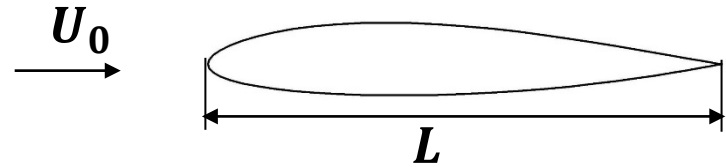


AEROELASTICITY AND FLUID-STRUCTURE INTERACTION

Chapter 4

Quasi-static Aeroelasticity Static Instability - Divergence

Time scales in solid and fluid domains:



$$T_{fluid} = \frac{L}{U_0} \quad : \text{Travel time over a distance } L \text{ at velocity } U_0$$

$$T_{solid} = \frac{L}{\sqrt{E/\rho_s}} = \frac{L}{c} \quad : \text{Travel time over a distance } L \text{ at the elastic waves celerity}$$

or

$$T_{solid} = \sqrt{m/k} \quad : \text{Oscillation period of a given mode shape}$$

Hypothesis : Large reduced velocity

$$U_R = \frac{T_{solid}}{T_{fluid}} \gg 1 \quad \Rightarrow \quad T_{solid} \gg T_{fluid}$$

$$U_R \ll 1$$

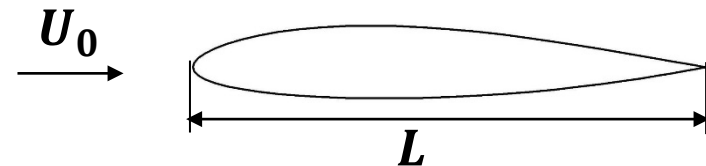
The solid evolves in an almost still fluid



→ Added stiffness & Added mass
(See Chapter 2)

$$U_R \gg 1$$

The fluid evolves with an almost fixed solid



$U_0 \sim 100 \text{ m/s}; L \sim 1 \text{ m}; T_{\text{fluid}} \sim 0.01 \text{ s}$
 $\text{Torsional frequency} \sim 1 \text{ Hz} \rightarrow T_{\text{solid}} \sim 1 \text{ s}$
→ $U_R \sim 100 \gg 1$

Dimensionless numbers for large reduced velocity :

$$Re = \frac{\rho U_0 L}{\mu} \quad Fr = \frac{U_0}{\sqrt{gL}} \quad Cy = \frac{\rho U_0^2}{E} \quad U_R = \frac{U_0}{c}$$

U_0 is relevant with the hypothesis of large reduced velocity

T_{solid} is no more relevant with the hypothesis of $U_R \gg 1$

→ Alternate choice : T_{fluid} instead of T_{solid}

Non dimensional variables:

$$t_f = \frac{t}{T_{fluid}} = \frac{tU_0}{L} \quad U_f = \frac{U}{U_0} \quad p_f = \frac{p}{\rho U_0^2}$$

Dimensionless equations of fluid and solid motions

- *On the fluid side: Navier-Stokes equations*

$$\nabla U_f = 0 \quad \frac{dU_f}{dt_f} = -\frac{1}{Fr^2} e_z - \nabla p_f + \frac{1}{Re} \Delta U_f$$

- *On the solid side (single mode approximation):*

$$\xi_s(x_s, t_s) = Dq_s(t_s)\phi(x_s)$$

$$U_R^2 \frac{d^2 q_s}{dt_f^2} + q_s = f_s$$

Dimensionless equations of fluid and solid motions

- **At the Fluid-Solid interface:**

- **Kinematic conditions:**

$$U = \frac{d\xi}{dt} \quad \Rightarrow \quad U_f = \frac{d\xi_s}{dt_f} \approx \mathbf{0} \left(\frac{\frac{\xi_0}{L}}{\frac{T_{solid}}{T_{fluid}}} \right) = \mathbf{0} \left(\frac{D}{U_R} \right)$$

- **Assumption of large reduced velocity:**

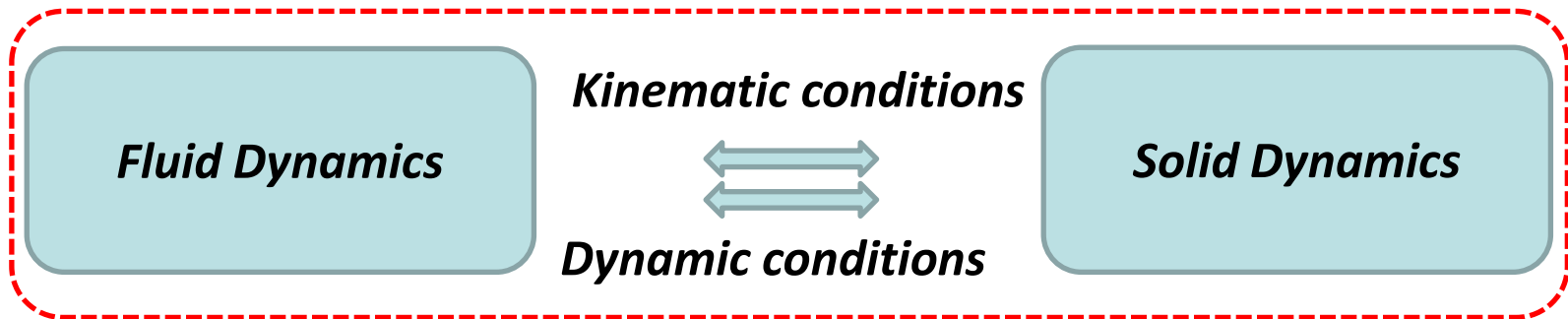
$$U_R \gg D \Rightarrow U_f \approx \mathbf{0} \quad \underline{\text{At the interface}}$$

- **Dynamic conditions along the interface (I):**

$$C_y \int_I \left\{ \left[-p_f \mathbf{I} + \frac{1}{Re} (\nabla U_f + \nabla^t U_f) \right] \cdot \mathbf{n} \right\} \cdot \boldsymbol{\phi} dS = D \mathbf{f}_s$$

Quasi-Static Aeroelasticity

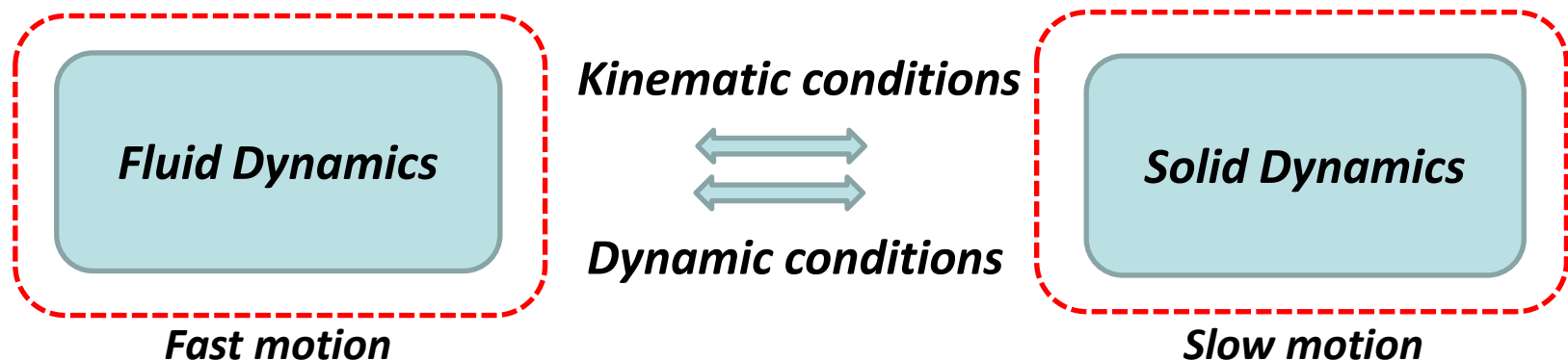
- **General case:**
 - *The fluid-solid interface has a deformation and a velocity*



- *The motions in the fluid and in the solid must be solved simultaneously*

Quasi-Static Aeroelasticity

- $U_R \gg D \rightarrow$ The fluid-solid interface deforms “without” velocity
 - \rightarrow Within the fluid time scale, the motion of the solid is so slow that we can neglect it \rightarrow the solid does not move
 - \rightarrow This case is defined as “**Quasi-static Aeroelasticity**”



- \rightarrow *Extremely different time scales* \rightarrow *no need to solve the solid and fluid motions simultaneously. We can solve one domain at a time (iterations)*

Static Instability - Divergence

- **Fluid loading on the solid:**
 - **Single mode approximation:**

$$\xi_s(x_s, t_s) = Dq_s(t_s)\phi(x_s)$$

- **Fixed interface \rightarrow At a given time, the flow depends only on the position of the interface at that time (steady state) :**

$$p_f(Dq_s) \text{ and } U_f(Dq_s)$$

Instantaneous position
of the interface

$$Df_s = C_y \int_I \left\{ \left[-p_f I + \frac{1}{Re} (\nabla U_f + \nabla^t U_f) \right] \cdot n \right\} \cdot \phi dS = C_y F(Re, Dq_s)$$

Static Instability - Divergence

- ***Expansion of fluid loading (small displacement Dq_s):***
 - ***Flow induced force from the motion of the interface***

$$C_y F(Re, Dq_s) = C_y F^0 + C_y \left(\frac{\partial F}{\partial Dq_s} \right) Dq_s + \dots$$

- ***Proportional to modal displacement $q_s \rightarrow$ Stiffness force***
 \rightarrow The solid behaves like if it was attached to a spring with a stiffness

$$k_f = -C_y \left(\frac{\partial F}{\partial Dq_s} \right)$$

- ***The flow induced stiffness depends only on how the fluid reacts to the solid displacement and may be positive or negative***

Static Instability - Divergence

- ***Solid motion:***

$$U_R^2 \frac{d^2 q_s}{dt_f^2} + q_s = f_s$$

- ***Using t_s instead of t_f for the reference time***

$$\Rightarrow \frac{d^2 q_s}{dt_s^2} + q_s = f_s$$

- ***After incorporating the flow induced stiffness:***

$$\frac{d^2 q_s}{dt_s^2} + \left(1 - C_y \left(\frac{\partial F}{\partial D q_s} \right) \right) q_s = 0$$

Static Instability - Divergence

- ***Solid motion:***

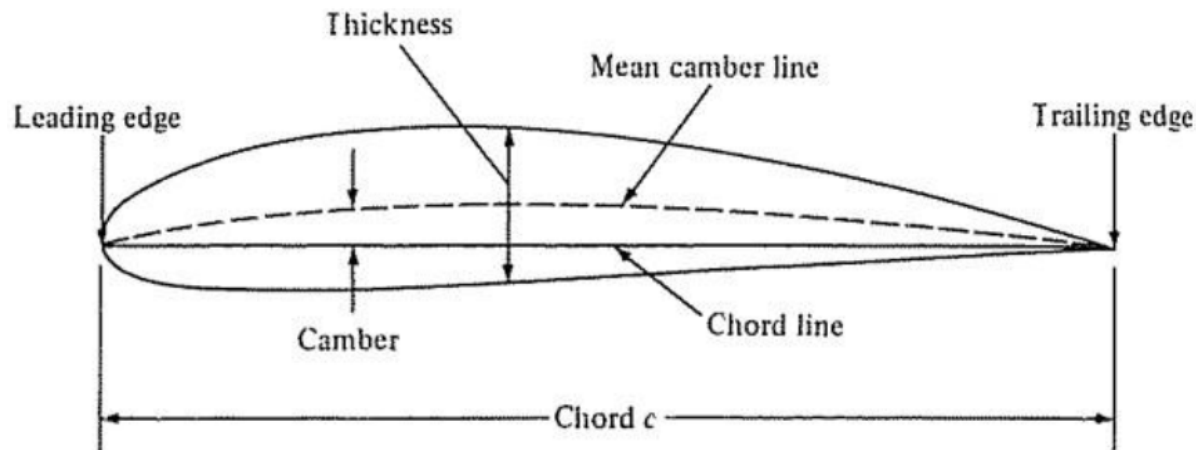
$$\frac{d^2 q_s}{dt_s^2} + \left(1 - C_y \left(\frac{\partial F}{\partial D q_s} \right) \right) q_s = 0$$

- ***Flow induced stiffness proportional to U_0^2 ($C_y = \rho U_0^2 / E$)***
- ***If $\left(\frac{\partial F}{\partial D q_s} \right) > 0$, as the flow velocity is increased, the total stiffness decreases as well as the resonance frequency***
- ***When $1 - C_y \left(\frac{\partial F}{\partial D q_s} \right) \leq 0$***
 - Static instability, also called divergence or buckling***
 - Unbounded increase of oscillation amplitude***

Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder*):

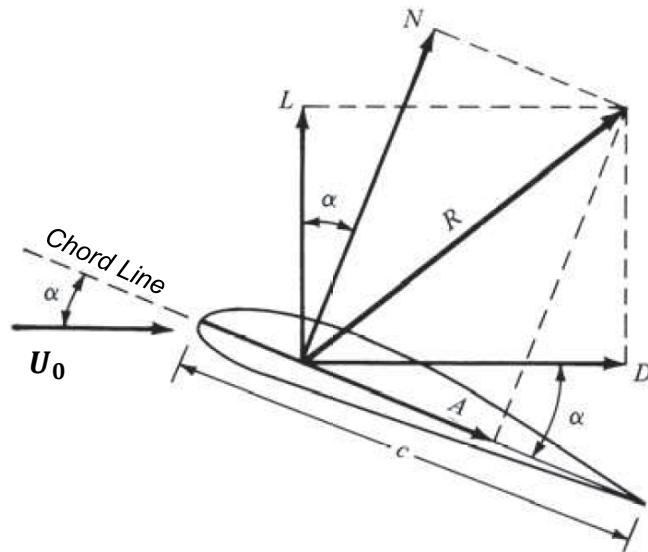
- **Nomenclature related to airfoils:**
 - **Mean camber line:** located halfway between upper and lower surfaces
 - **Chord line:** Straight line connecting the leading and trailing edges
 - **Chord length:** Distance between leading and trailing edges (along the chord line)
 - **Camber:** Maximum distance between mean camber line and chord line Measured perpendicularly to the chord line.



Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder):

- **Lift and drag:**
 - **Angle of attack, α :** Angle between relative wind and chord line
 - **Total aerodynamic force, R ,** can be decomposed into 2 components:
 - **Lift, L :** Component of aerodynamic force perpendicular to relative wind
 - **Drag, D :** Component of aerodynamic force parallel to relative wind



Lift coefficient:

$$C_L = \frac{L}{\frac{1}{2} \rho U_0^2 S} = \frac{L}{q_\infty S}$$

Drag coefficient:

$$C_D = \frac{D}{\frac{1}{2} \rho U_0^2 S} = \frac{D}{q_\infty S}$$

With

$$q_\infty = \frac{1}{2} \rho U_0^2$$

Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder):

- Lift and drag:

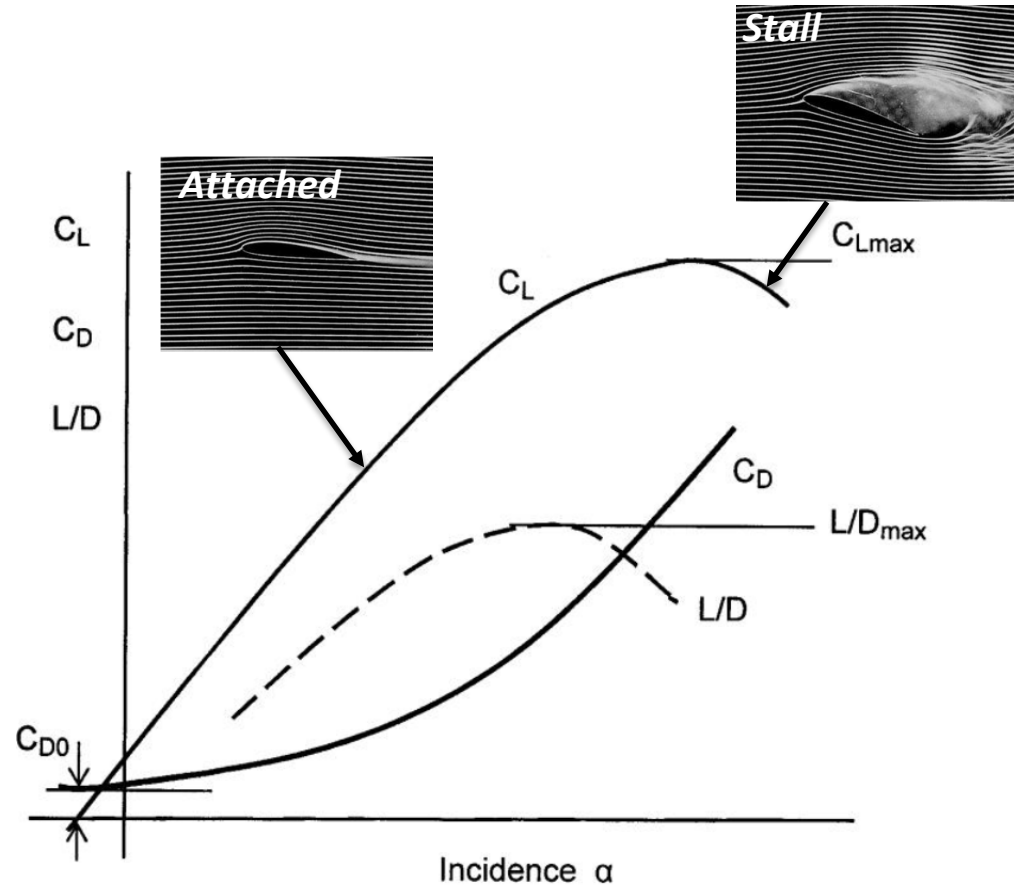
Lift coefficient (per unit length):

$$c_L = \frac{L}{\frac{1}{2} \rho U_0^2 c} = \frac{L}{q_\infty c}$$

Drag coefficient (per unit length):

$$c_D = \frac{D}{\frac{1}{2} \rho U_0^2 c} = \frac{D}{q_\infty c}$$

- Aerodynamic efficiency: $\frac{L}{D} = \frac{c_L}{c_D}$

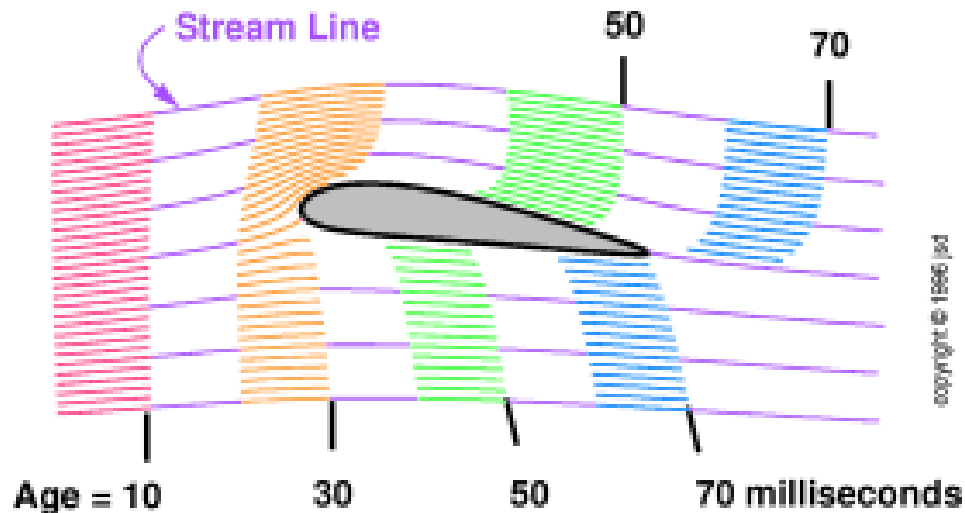


- For a given geometry, c_L and c_D are functions of incidence angle and Re number

Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder):

- **Mechanism of lift generation:**
 - *Illustration of the fluid displacement around a foil using transient injections of colored smokes (during 10 msec)*



- *The flow is accelerated on the upper surface (suction side) with respect to the lower surface (pressure side) → pressure difference → lift generation*

Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder):

- **Thin Airfoil Theory (TAT):**

- Airfoil thickness $< 12\%$ of the chord length
- Small incidence angle ($\alpha \ll \alpha_{stall}$)
- Inviscid and incompressible flow

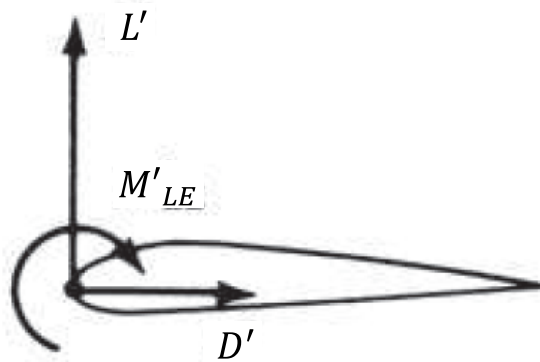
$$\rightarrow \frac{dC_L}{d\alpha} = 2\pi$$

- ***Center of pressure (CP)***: point where the moment of aerodynamic forces is zero. Its location depends on incidence angle and may be located outside of the foil. Not very practical for engineers. The center of pressure is to pressure forces what the center of gravity is to gravitational forces
- ***Aerodynamic center (AC)***: point where the moment of aerodynamic forces does not depend on incidence angle.
TAT \rightarrow AC is located $c/4$ away from the leading edge (c : chord length)

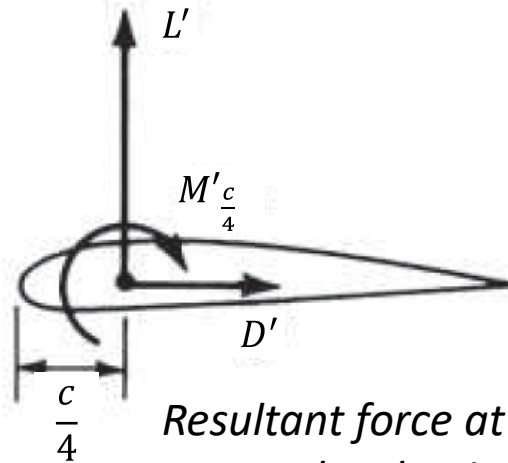
Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder):

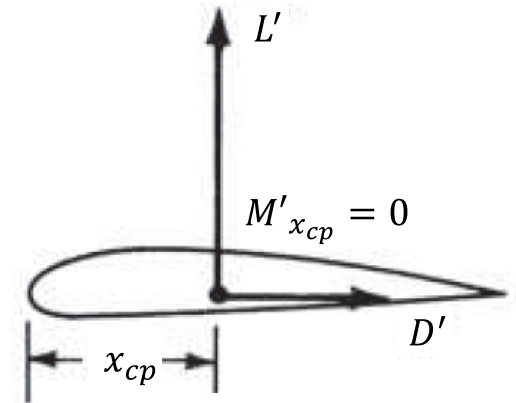
- Thin Airfoil Theory (TAT):



Resultant force at leading edge



Resultant force at quarter-chord point

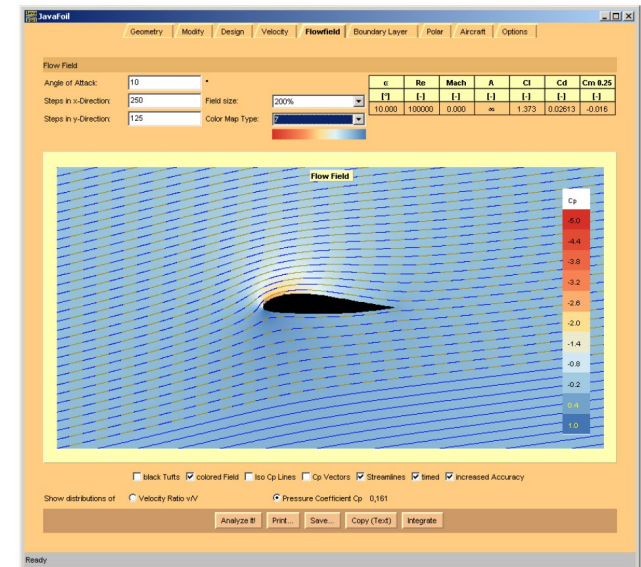


Resultant force at center of pressure

Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder):

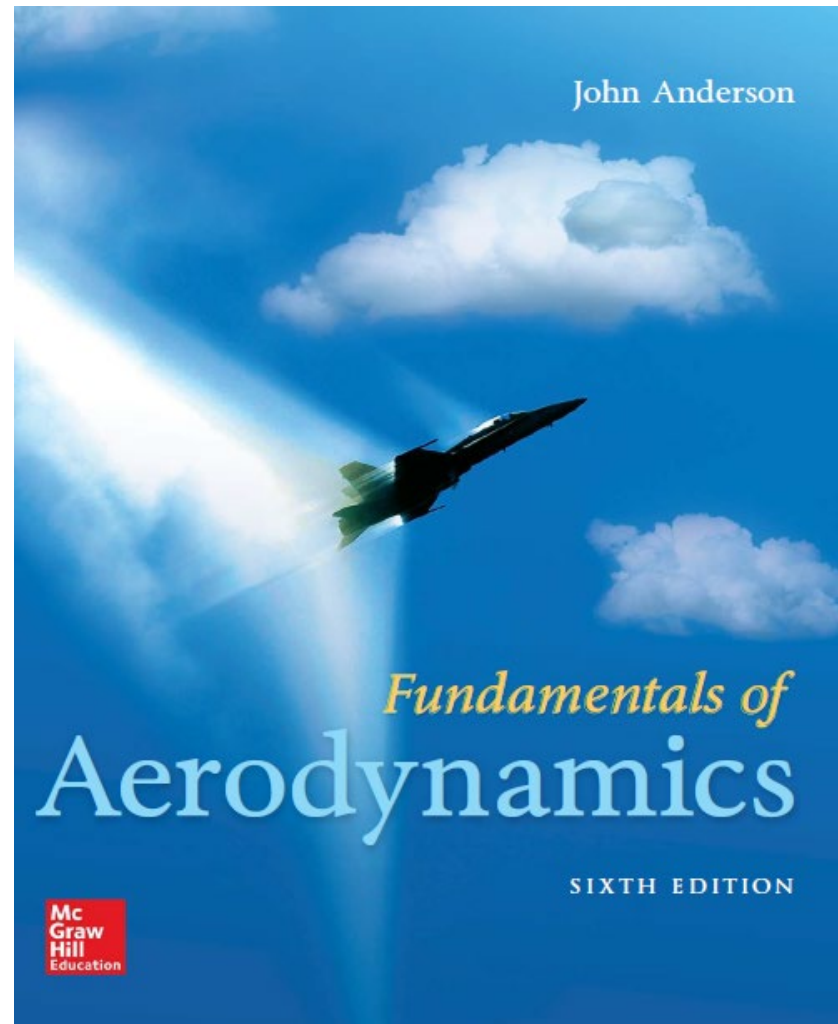
- **JAVAFOIL solver: is a relatively simple program, which uses several traditional methods for the analysis of a subsonic flow around a profiled body and provides a quick and fair estimation of the velocity and pressure distributions along with the lift and drag forces.**
- **Training:**
 - **Install JAVAFOIL in your computer and use it for a foil of simple geometry (NACA)**
 - **Analyze the flow for different operating parameters:**
 - **Plot velocity and pressure fields for different incidence angles**
 - **Plot $C_L(\alpha)$, $C_D(\alpha)$ and $C_L(C_D)$**
 - **Verify that $\frac{dC_L}{d\alpha} = 2\pi$**
 - **Verify that the moment of aerodynamic forces is independent on incidence angle, when computed at the aerodynamic center.**
 - **Examine the influence of upstream velocity**
 - **Reference:**
 - **JAVAFOIL User's Guide, M. Hepperle, 2017**
 - **https://www.mh-aerotoools.de/airfoils/jf_applet.htm**



Application: Torsional divergence of an airfoil

Basics of aerodynamics (Reminder):

- *Good reference for aerodynamics:*



Application: Torsional divergence of an airfoil

- **The case study:**

- We consider a two-dimensional airfoil restrained by a rotation spring (stiffness k_α), placed in air stream of velocity U_0 . The angle of attack, α , is the sum of the rigid foil angle, α_r , and the structural twist angle, θ . The aerodynamic center (resp. gravity center) is located at a distance e (resp. d) of the elastic center, counted positive towards the leading edge.

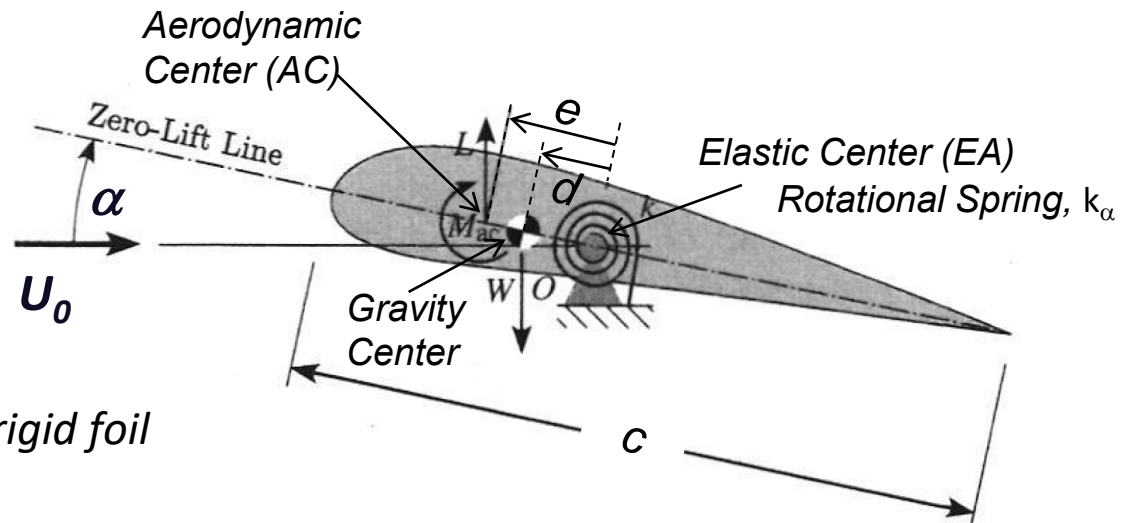
L : Lift force

W : Weight

$$\alpha = \alpha_r + \theta$$

α_r : incidence angle of a rigid foil

θ : elastic twist angle



Application: Torsional divergence of an airfoil

- **Assumption: Incompressible flow**
 - Compressibility may be taken into account using Prandtl-Glauert corrections for Mach Nb up to ~ 0.8
- **The lift always acts upstream of the elastic center ($e > 0$)**
 - an increase of the lift force leads to an increase of α , which in its turn, increases the lift !
 - Unstable

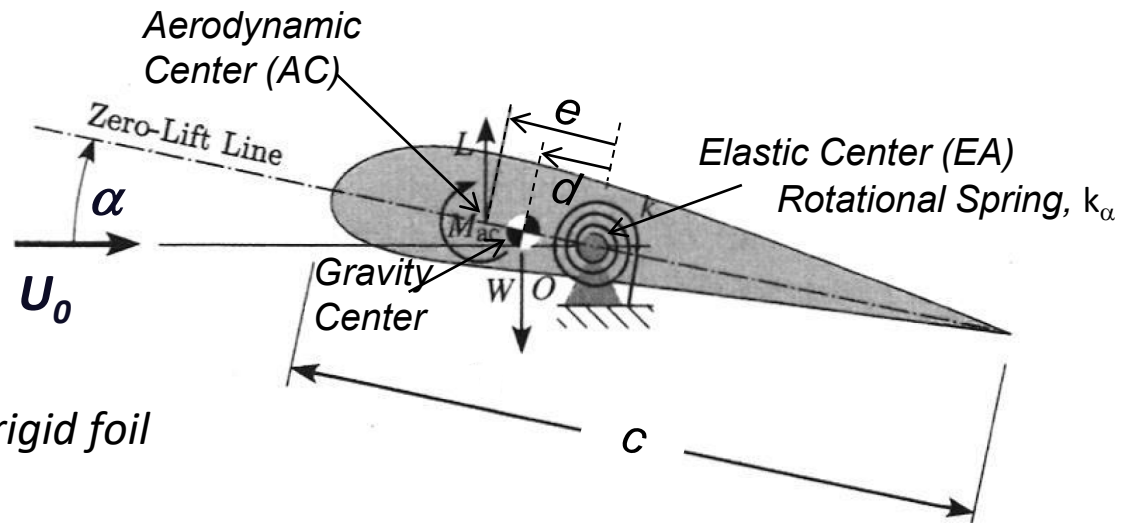
L : Lift force

W : Weight

$$\alpha = \alpha_r + \theta$$

α_r : incidence angle of a rigid foil

θ : elastic twist angle



Application: Torsional divergence of an airfoil

- We assume the airfoil is pivoted about its elastic center (EC) with an angle of attack sufficiently small, so that $\cos\alpha \sim 1$ and $\sin\alpha \sim \alpha$
Equilibrium equation (expressed at the elastic center):

$$\sum \text{moments} = 0 \quad \rightarrow \quad M_{ac} + Le - Wd - k_{\alpha}\theta = 0$$

Lift for an elastic foil: $L = q_{\infty} S C_L = q_{\infty} S C_{L,\alpha} \alpha = q_{\infty} S C_{L,\alpha} (\alpha_r + \theta)$

with: $q_{\infty} = \frac{1}{2} \rho U_0^2$, $C_{L,\alpha} = \frac{dC_L}{d\alpha}$ and S is the airfoil area

Moment of aerodynamic forces: $M_{ac} = q_{\infty} S c C_{M_{ac}}$

Thin airfoil assumption: $C_{L,\alpha}$ & $C_{M_{ac}}$ constant ($C_{M_{ac}} = 0$ for symmetric foil)

Application: Torsional divergence of an airfoil

- The equilibrium equation becomes:

$$q_{\infty} S c C_{M_{ac}} + q_{\infty} S e C_{L,\alpha} (\alpha_r + \theta) - W d = k_{\alpha} \theta$$

$$\rightarrow \theta = \frac{q_{\infty} S c C_{M_{ac}} + q_{\infty} S e C_{L,\alpha} \alpha_r - W d}{k_{\alpha} - q_{\infty} S e C_{L,\alpha}}$$

- Divergence occurs when the denominator vanishes:

$$\rightarrow q_{\infty} = q_d = \frac{k_{\alpha}}{S e C_{L,\alpha}}$$

- *Thin airfoil theory (2D airfoil) :*

$$C_{L,\alpha} = 2\pi \implies q_d = \frac{k_{\alpha}}{2\pi S e}$$

Application: Torsional divergence of an airfoil

- Divergence speed U_D :

$$q_D = \frac{1}{2} \rho U_D^2 = \frac{k_\alpha}{SeC_{L,\alpha}}$$

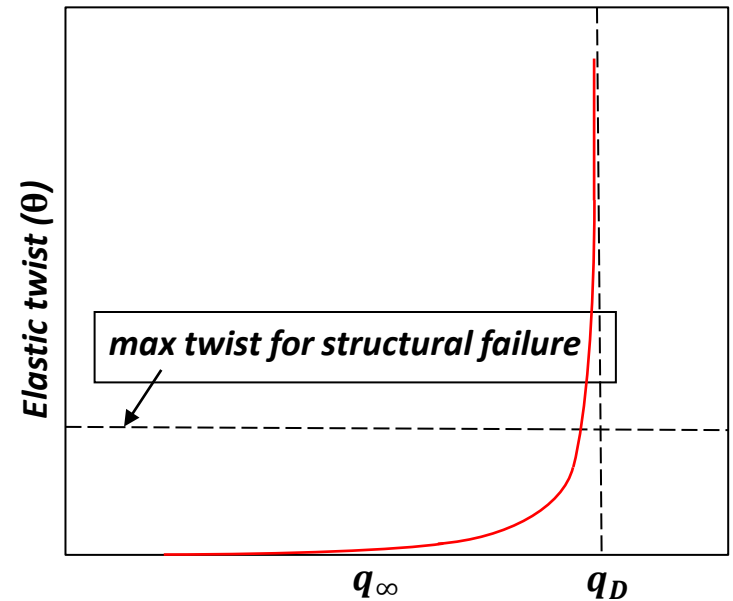
$$U_D = \sqrt{\frac{2k_\alpha}{\rho SeC_{L,\alpha}}}$$

- Assumption:

- Symmetric foil : $M_{ac} = 0$
- Gravity center = elastic center ($d = 0$)

Then the twist angle may be written as:

$$\theta = \frac{\alpha_r}{\frac{q_D}{q_\infty} - 1} \quad \lim_{q_\infty \rightarrow q_D} \theta = +\infty$$



Elastic twist vs. Dynamic pressure

Application: Torsional divergence of an airfoil

Interpretation of divergence:

- **Twist angle:**

$$\theta = \frac{q_\infty Sc C_{M_{ac}} + q_\infty Se C_{L,\alpha} \alpha_r - Wd}{k_\alpha - q_\infty Se C_{L,\alpha}}$$

$$\Rightarrow \theta = \frac{q_\infty Sc C_{M_{ac}} - Wd}{k_\alpha - q_\infty Se C_{L,\alpha}} + \frac{q_\infty Se C_{L,\alpha}}{k_\alpha - q_\infty Se C_{L,\alpha}} \alpha_r$$

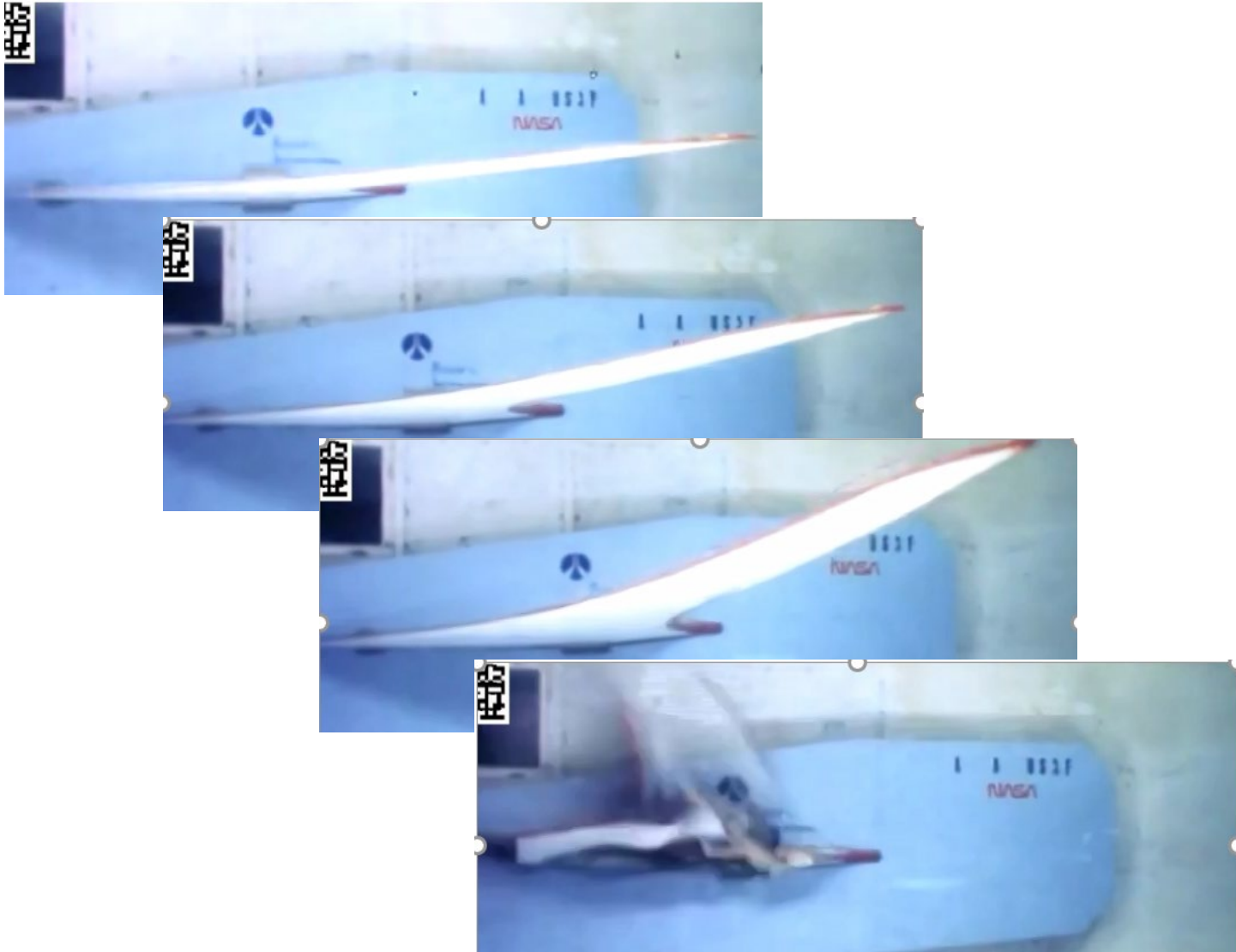
- **Assumption:**

- **Symmetric foil : $M_{AC} = 0$ and Gravity center = elastic center ($d = 0$)**
Then the twist angle may be written as:

$$\theta = \frac{q_\infty Se C_{L,\alpha}}{k_\alpha - q_\infty Se C_{L,\alpha}} \alpha_r$$

- **For a given q_∞ , the twist angle varies linearly with α_r , but the slope tends to infinity as q_∞ tends to q_D**

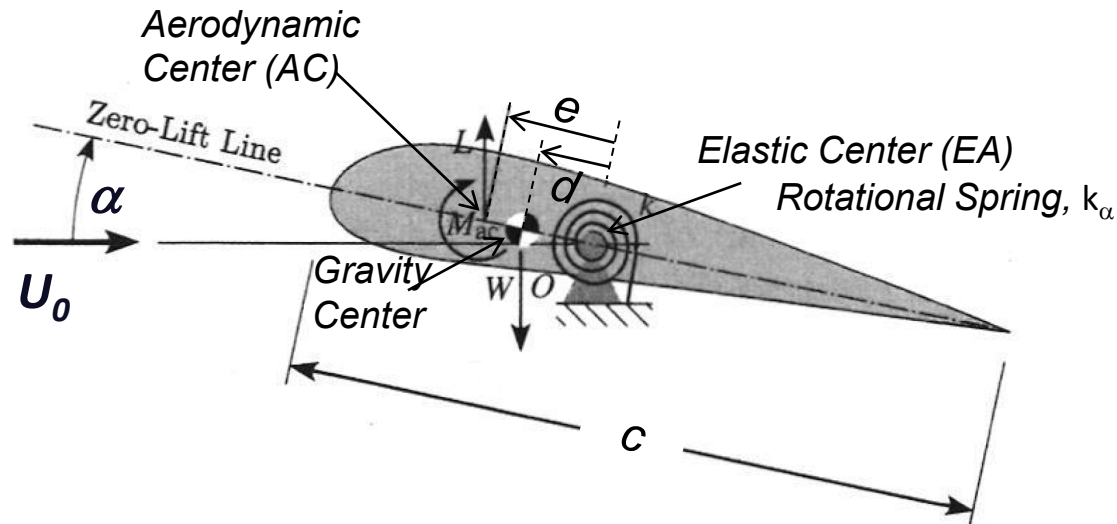
Illustration of a static instability (divergence) of an airfoil



Application: Torsional divergence of an airfoil

- Divergence might be avoided by locating the aerodynamic center between the elastic center and the trailing edge ($e < 0$)
 - In this case, an increase of the lift decreases α , which in its turn decreases the lift (Stable)
 - This solution is hardly feasible in practice

$$q_d = \frac{k_\alpha}{SeC_{L,\alpha}}$$



Application: Torsional divergence of an airfoil

- Divergence might be avoided by locating the aerodynamic center between the elastic center and the trailing edge ($e < 0$)
 - In this case, an increase of the lift decreases α , which in its turn decreases the lift (Stable)
 - This solution is hardly feasible in practice
- Another alternative is to increase the wing twist stiffness, although this solution typically penalizes the total weight of the wing
- Divergence is disastrous and design rules in aeronautics require a divergence speed 1.15 times higher than the maximum diving speed

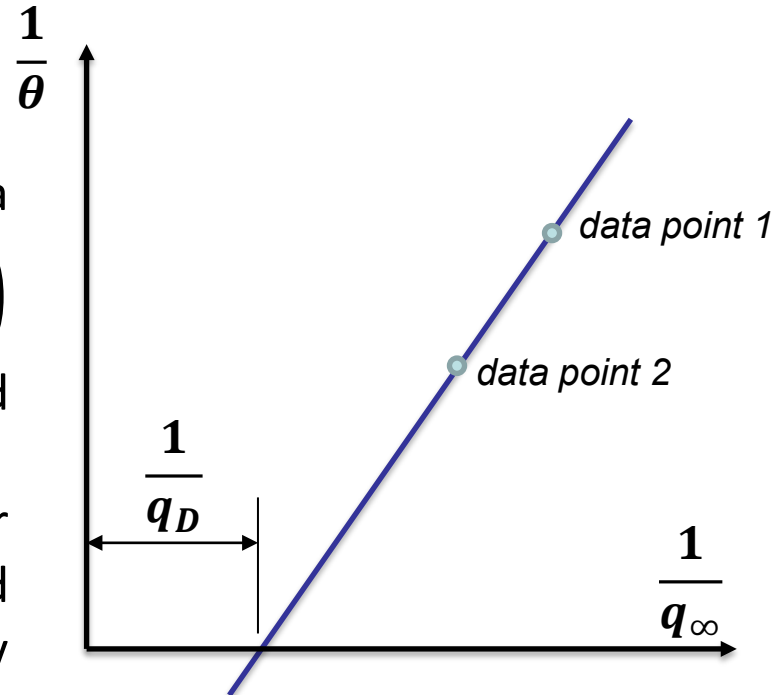
$$q_d = \frac{k_\alpha}{SeC_{L,\alpha}}$$

Application: Torsional divergence of an airfoil

- Experimental evaluation of divergence speed ? (non destructive method)
 - Inverting the expression of the twist angle leads to a linear relationship between $\frac{1}{\theta}$ and $\frac{1}{q_\infty}$, whose slope is $\frac{q_D}{\alpha_r}$:

$$\theta = \frac{\alpha_r}{\frac{q_D}{q_\infty} - 1} \Rightarrow \frac{1}{\theta} = \frac{q_D}{\alpha_r} \left(\frac{1}{q_\infty} - \frac{1}{q_D} \right)$$

- Non destructive tests may be carried out in a wind tunnel to plot the line $\frac{1}{\theta} = f\left(\frac{1}{q_\infty}\right)$ (known as divergence Southwell Plot) and estimate the divergence speed.
- 2 experimental data points (at speeds lower than U_D) are enough to draw the line and have an estimate q_D in a non destructive way

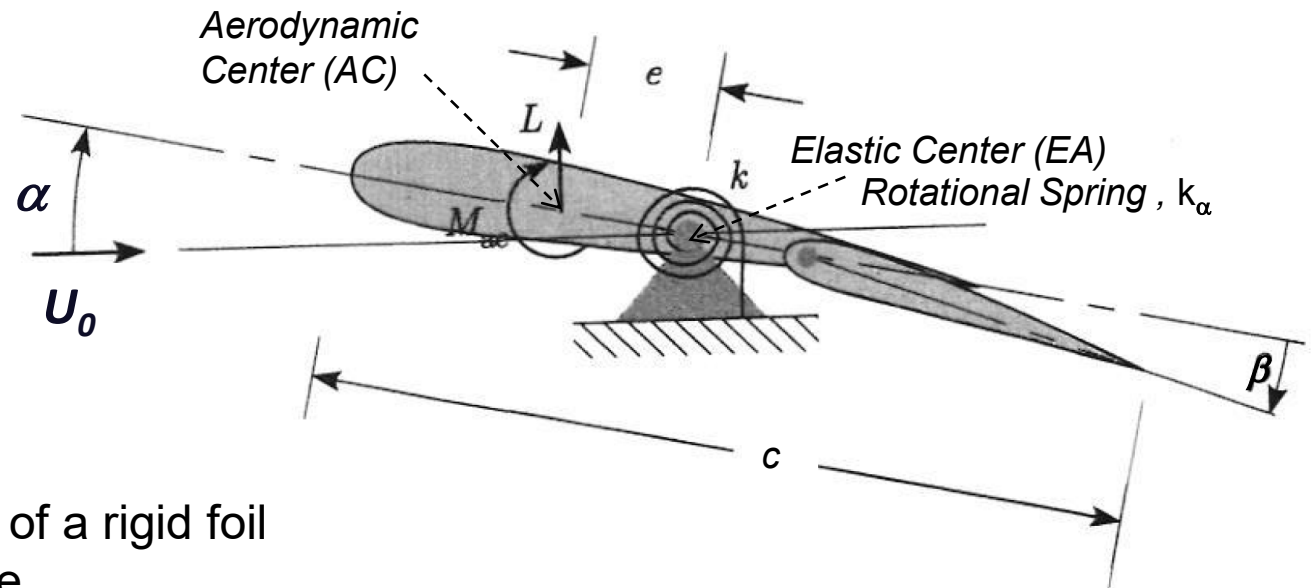


Application: Torsional divergence of an airfoil Aileron reversal

- The so-called “Aileron Reversal” problem occurs when the response of an aileron is opposite to the one expected, because of structural deformation of the wing:
 - For example, wing torsional flexibility, can cause ailerons to lose their effectiveness as the dynamic pressure increases beyond “reversal dynamic pressure”
- Dangerous consequences:
 - The pilot cannot control the aircraft in the usual way.
 - Impossible to execute evasive maneuvers.

Application: Torsional divergence of an airfoil Aileron reversal

- We consider the same 2D wing, with a flap attached to its trailing edge at an angle β , which may be arbitrarily set by the flight-control system.
- For the sake of simplicity, the weight is ignored.



L: Lift force

$$\alpha = \alpha_r + \theta$$

α_r : incidence angle of a rigid foil

θ : elastic twist angle

Application: Torsional divergence of an airfoil Aileron reversal

- Moment equilibrium about the pivot: $M_{ac} + Le = k_\alpha \theta$

With: $L = q_\infty S C_L$ $M_{ac} = q_\infty S c C_{M_{ac}}$

The flap changes the effective camber of the wing. Owing to linear theory (small α and β), the resulting changes in lift and pitching moment may be approximated as follows:

$$C_L = C_{L,\alpha} \alpha + C_{L,\beta} \beta \quad \text{and} \quad C_{M_{ac}} = C_{M_0} + C_{M,\beta} \beta$$

$\alpha = \alpha_r + \theta$ \uparrow
(Foil without the flap)

Application: Torsional divergence of an airfoil Aileron reversal

- After combining the previous relations, with the assumption of a symmetric wing ($\mathbf{C}_{M_0} = \mathbf{0}$), the twist angle θ may be written as follows:

$$\theta = \frac{q_\infty S [eC_{L,\alpha} \alpha_r + (eC_{L,\beta} + cC_{M,\beta}) \beta]}{k_\alpha - eq_\infty S C_{L,\alpha}}$$

- The twist angle θ , due to the flexibility of the wing, is a function of β
- Note: The divergence dynamic pressure ($q_D = \frac{k_\alpha}{eS C_{L,\alpha}}$) is unaffected by the flap since it does not depend on β .

Application: Torsional divergence of an airfoil Aileron reversal

- The lift equation ($C_L = C_{L,\alpha}(\alpha_r + \theta) + C_{L,\beta}\beta$) leads to the following relation of the lift force, using the previous expression of θ :

$$L = \frac{q_\infty S \left[C_{L,\alpha} \alpha_r + \left(C_{L,\beta} + \frac{c q_\infty S C_{L,\alpha} C_{M,\beta}}{k_\alpha} \right) \beta \right]}{1 - \frac{e q_\infty S C_{L,\alpha}}{k_\alpha}}$$

- From the coefficient of β , the lift is a function of β in 2 counteracting ways:
 - The 1st term is purely aerodynamic: an increase of β leads to an increase of lift because of a change in the effective camber
 - 2nd term: aeroelastic effects
 Since $C_{M,\beta} < 0$, an increase of β induces a nose down pitching moment, which tends to decrease θ and consequently the lift

Application: Torsional divergence of an airfoil Aileron reversal

- At low speed, the purely aerodynamic increase in lift overpowers the aeroelastic tendency to decrease the lift → the lift increases with β (expected behavior)
- As the speed increases, the aeroelastic term rapidly increases and there is a risk that it cancels the aerodynamic effect (reversal):

$$\frac{\partial L}{\partial \beta} = \frac{q_\infty S \left(C_{L,\beta} + \frac{c q_\infty S C_{L,\alpha} C_{M,\beta}}{k_\alpha} \right)}{1 - \frac{e q_\infty S C_{L,\alpha}}{k_\alpha}} = 0$$

→ Reversal dynamic pressure, q_r , and reversal speed, U_r , read:

$$q_r = -\frac{k_\alpha C_{L,\beta}}{c S C_{L,\alpha} C_{M,\beta}} \quad U_r = \sqrt{-\frac{2 k_\alpha C_{L,\beta}}{\rho c S C_{L,\alpha} C_{M,\beta}}}$$

- Note: q_r and U_r do not depend on the location of the aeroelastic center (e)

Application: Torsional divergence of an airfoil Aileron reversal

- The lift may be written as follows:

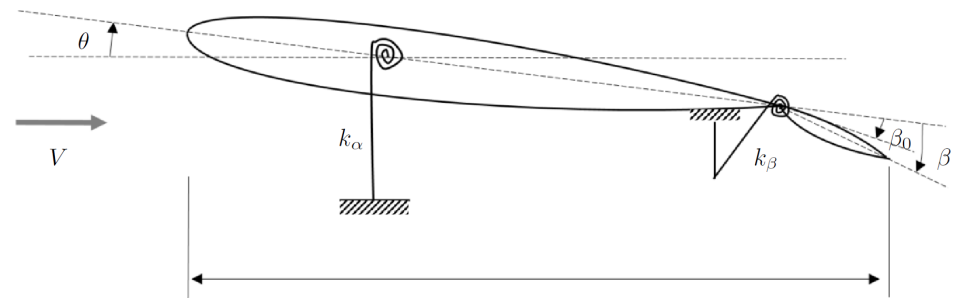
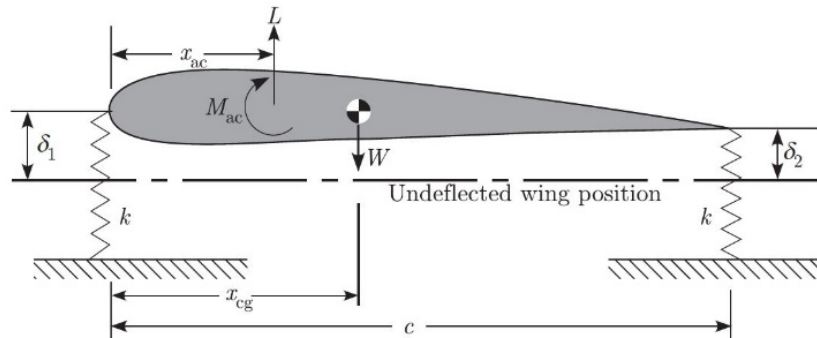
$$L = \frac{q_\infty S \left[C_{L,\alpha} \alpha_r + C_{L,\beta} \left(1 - \frac{q_\infty}{q_R} \right) \beta \right]}{1 - \frac{q_\infty}{q_D}}$$

- Aileron's lift efficiency η :
 - Definition: Ratio of lift changes for aeroelastic and rigid wings

$$\eta = \frac{\left(\frac{\Delta L}{\Delta \beta} \right)_{elastic}}{\left(\frac{\Delta L}{\Delta \beta} \right)_{rigid}} = \frac{1 - \frac{q_\infty}{q_R}}{1 - \frac{q_\infty}{q_D}}$$

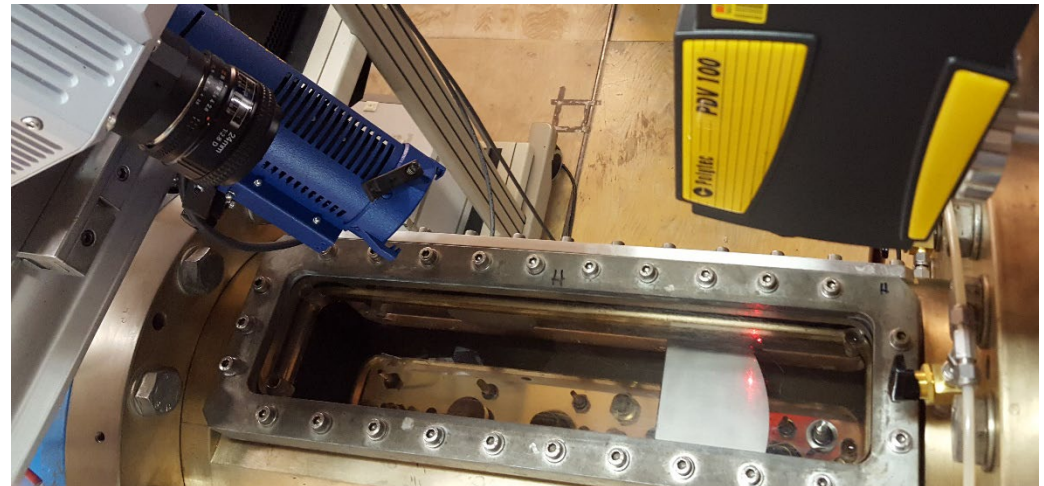
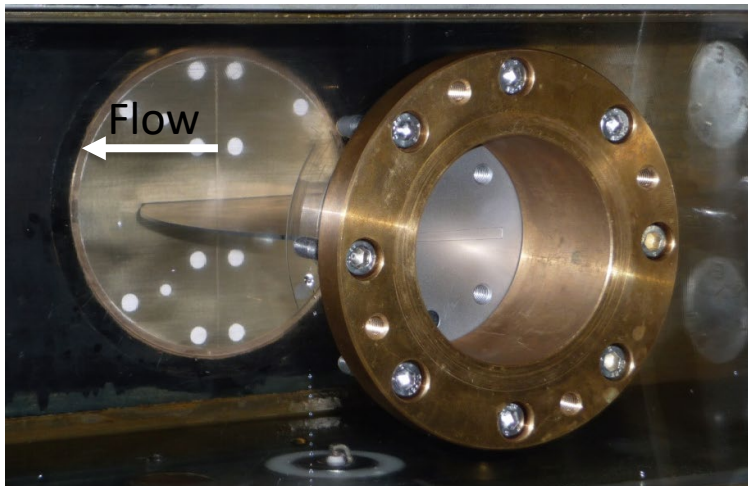
Application: Torsional divergence of an airfoil

- **Other configurations of airfoil divergence (Exercise session)**



Torsional divergence in real life

- Static instability (divergence) is always associated with aeroelasticity:
 - Flexible structures in air flow (e.g. wings of airplanes, ...)
 - Divergence is almost “never” considered in hydrodynamic applications:
 - Marine propellers, hydraulic turbines and pumps
 - The question is recently raised because the blades of hydraulic machines are getting thinner and thinner.
 - Tests are conducted in hydrodynamic tunnel for a simplified geometry
 - Reproduce and investigate the occurrence of divergence
 - Validation of numerical simulation



Torsional divergence in real life

- Thin leading edge hydrofoil (SP 80 foil):
 - Foil with sharp leading edge (supercavitating foil)
 - Tested in EPFL Cavitation Tunnel with air injection
 - As the flow speed is increased, divergence occurs and the foil is severely bent

