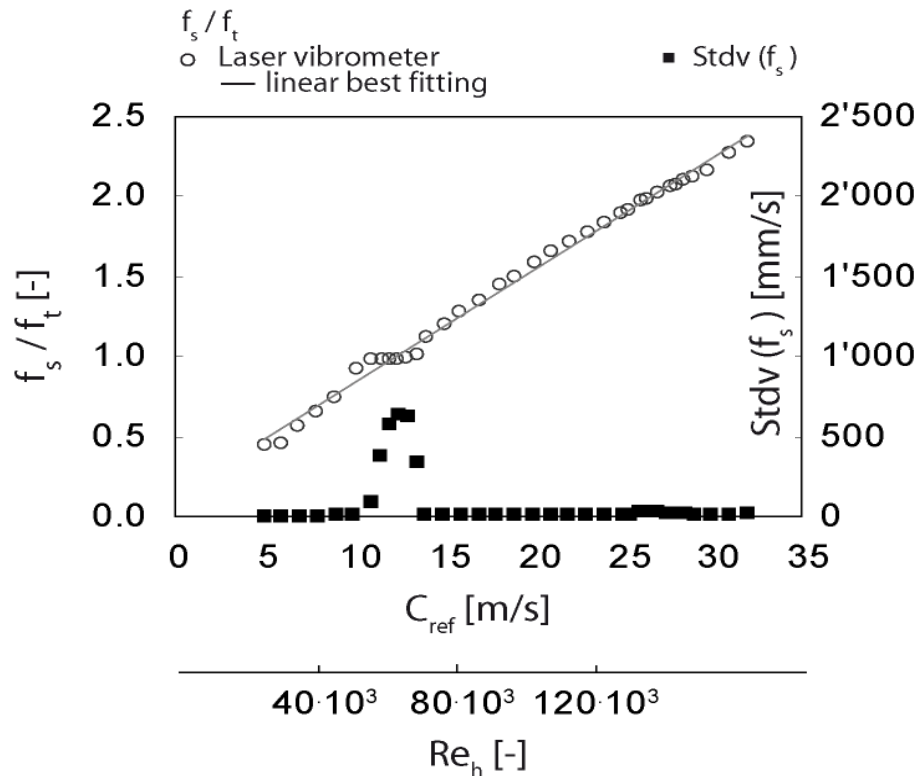
$$\overline{St}_{Smooth} = 0.24$$

$$\overline{St}_{Rough} = 0.18$$

Vortices – Source of cavitation and vibration

Effect of the boundary layer on vortex shedding

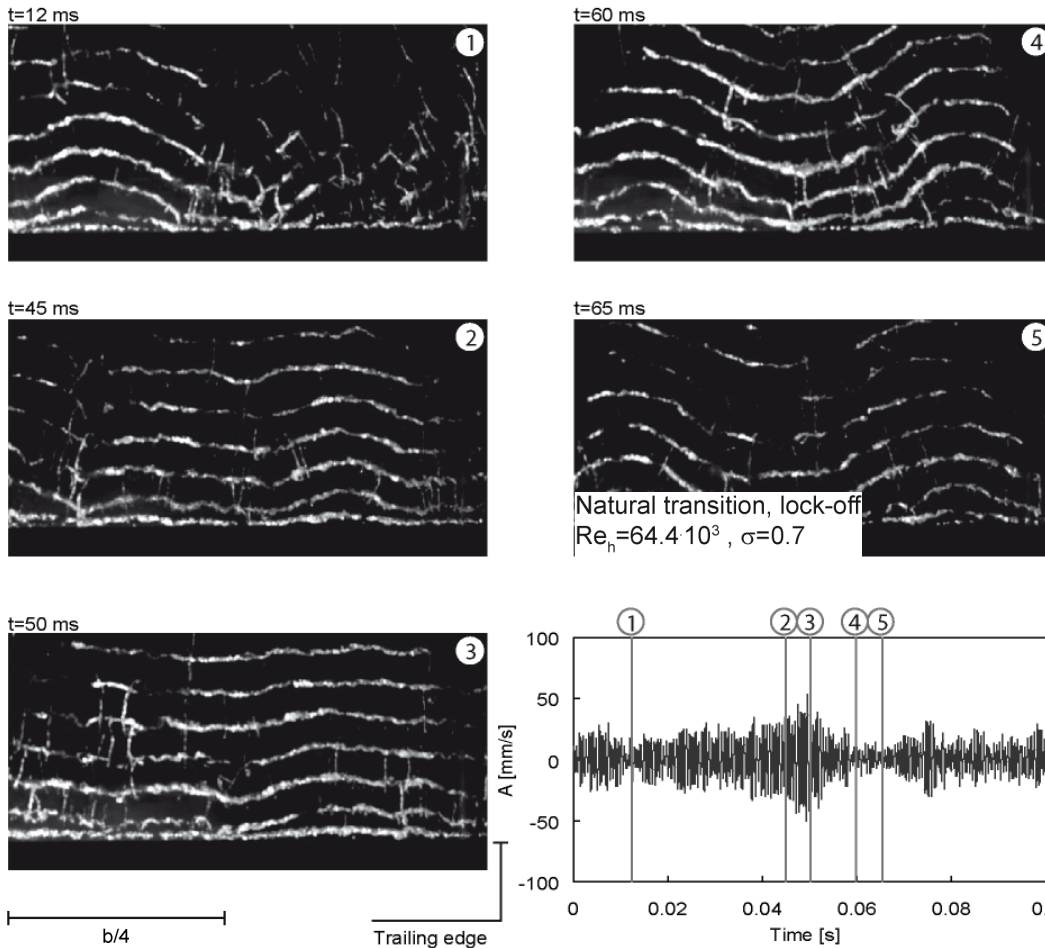
- **Smooth leading edge – Lock-in**
 - **1st torsional mode : Large vibration amplitude**



Vortices – Source of cavitation and vibration

Effect of the boundary layer on vortex shedding

- Wake structure – **Natural transition** – **Lock-off**



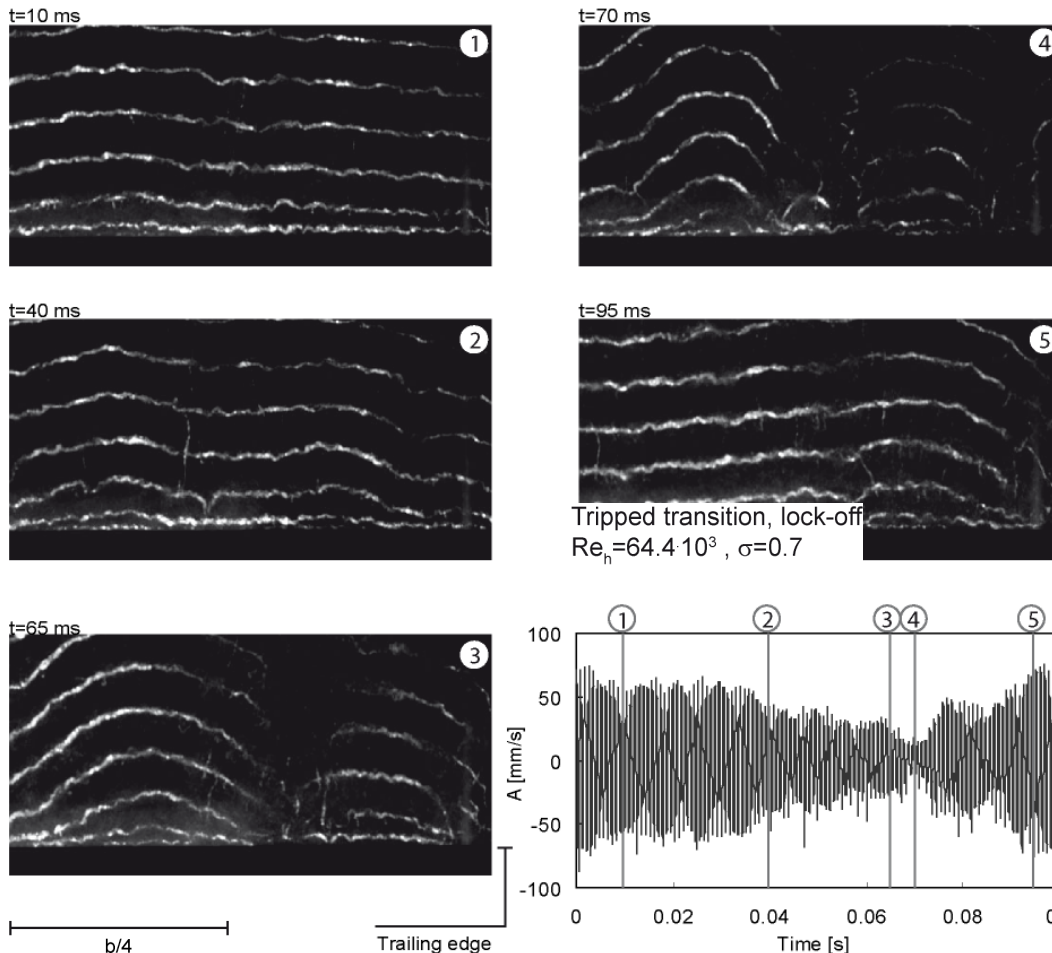
Vibration amplitude correlates well with vortex patterns:

- **3D pattern** → lower vibration
 - Loss of coherence
- **2D pattern** → Higher vibration
 - More coherent structures

Vortices – Source of cavitation and vibration

Effect of the boundary layer on vortex shedding

- **Wake structure – Tripped transition – Lock-off**



Tripped transition:

→ Larger vibration amplitude

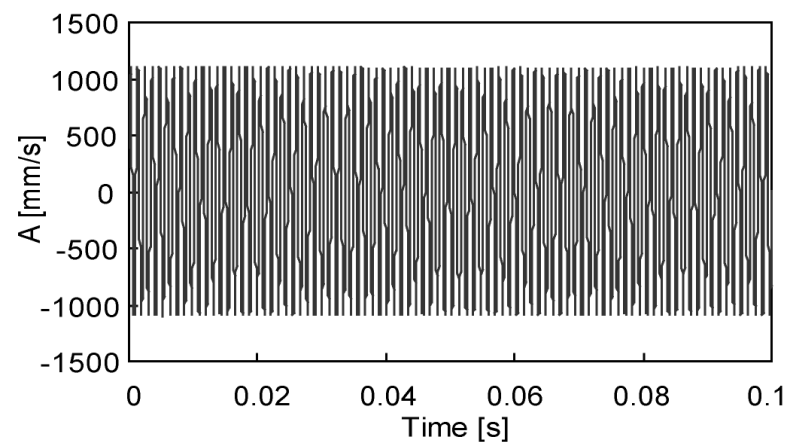
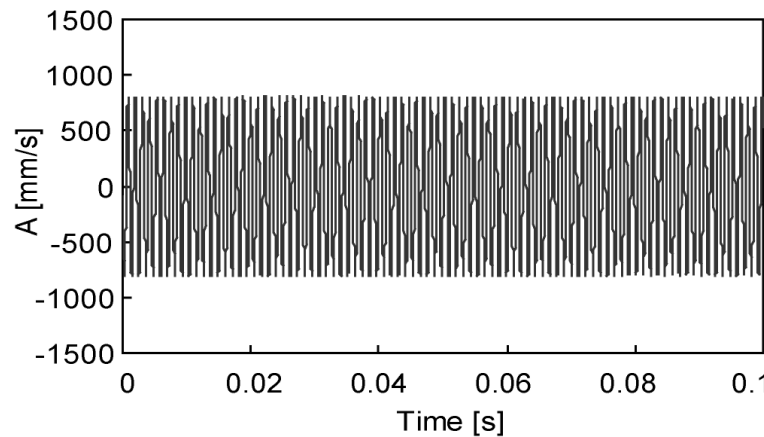
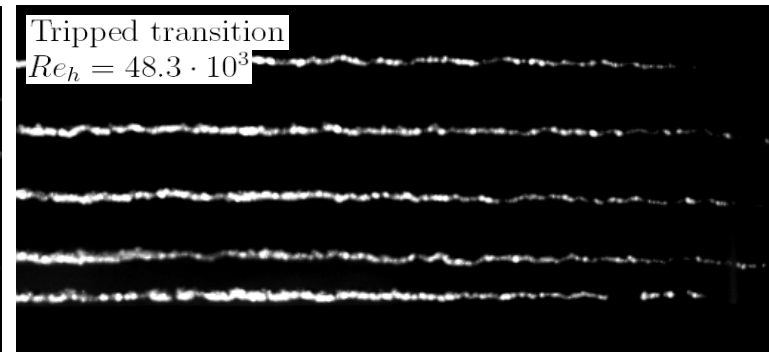
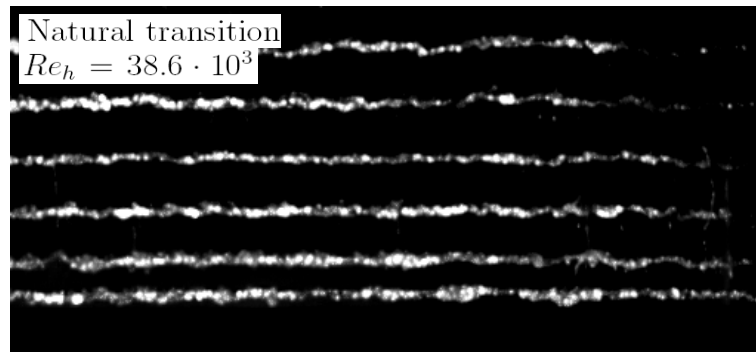
→ Enhance vortex spanwise organization

→ Promotes parallel vortex shedding

Vortices – Source of cavitation and vibration

Effect of the boundary layer on vortex shedding

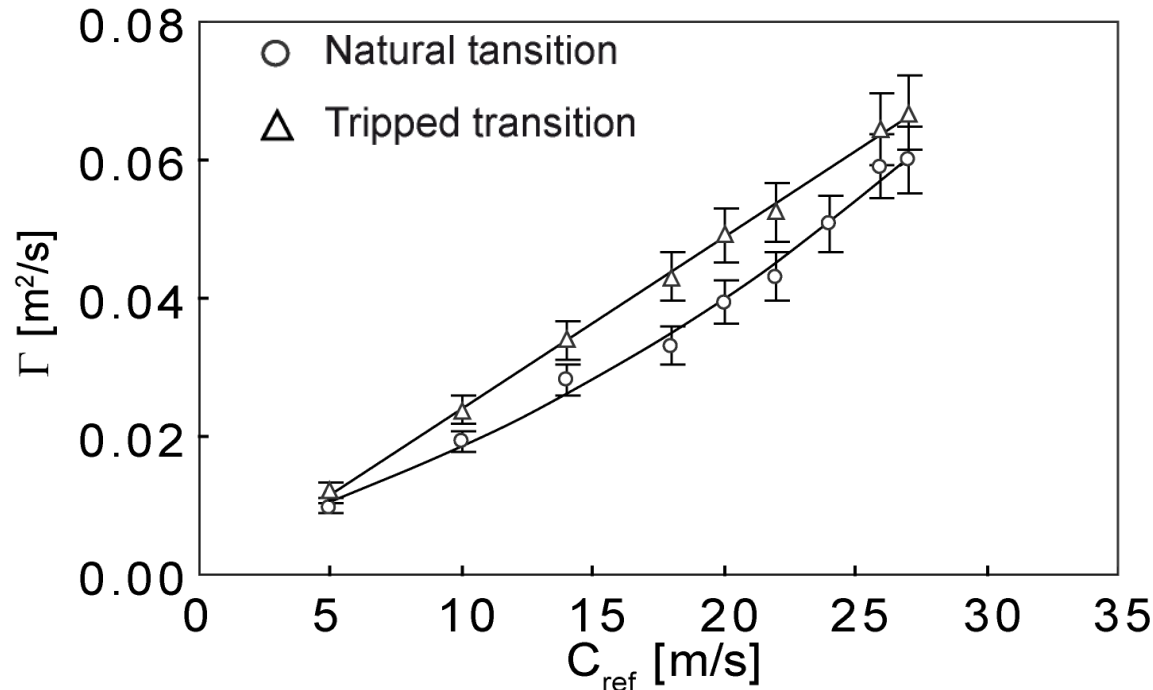
- **Wake structure – Lock-in - Natural vs. Tripped transition:**
 - **Vibration signal almost sinusoidal and parallel vortex shedding for both BL transitions**
 - **Vibration amplitude larger for tripped transition**



Vortices – Source of cavitation and vibration

Effect of the boundary layer on vortex shedding

- **Vortex strength:**
 - **Using Rankine model for vortex velocity distribution**



- **Tripped transition: Vortices have more strength because of the enhanced spanwise organization → increased vibration**

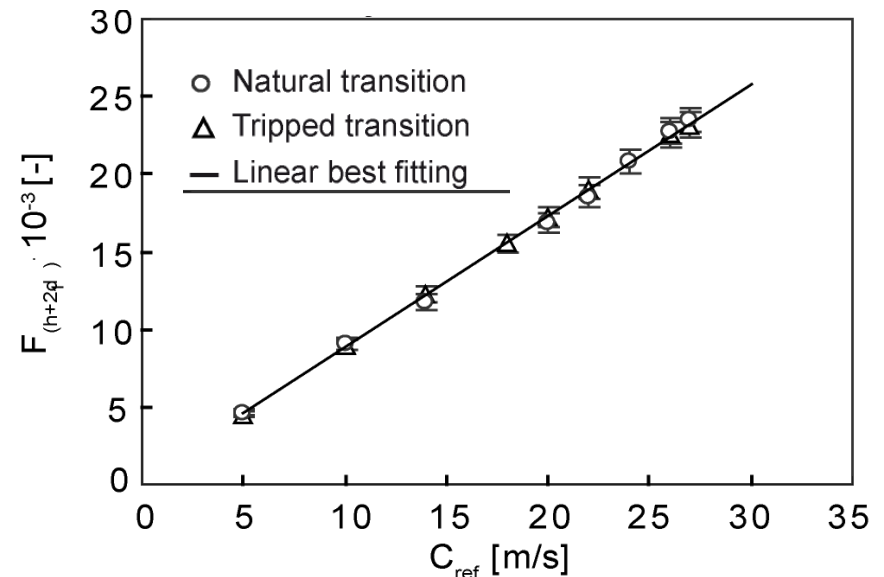
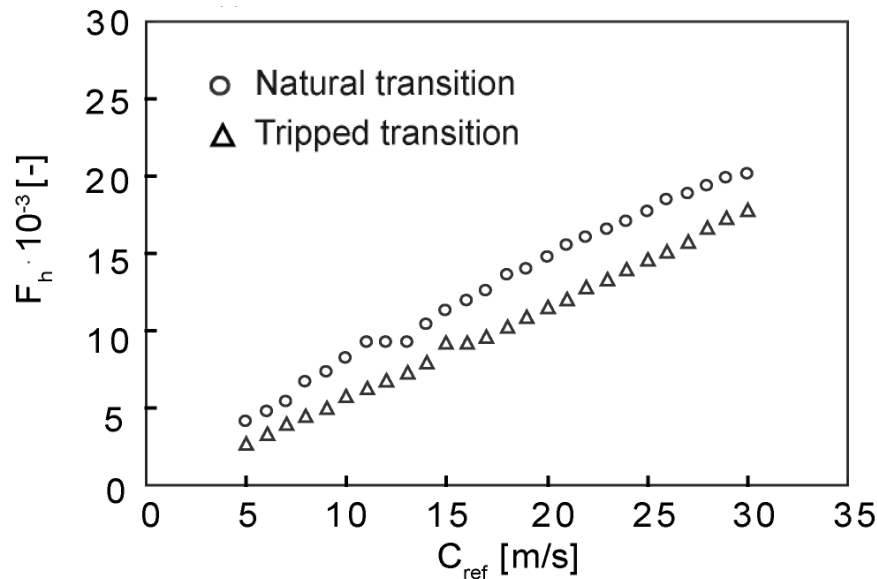
Vortices – Source of cavitation and vibration

Effect of the boundary layer on vortex shedding

- **Universal scaling law (Roshko Number):**
 - **Replace the inertia time (h/C_{ref}) by diffusion time (h^2/ν)**

$$F_h = \frac{f_s h^2}{\nu}$$

$$F_{(h+2\delta_1)} = \frac{f_s (h + 2\delta_1)^2}{\nu}$$

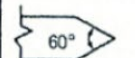
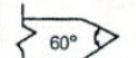



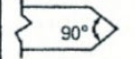
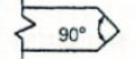
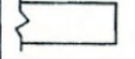
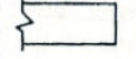
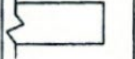
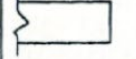
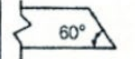
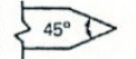
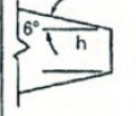
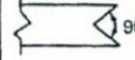
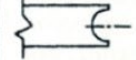
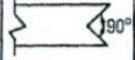
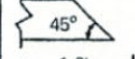
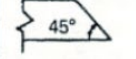

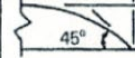


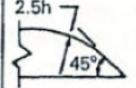
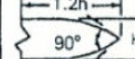
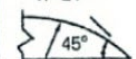
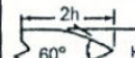

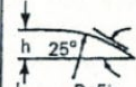

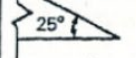


- ✓ **Natural and tripped transitions: Single linear law when d_1 is taken into account**
- ✓ **Rosko number more appropriate for the scaling than Strouhal number**
- ✓ **But not straightforward (boundary layer thickness is not always known)**

Vortices – Source of cavitation and vibration

Effect of trailing edge shape on vortex induced vibration

- A variety of trailing edge (T.E.) geometries have been tested
- Relative vibration amplitude is reported in comparison with baseline hydrofoil with blunt T.E.
- Some of the shapes cut the vibration amplitude by 2 orders of magnitude

DONALDSON		HESKESTAD & OLBERTS		IPPEN		BLAKE ET AL			
GEOMETRY	REL. AMP.	GEOMETRY	REL. AMP.	GEOMETRY	REL. AMP.	GEOMETRY	REL. AMP.	$\gamma f/h$	U_g/U_∞
	360%		380%		320%				
	260				230				
	230		190						
	100		100		100		100	0.9	1.25
	48		43				100	1.0	1.05
	22		31		80				
	20		38		70				
	> 1		3		60		0.3-1.5	0.5	1.05
	> 1		< 1						
	> 1		< 1				1.0	0.5	1.03
			< 1				0.5-1.7	0.8	1.2

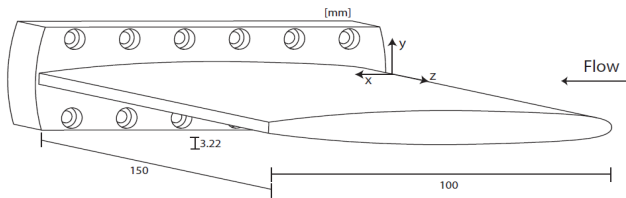
Ref.: Donaldson, J. of Eng. for Power, 1956

Vortices – Source of cavitation and vibration

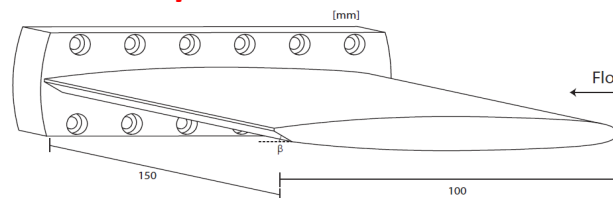
Effect of trailing edge shape on vortex induced vibration

- Experimental Setup – Case study
 - 3 NACA0009 hydrofoils with:
 1. Blunt (truncated) trailing edge
 2. Oblique trailing edge
 3. Rounded (Donaldson) trailing edge
 - Natural and tripped boundary layer transition

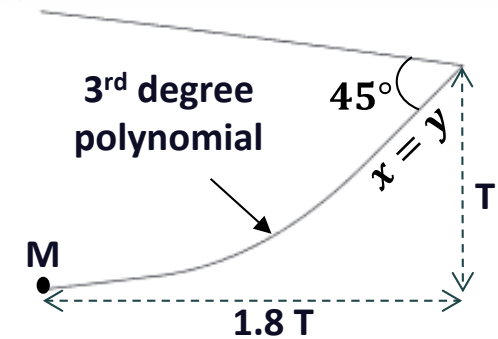
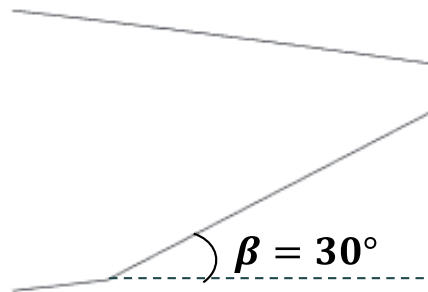
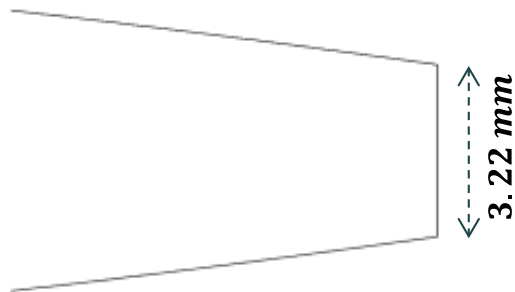
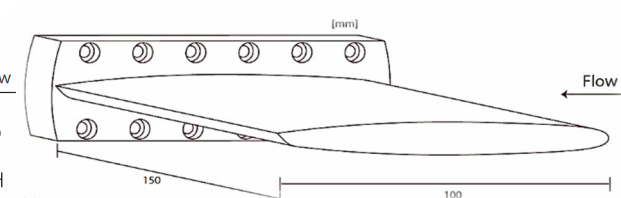
1. Blunt T.E.



2. Oblique T.E.



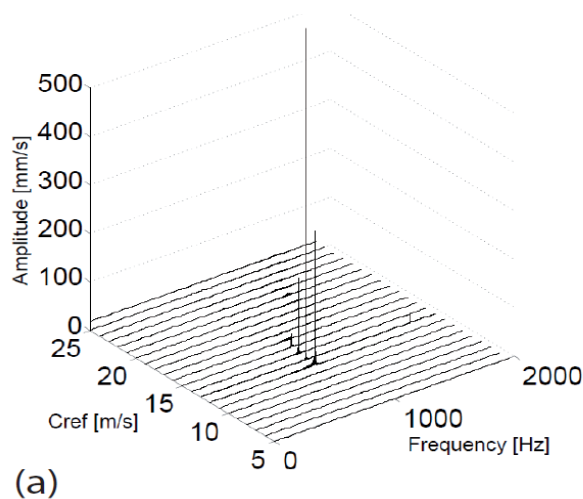
3. Donaldson T.E.



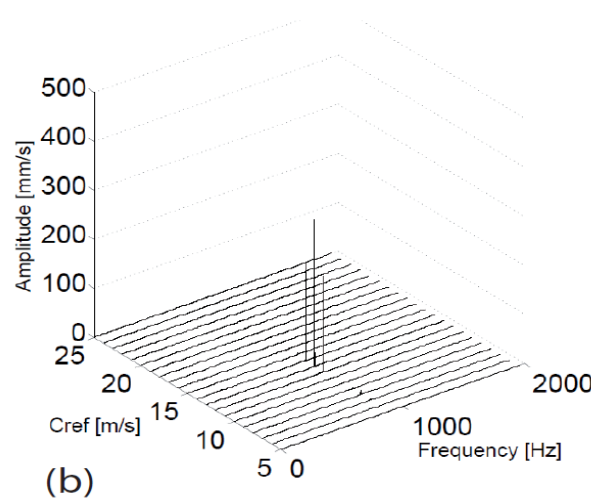
Vortices – Source of cavitation and vibration

Effect of trailing edge shape on vortex induced vibration

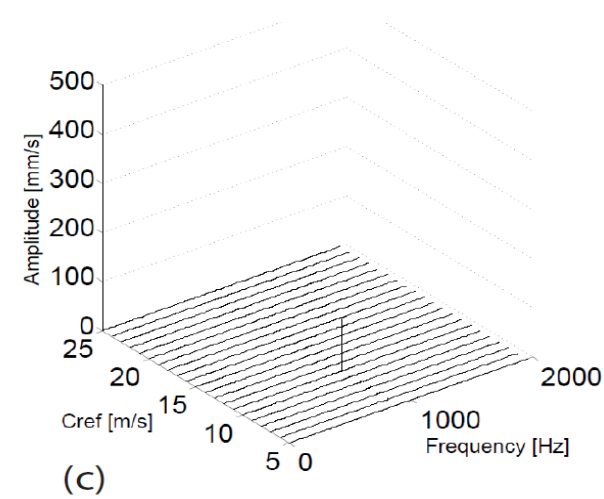
Truncated Trailing Edge



Oblique Trailing Edge



Donaldson Trailing Edge

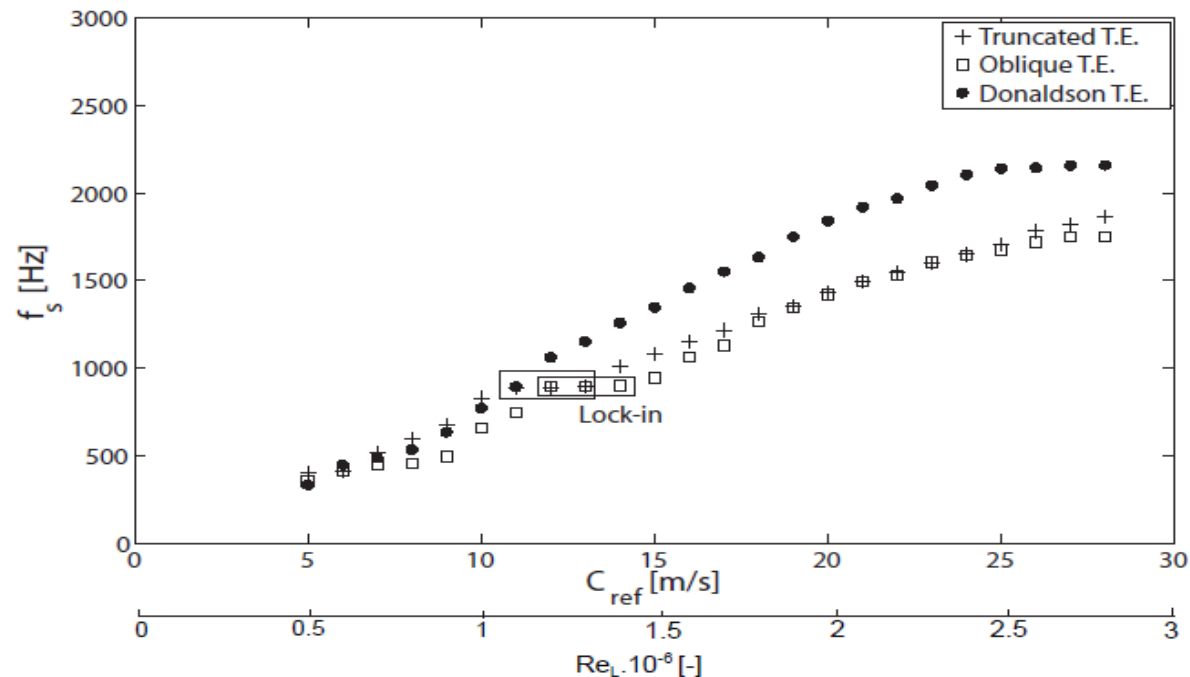


- **Maximum vibration with the truncated trailing edge**
- **Minimum vibration with Donaldson Trailing Edge**
- **No lock-in (Hydro-elastic coupling) with Donaldson trailing edge**

Vortices – Source of cavitation and vibration

Effect of trailing edge shape on flow shedding frequency

- Linear relationship between shedding frequency and upstream velocity, except under Lock-in condition for truncated and oblique T. E.
- Lock-in (shedding frequency locked on the torsion mode of the hydrofoil) :
 - Truncated TE: 11- 14 m/s
 - Oblique TE : 12-15 m/s
 - Donaldson TE: No lock-in



Vortices – Source of cavitation and vibration

Effect of trailing edge shape on wake dynamics

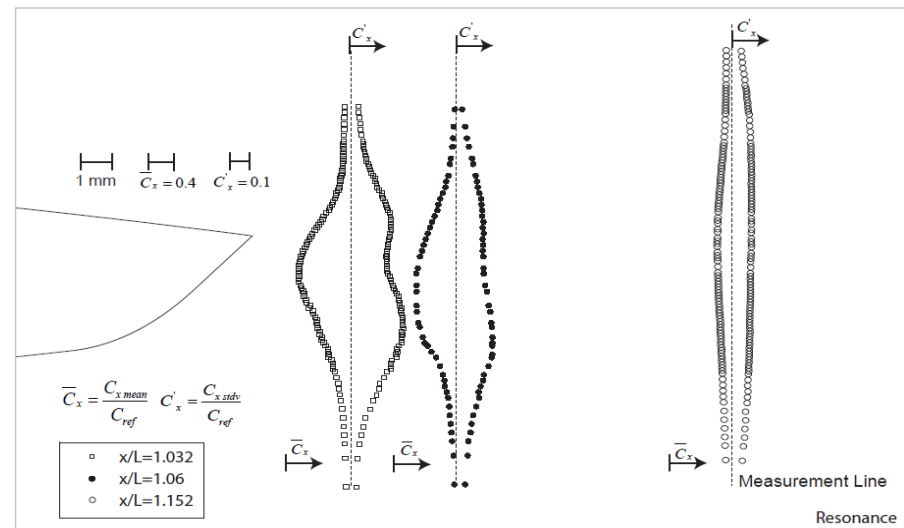
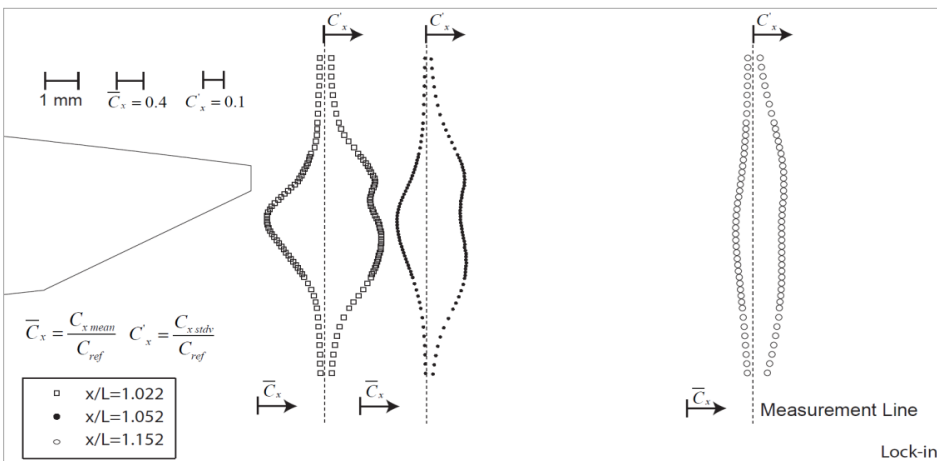
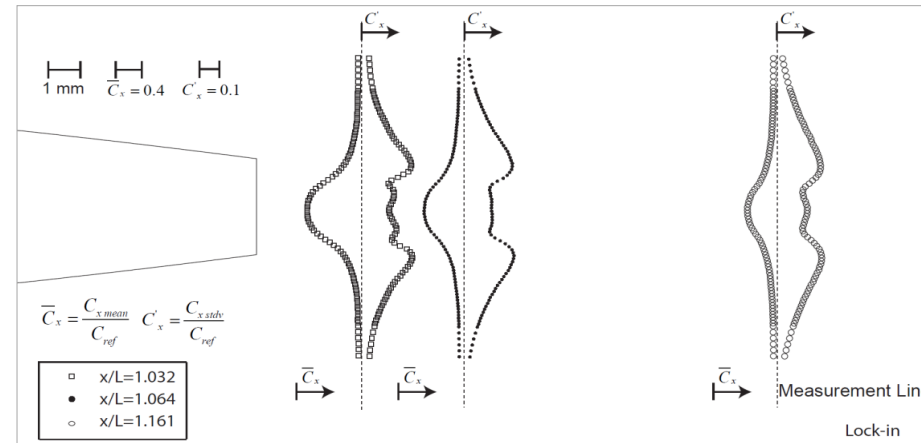
Velocity Profiles along the wake (Lock-in)

For all trailing edges:

- Decrease of velocity deficit along the wake more pronounced with Donaldson TE

Truncated TE: Symmetric wake

Oblique and Donaldson TE: Asymmetric wake



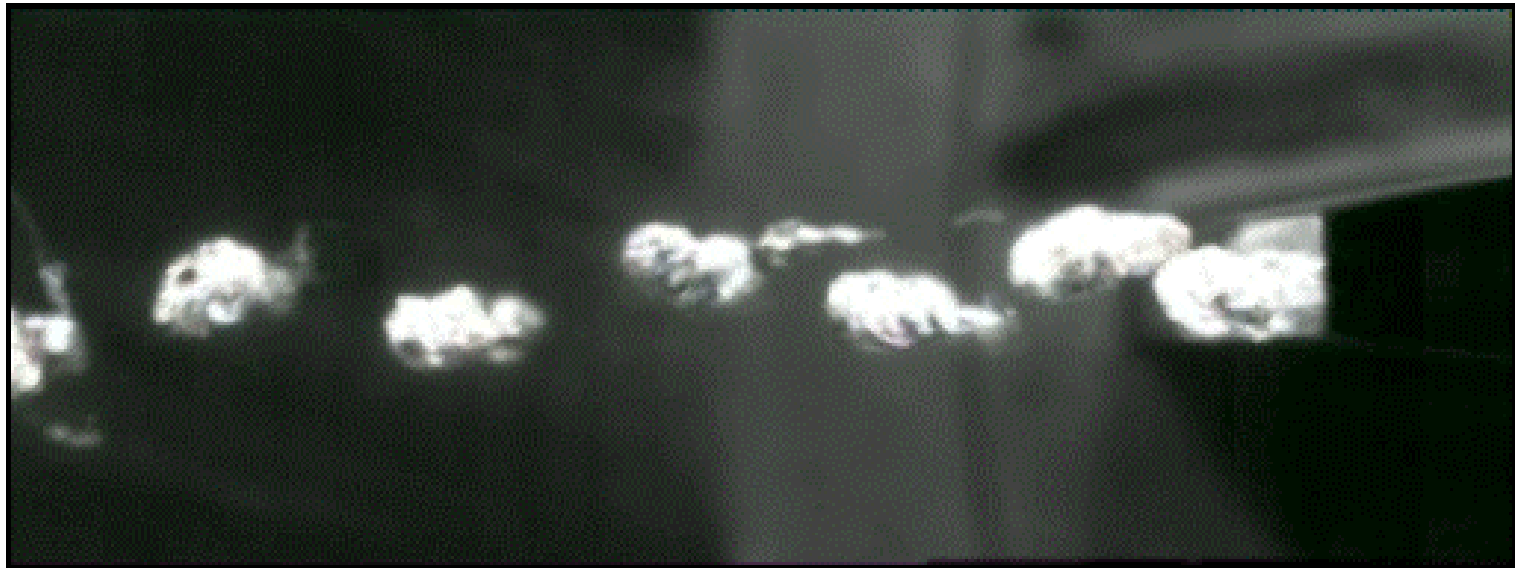
Vortices – Source of cavitation and vibration

Effect of trailing edge shape on wake dynamics

High speed visualization of cavitating vortices (Truncated TE., **Lock-in**)

- Alternate shedding with lower and upper vortices of the same size.

$$C_{ref}=12 \text{ m/s}, \sigma = 0.87$$



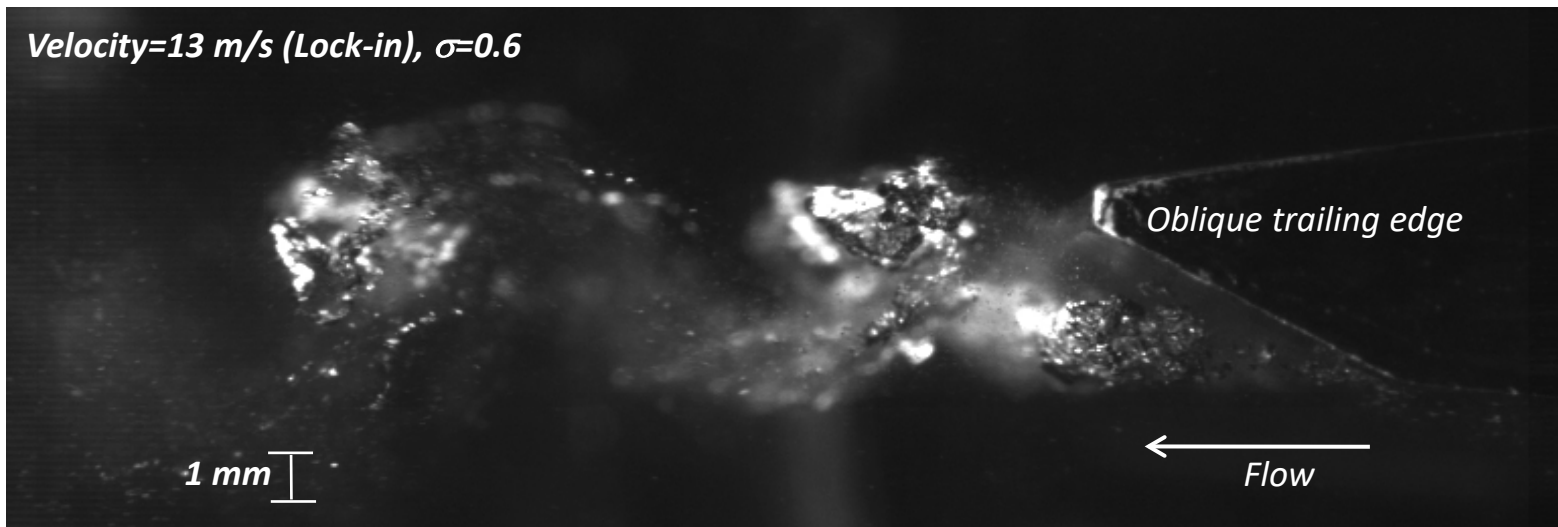
Vortices – Source of cavitation and vibration

Effect of trailing edge shape on wake dynamics

High speed visualization of cavitating vortices (Oblique T. E., **Lock-in**)

- Upper vortex coincides with the passage of lower vortex during the rolling up
Spatial shift → collision between upper and lower vortices (cancellation)
- The vorticity less concentrated within the core of Karman vortices.

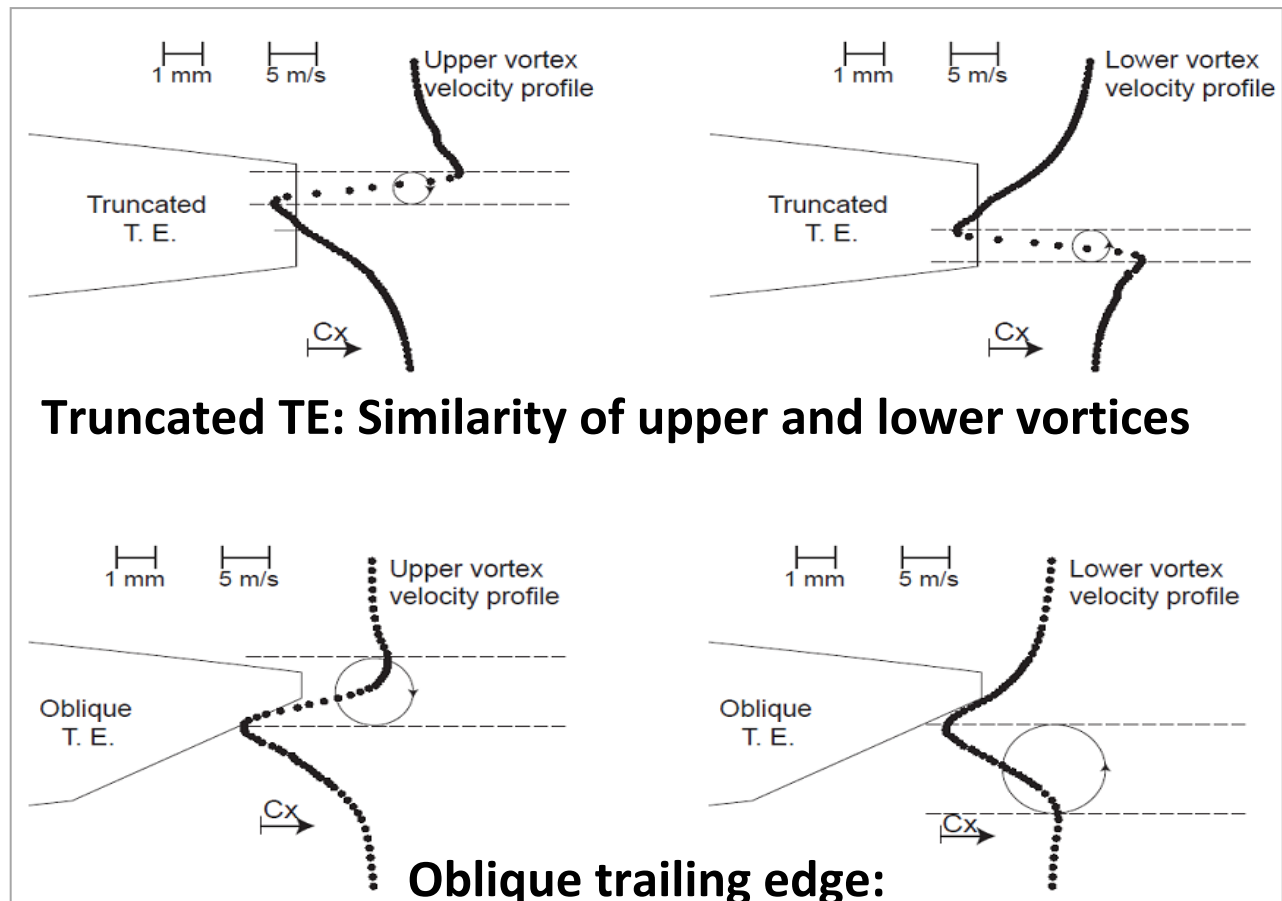
→ Decrease of induced vibration level



Vortices – Source of cavitation and vibration

Effect of trailing edge shape on wake dynamics

- Instantaneous velocity profiles (LDV)



Larger vortex core diameter in comparison with truncated T. E.
Larger vortex core of lower vortices compared to upper vortices

Vortices – Source of cavitation and vibration

Effect of trailing edge shape on wake dynamics

High speed visualization of the wake dynamic for Donaldson TE

- Strong collision between upper and lower vortices
- Most part of the lower vortex is destroyed by upper vortex
- Less vorticity concentration within the core of vortices
- Very efficient in reducing flow induced vibration

