AEROELASTICITY AND FLUID-STRUCTURE INTERACTION

Chapter 3:

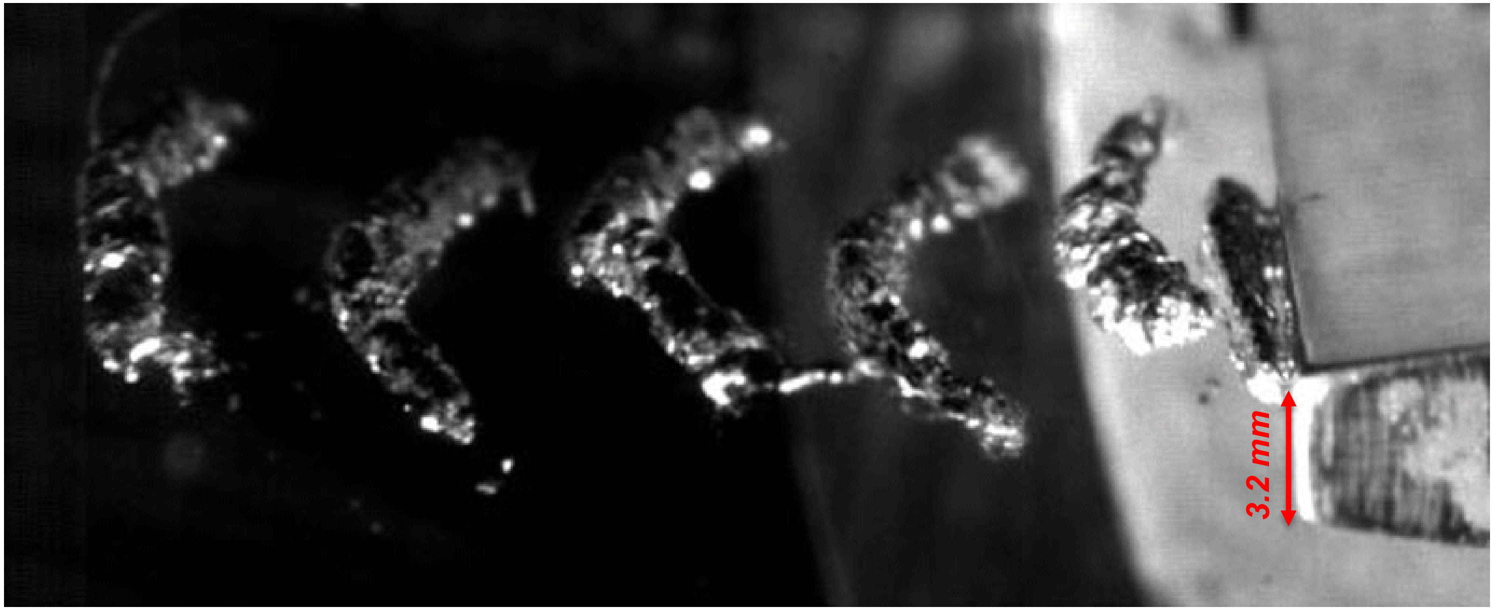
Small Reduced Velocity: Added Mass, Added Stiffness

Introduction

- ***Added mass and added stiffness are of major importance in mechanical engineering design:***
  - ***May induce a significant alteration of the mechanical response (mode shape: amplitude and frequency)***
  - ***Added mass: always defined as the mass of displaced fluid***
    - ***While accelerating or vibrating, a structure must displace the surrounding fluid (air, water, …)***
    - ***→ added inertia***

Introduction

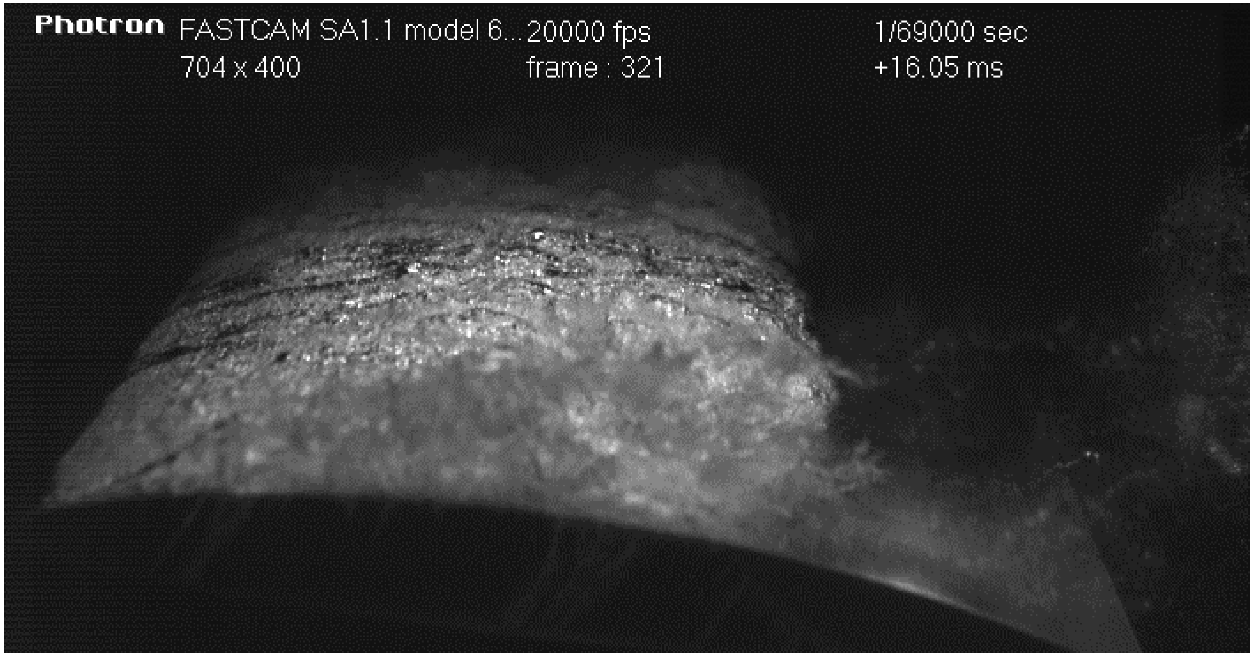
Case of vortex induced vibration in water flow:
Strong interaction of vortex shedding and 1st torsional mode of vibration



Influence of the flow on the mechanical response (resonance frequencies) ?

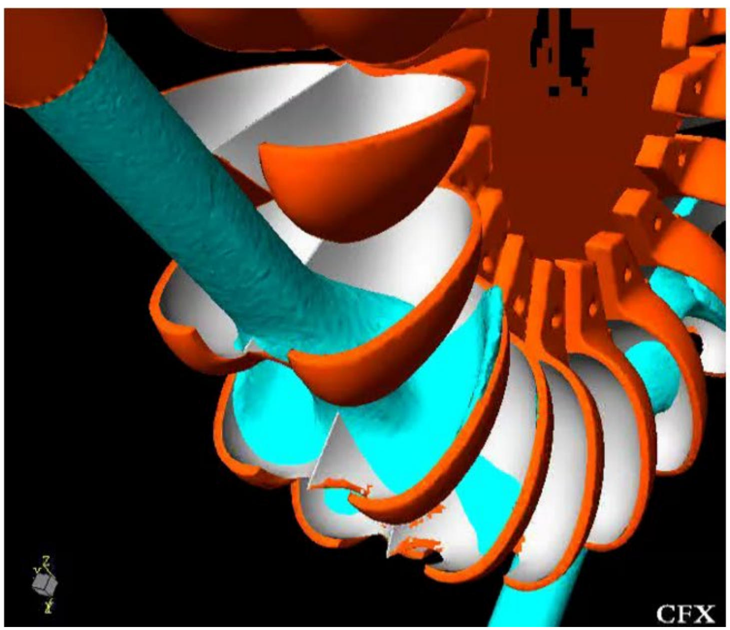
Introduction
Case of leading edge cavitation

- ***The presence of an unstable vapor cavity in a liquid flow may lead to a significant change in the mechanical response of the hydrofoil***
- ***As it vibrates, the hydrofoil displaces a fluid with a density that varies in time***



Introduction
Case of Pelton Turbine

The buckets of a pelton turbine are sequentially hit by the jet
→ They are alternately wet and dry → Mechanical response ?



Introduction
Case of sailing boats

Case of lifting foils with variable immersion:
The mechanical response is highly dependent on the level of immersion

Small Reduced Velocity: Added mass, added stiffness

Time scales in solid and fluid domains:

- ***Fluid: Travel time over a distance L at velocity U_0 :***

$$T_{fluid} = \frac{L}{U_0}$$

- ***Solid:***
  - ***Travel time over a distance L at the elastic wave celerity***

$$T_{solid} = \frac{L}{\sqrt{E/\rho_s}} = \frac{L}{c}$$

  - ***Or, the oscillation period of a given mode shape***

$$T_{solid} = \sqrt{m/k}$$

Small Reduced Velocity: Added mass, added stiffness

Small reduced velocity: $\boldsymbol{U_R \ll 1}$

$$U_R = \frac{T_{solid}}{T_{fluid}} = \frac{U_0}{c} \quad or \quad U_R = \frac{\sqrt{m/k}}{L/U_0} = \frac{U_0}{L}\sqrt{\frac{m}{k}}$$

$$U_R \ll 1 \qquad \Rightarrow \qquad T_{solid} \ll T_{fluid}$$

$\boldsymbol{U_R \ll 1}$ $\Rightarrow$ ***The solid evolves in an almost still fluid***

Example:

- ***Oscillation of a boat in a harbor***
- ***Vibration of a hydraulic dam***

Small Reduced Velocity: Added mass, added stiffness

Small reduced velocity:

Boundary conditions:

- ***At the solid-fluid interface (Kinematic conditions):***

$$U = \frac{\partial \xi}{\partial t} \approx \frac{\xi_0}{T_{solid}}$$

ξ_0: ***Reference Displacement***
U : ***Fluid velocity @ fluid/solid interface***

- ***At the fluid boundary:***

$$U = O(U_0)$$

$$U_R \ll 1 \Rightarrow U_0 \ll \frac{\xi_0}{T_{solid}} \Rightarrow \frac{U_0 T_{solid}}{L} \ll \frac{\xi_0}{L} \Rightarrow U_R \ll D$$

Displacement Nb

Small Reduced Velocity: Added mass, added stiffness

Small reduced velocity:

Dimensionless numbers :

$$Re = \frac{\rho U_0 L}{\mu} \qquad Fr = \frac{U_0}{\sqrt{gL}} \qquad Cy = \frac{\rho U_0^2}{E} \qquad U_R = \frac{U_0}{c}$$

U_0 is no more relevant with the hypothesis of small reduced velocity
→ Alternate choice for dimensionless parameters:

Instead of U_0, we may use the wave celerity in the solid c or $\frac{L}{T_{solid}}$

Small Reduced Velocity: Added mass, added stiffness

Small reduced velocity:

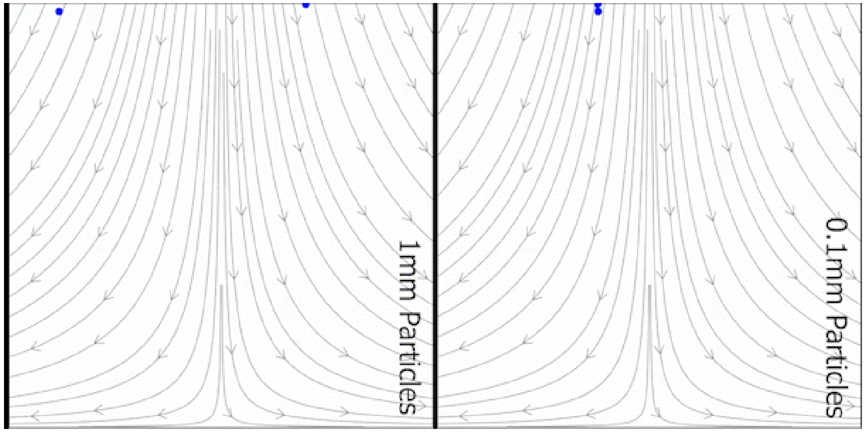
Alternative for dimensionless numbers (using c instead of U_0):

$$Re = \frac{\rho U_0 L}{\mu} \rightarrow \quad S_T = \frac{\rho c L}{\mu} = \frac{Re}{U_R} \quad : Stokes\ Nb$$

- ***Stokes Nb is the ratio of the characteristic time of a solid particle (or drop) to the characteristic time of the flow. It is also defined as the ratio of kinetic energy of the solid to the energy dissipated in the fluid by viscosity.***
- ***Example: in PIV measurements, Stokes Nb is used to measure the ability of solid particles to follow the flow streamlines:***

$S_T \ll 1 \Longrightarrow$ ***the particles follow the streamlines and can trace the flow***

$S_T \gg 1 \Longrightarrow$ ***the particles detach from the flow as it accelerates/decelerate***



Small Reduced Velocity: Added mass, added stiffness

Small reduced velocity:

Alternative for dimensionless numbers (using c instead of U_0):

$$Re = \frac{\rho U_0 L}{\mu} \rightarrow \quad S_T = \frac{\rho c L}{\mu} = \frac{Re}{U_R} \qquad : \textit{Stokes Nb}$$

$$Fr = \frac{U_0}{\sqrt{gL}} \rightarrow \quad F_D = \frac{c}{\sqrt{gL}} = \frac{Fr}{U_R} \qquad : \textit{Dynamic Froude Nb}$$

$$Cy = \frac{\rho {U_0}^2}{E} \rightarrow \quad M = \frac{\rho c^2}{E} = \frac{Cy}{U_R^2} = \frac{\rho}{\rho_s} \qquad : \textit{Mass Nb}$$

$$U_R = \frac{U_0}{c} = \frac{U_0}{L/T_{solid}} \qquad : \textit{Reduced Velocity}$$

Small Reduced Velocity: Added mass, added stiffness

Small reduced velocity:

Alternative for dimensionless numbers (using L/T_{solid} instead of U_0):

$$Re = \frac{\rho U_0 L}{\mu} \rightarrow \quad S_T = \frac{\rho L^2}{\mu T_{solid}} = \frac{Re}{U_R} \qquad : \textit{Stokes Nb}$$

$$Fr = \frac{U_0}{\sqrt{gL}} \rightarrow \quad F_D = \frac{L}{T_{solid}\sqrt{gL}} = \frac{Fr}{U_R} \qquad : \textit{Dynamic Froude Nb}$$

$$Cy = \frac{\rho {U_0}^2}{E} \rightarrow \quad M = \frac{\rho c^2}{E} = \frac{Cy}{U_R^2} = \frac{\rho}{\rho_s} \qquad : \textit{Mass Nb}$$

$$U_R = \frac{U_0}{c} = \frac{U_0}{L/T_{solid}} \qquad : \textit{Reduced Velocity}$$

Small Reduced Velocity: Added mass, added stiffness

Small reduced velocity:

- ***Equations of motion of the fluid and solid:***
    - ***Dimensionless variables:***

Fluid

$$x_f = \frac{x}{L}$$

$$U_f = \frac{U}{U_0} \rightarrow U_f = \frac{U}{c}$$

$$p_f = \frac{p}{\rho U_0^2} \rightarrow p_f = \frac{p}{\rho c^2}$$

$$t_s = \frac{t}{T_{solid}}$$

Solid

$$x_s = \frac{x}{L}$$

$$q_s = \frac{q}{\xi_0}$$

$$f_s = \frac{f}{k\xi_0}$$

- ***In the Solid:***

$$\frac{d^2 q_s}{dt_s^2} + q_s = f_s$$

Small Reduced Velocity: Added mass, added stiffness

Small reduced velocity:

- ***Equations of motion of the fluid and solid:***
  - ***In the fluid (Navier-Stokes):***

$$\vec{\nabla}\vec{U} = 0 \qquad \rho\frac{d\vec{U}}{dt} = -\rho g\vec{e_z} - \vec{\nabla}p + \mu\Delta\vec{U}$$

Dimensionless equations – General formulation (See Chapter 2):

$$\vec{\nabla}\overrightarrow{U_f} = 0 \qquad \frac{1}{U_R}\frac{d\overrightarrow{U_f}}{dt_s} = -\frac{1}{Fr^2}\vec{e_z} - \vec{\nabla}p_f + \frac{1}{Re}\Delta\overrightarrow{U_f}$$

Small reduced velocity, $U_R \ll 1$, (with $U_f = U/c$ and $p_f = p/\rho c^2$)

$$\Longrightarrow \quad \vec{\nabla}\overrightarrow{U_f} = 0 \qquad \frac{d\overrightarrow{U_f}}{dt_s} = -\frac{1}{F_D^2}\vec{e_z} - \vec{\nabla}p_f + \frac{1}{S_T}\Delta\overrightarrow{U_f}$$

Dimensionless equations of fluid and solid motions

- ***Kinematic & dynamic conditions at the fluid-solid interface (see Chap. 2):***

$$U_R U_f = D\frac{dq_s}{dt_s}\phi(x)$$

$$\int_{Interface}\left\{Cy\left[-p_f I + \frac{1}{Re}\left(\nabla U_f + \nabla^t U_f\right)\right].n\right\}.\phi dS = Df_s$$

- ***After replacing U_0 by c ($U_f = U/c$ and $p_f = p/\rho c^2$)***

$$U_f = D\frac{dq_s}{dt_s}\phi(x)$$

$$\int_{Interface}\left\{M\left[-p_f I + \frac{1}{S_T}\left(\nabla U_f + \nabla^t U_f\right)\right].n\right\}.\phi dS = Df_s$$

Dimensionless equations of fluid and solid motions

- ***Additional assumption:***
  - ***Small displacement number:*** $$D = \frac{\xi_0}{L} \ll 1$$

"Characteristic displacement/deformation of the solid (ξ_0) small, compared to its characteristic length (L)"

→ Expansions in D
In the fluid (pressure and velocity) and in the solid (displacement):

$$P = P_0 + Dp + \cdots$$
$$U = U_0 + Du + \cdots = 0 + Du + \cdots$$

$$\xi = 0 + Dq\phi$$

Small Reduced Velocity: Added mass, added stiffness

Dimensionless equations of fluid and solid motions

- ***Linearized form - Zero order in D:***

$$\begin{cases} \nabla U_f = 0 \\ \dfrac{dU_f}{dt_s} = -\dfrac{1}{F_D^2}\overrightarrow{e_z} - \nabla p_f + \dfrac{1}{S_T}\Delta U_f \end{cases} \Rightarrow \begin{cases} \nabla U_f = \nabla U_0 = \nabla 0 = 0 \\ 0 = -\dfrac{1}{F_D^2}\overrightarrow{e_z} - \vec{\nabla} P_0 + 0 \end{cases}$$

$$\underbrace{-\frac{1}{F_D^2}\overrightarrow{e_z} - \vec{\nabla} P_0 = 0}_{\textit{Non dimensional}} \quad \Rightarrow \quad \underbrace{\frac{gL}{c^2} = -\frac{L}{\rho c^2}\frac{\partial p}{\partial z}}_{\textit{Dimensional}} \Rightarrow \quad \frac{\partial p}{\partial z} = -\rho g$$

- ***We obtain the hydrostatic equation relating the pressure gradient with the gravity (no fluid motion)***

Dimensionless equations of fluid and solid motions

- ***Linearized form - 1st order in D:***

$$\begin{cases} \nabla \boldsymbol{U_f} = \mathbf{0} \\ \dfrac{d\boldsymbol{U_f}}{d\boldsymbol{t_s}} = -\dfrac{\mathbf{1}}{\boldsymbol{F_D^2}}\overrightarrow{\boldsymbol{e_z}} - \nabla \boldsymbol{p_f} + \dfrac{\mathbf{1}}{\boldsymbol{S_T}}\Delta \boldsymbol{U_f} \end{cases}$$

By substituting the linearized form and keeping only the 1st order terms

$$\Rightarrow \begin{cases} \nabla \boldsymbol{u_f} = \mathbf{0} \\ \dfrac{\partial \boldsymbol{u_f}}{\partial \boldsymbol{t_s}} = -\nabla \boldsymbol{p_f} + \dfrac{\mathbf{1}}{\boldsymbol{S_T}}\Delta \boldsymbol{u_f} \end{cases}$$

- $\dfrac{d\boldsymbol{U_f}}{d\boldsymbol{t_s}}$ ***becomes*** $\dfrac{\partial \boldsymbol{u_f}}{\partial \boldsymbol{t_s}}$ ***because the convective term vanishes since it is 2nd order in D***
- ***The gravity is no more present (zero order)***

Dimensionless equations of fluid and solid motions

- ***Linearized form - 1st order in D:***
  - ***Kinematic conditions at the interface:***

$$U_f = D\frac{dq_s}{dt_s}\phi = 0 + Du_f + \cdots \quad \Rightarrow \quad u_f = \frac{dq_s}{dt_s}\phi$$

  - ***Dynamic conditions at the interface (I)***
    - ***A priori, the dynamic condition is an integral over a moving/deforming interface (beyond the scope of the course)***
    - ***In the case of pure translation of the interface ($\phi = cste$):***

$$\int_I \left\{ M\left[-P_f I + \frac{1}{S_T}\left(\nabla U_f + \nabla^t U_f\right)\right].n \right\}.\phi dS = M\phi \int_I \left[-P_f I + \frac{1}{S_T}\left(\nabla U_f + \nabla^t U_f\right)\right].n dS$$

Dimensionless equations of fluid and solid motions

- ***Linearized form - 1st order in D:***
  - ***Dynamic conditions at the interface (I)***
    - ***In the case of pure translation of the interface $(\phi) = cste$:***

$$M\phi \int_I \left[-P_f I + \frac{1}{S_T}\left(\nabla U_f + \nabla^t U_f\right)\right].n dS = D f_s$$

$$P(X + Dq\phi) = P_0(X + Dq\phi) + Dp(X + Dq\phi) + \cdots$$
$$\approx P_0(X) + Dq\phi.\nabla P_0 + Dp(X) + \cdots$$

$$U(X + Dq\phi) = 0 + Du(X) + \cdots$$

    - ***Using these relations and keeping only the 1st order in D, one may show that the dynamic condition reads:***

$$M\phi \int_I \left[-p_f I + \frac{1}{S_T}\left(\nabla u_f + \nabla^t u_f\right)\right].n dS - M q_s \phi \int_I (\nabla P_0.\phi).n dS = f_s$$

Note: The integral is now computed on the original interface where the variables are taken

Dimensionless equations of fluid and solid motions

- ***Linearized form - 1st order in D:***
  - ***Dynamic conditions at the interface ($\boldsymbol{I}$)***
    - ***In more general case (arbitrary motion of the interface) It may be demonstrated that:***

$$\underbrace{M\int_I \boldsymbol{\phi}.\left[-p_f I + \frac{1}{S_T}\left(\nabla u_f + \nabla^t u_f\right)\right].n dS}_{f_1} - \underbrace{M q_s \int_I (\nabla P_0.\boldsymbol{\phi}).(\boldsymbol{\phi}.n) dS}_{f_2} = f_s$$

$\boldsymbol{f_1}$:
Pressure & viscous forces projected on the solid (due to the flow induced by solid motion)

$\boldsymbol{f_2}$:
Depends only on 0-order of pressure Related to motion of the solid In a pre-stressed fluid

Small Reduced Velocity: Added mass, added stiffness

Dimensionless equations of fluid and solid motions

- ***Linearized form - 1st order in D:***
  - ***2nd term (f_2) in the dynamic condition at the interface (I):***

Modal displacement

Modal shape

$$\boldsymbol{f}_2 = \boldsymbol{M}\boldsymbol{q}_s \int_I (\nabla \boldsymbol{P}_0 . \boldsymbol{\phi}).(\boldsymbol{\phi}.\boldsymbol{n})\boldsymbol{dS}$$

Mass Nb

Hydrostatic pressure

$$\mathbf{0} = -\frac{\mathbf{1}}{\boldsymbol{F}_D^2}\boldsymbol{e}_z - \nabla \boldsymbol{P}_0 \Rightarrow \boldsymbol{f}_2 = \boldsymbol{q}_s \frac{\boldsymbol{M}}{\boldsymbol{F}_D^2}\int_I (\boldsymbol{\phi}.\boldsymbol{e}_z).(\boldsymbol{\phi}.\boldsymbol{n})\boldsymbol{dS}$$

Small Reduced Velocity: Added mass, added stiffness

Added Stiffness

- ***Linearized form - 1st order in D:***
  - ***2nd term (f_2) in the dynamic condition at the interface (I):***

$$f_2 = -q_s k_f \qquad \textit{with} \qquad k_f = -\frac{M}{F_D^2}\int_I (\phi . e_z).(\phi . n)dS$$

  - ***f_2 is proportional to the displacement (stiffness force)***
    k_f : Fluid Stiffness or Added Stiffness
  - ***k_f does not depend on the flow generated by the solid***
  - ***Needs to be determined once***

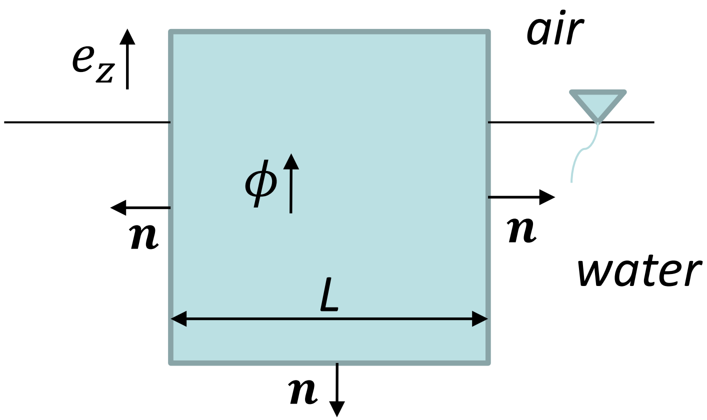
From the solid viewpoint, moving in a fluid with a pressure gradient is equivalent to be attached to an elastic spring of stiffness k_f

Evaluation of added stiffness

- ***Example: a cube floating on water (boat, iceberg, …)***

$$\int_{I} (\boldsymbol{\phi}.\boldsymbol{e_z}).(\boldsymbol{\phi}.\boldsymbol{n})d\boldsymbol{S} = \int_{Sides} (\boldsymbol{\phi}.\boldsymbol{e_z}).(\boldsymbol{\phi}.\boldsymbol{n})d\boldsymbol{S} + \int_{Bottom} (\boldsymbol{\phi}.\boldsymbol{e_z}).(\boldsymbol{\phi}.\boldsymbol{n})d\boldsymbol{S}$$

$$\int_{I} (\boldsymbol{\phi}.\boldsymbol{e_z}).(\boldsymbol{\phi}.\boldsymbol{n})d\boldsymbol{S} = \boldsymbol{0} - \boldsymbol{1} = -\boldsymbol{1}$$



L: Reference Length

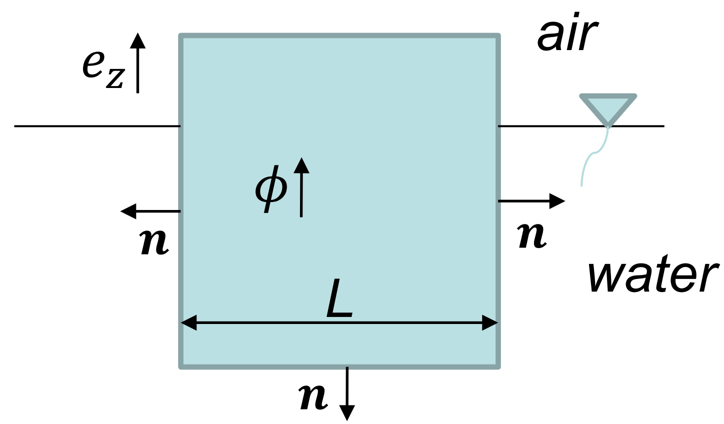
$\boldsymbol{\phi}$ *aligned on the vertical direction:* $\boldsymbol{\phi} = (\boldsymbol{0}, \boldsymbol{0}, \boldsymbol{1})$

Notice that the above integral is zero for horizontal modes (f_1=0).

Evaluation of added stiffness

- *Example: a cube floating on water (boat, iceberg, …)*

$$M = \frac{\rho_f}{\rho_s} \qquad F_D = \frac{Fr}{U_R} = \frac{1}{T_{solid}}\sqrt{\frac{L}{g}}$$



$$\int_I (\phi . e_z).(\phi . n) dS = -1$$

$$\Rightarrow k_f = -\frac{M}{F_D^2}\int_I (\phi . e_z).(\phi . n) dS = -\frac{M}{F_D^2} \times (-1) = \frac{\rho_f}{\rho_s}\frac{gT_{solid}^2}{L}$$

$$\Rightarrow \quad \frac{d^2 q_s}{dt_s^2} + q_s\left(\frac{\rho_f}{\rho_s}\frac{gT_{solid}^2}{L}\right) = 0$$

Dimensionless form

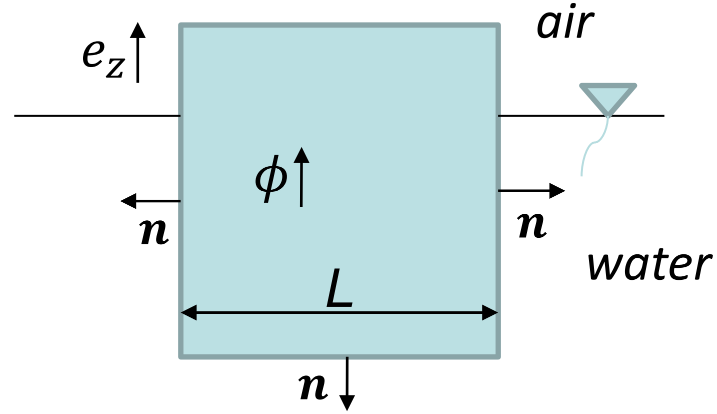
Evaluation of added stiffness

- ***Example: a cube floating on water (boat, iceberg, …)***

Dimensional form

$$t_s = \frac{t}{T_{solid}} \Rightarrow \quad \frac{d^2q}{dt^2} + q\left(\frac{\rho_f}{\rho_s}\frac{g}{L}\right) = 0$$

$$\Rightarrow \quad \omega = \sqrt{\frac{\rho_f}{\rho_s}\frac{g}{L}}$$



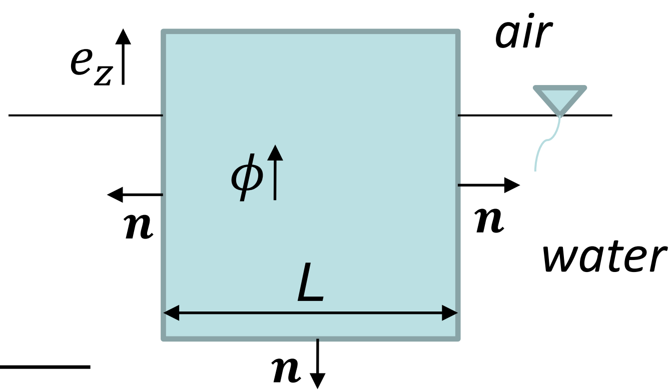
Case of an ice cube of 1 cm^3 in water → oscillation period T=0.2 second

Small Reduced Velocity: Added mass, added stiffness

Evaluation of added stiffness

- ***Example: a cube floating on water (boat, iceberg, …)***
  - ***Alternate method (Archimedes principle)***
  - ***Equation of motion:***
    q is the displacement counted from the position at rest (positive upward)

$$\rho_s L^3 \frac{d^2 q}{dt^2} = -\rho_s L^3 g - \rho_f L^2 \left(q - \underbrace{\frac{\rho_s}{\rho_f} L}_{\text{Depth at rest}} \right) g$$



$$\Rightarrow \quad \frac{d^2 q}{dt^2} + q\left(\frac{\rho_f}{\rho_s}\frac{g}{L}\right) = 0 \qquad and \qquad \omega = \sqrt{\frac{\rho_f}{\rho_s}\frac{g}{L}}$$

- ***Much more straightforward but the general formulation is a precious tool, which may help solving a variety of other more complex cases***

Dimensionless equations of fluid and solid motions

- ***Linearized form - 1st order in D:***

$$M\int_I \phi.\left[-p_f I + \frac{1}{S_T}\left(\nabla u_f + \nabla^t u_f\right)\right].ndS - Mq_s\int_I (\nabla P_0.\phi).(\phi.n)dS = f_s$$

- ***1st term (f_1) in the dynamic condition at the interface (I)***

$$f_1 = M\int_I \phi\left[-p_f I + \frac{1}{S_T}\left(\nabla u_f + \nabla^t u_f\right)\right].ndS$$

f_1 represents the reaction of the fluid to the solid motion.
It is a "feedback" term

Unlike f_2, f_1 needs the fluid motion to be solved

Dimensionless equations of fluid and solid motions

- ***Linearized form - 1st order in D:***
  - ***1st term (f_1) in the dynamic condition at the interface (I):***
  - ***Equation of fluid motion***

$$\nabla u_f = 0 \qquad \frac{\partial u_f}{\partial t_s} = -\vec{\nabla} p_f + \frac{1}{S_T} \Delta u_f$$

  - ***At the solid boundary:***

$$u_f = \dot{q}_s \phi$$

$$\underbrace{M \int_I \phi . \left[-p_f I + \frac{1}{S_T} \left(\nabla u_f + \nabla^t u_f \right) \right] . n dS}_{f_1\text{: Pressure \& Viscous load, Feedback term}} + \underbrace{q_s k_f}_{f_2\text{: Stiffness force}} = f$$

f_1: Pressure & Viscous load
Feedback term

f_2: Stiffness force

Dimensionless equations of fluid and solid motions

- ***Linearized form - 1st order in D:***
  - ***1st term (f_1) in the dynamic condition at the interface (I):***
  - ***Equation of fluid motion – Large Stokes number***

$$S_T = \frac{\rho c L}{\mu} = \frac{\rho L^2}{\mu T_{solid}}$$

Example:

$$\begin{cases} L \approx 1m \\ \mu/\rho = 10^{-6} \\ T_{solid} \approx 1s \end{cases} \Rightarrow \quad S_T = 10^6 \quad \Rightarrow \frac{1}{S_T} \ll 1$$

$$\frac{\partial u_f}{\partial t_s} = -\vec{\nabla} p_f + \cancel{\frac{1}{S_T} \Delta u_f} \quad \Rightarrow \frac{\partial u_f}{\partial t_s} = -\vec{\nabla} p_f$$

Viscous forces neglected

Dimensionless equations of fluid and solid motions

- ***Linearized form - 1st order in D:***
  - ***1st term (f_1) in the dynamic condition at the interface (I):***
  - ***Equation of fluid motion – Large Stokes number***

$$\nabla u_f = 0 \qquad \frac{\partial u_f}{\partial t_s} = -\vec{\nabla} p_f$$

  - ***With the boundary conditions at the interface (I):***

$$u_f . n = \dot{q}_s \phi . n \qquad M \int_I \left[-p_f . n\right] . \phi . dS + q_s k_f = f$$

Viscosity neglected

→ The kinematic boundary condition involves only normal components

Without viscosity, we cannot state any condition about tangential velocity

Dimensionless equations of fluid and solid motions

- ***1st term (f_1) in the boundary conditions***
  - ***At the interface:*** $\boldsymbol{u_f(x,t).n = \dot{q}_s(t)\phi(x).n}$
  - ***We assume a similar pattern in the entire fluid domain:***
    - ***For the velocity:***

$$\boldsymbol{u_f(x,t) = \dot{q}_s(t)\phi_u(x)}$$ ***Everywhere in the fluid***

$\boldsymbol{\phi_u(x)}$ ***is a vectorial function of space***

- ***For the pressure:***

$$\boldsymbol{p_f(x,t) = \ddot{q}_s(t)\phi_p(x)}$$ ***Everywhere in the fluid***

- $\boldsymbol{\phi_p(x)}$ ***is a scalar function of space***

Dimensionless equations of fluid and solid motions

- ***1st term (f_1) in the boundary conditions***
  - ***In the fluid domain:***

$$\begin{cases} \nabla u_f = 0 \\ \dfrac{\partial u_f}{\partial t_s} = -\nabla p_f \end{cases} \Rightarrow \begin{cases} \nabla \phi_u = 0 \\ \phi_u = -\nabla \phi_p \end{cases}$$

  - ***At the interface:***

$$\begin{cases} u_f(x,t).n = \dot{q}_s(t)\phi(x).n \\ M\displaystyle\int_I \left[-p_f.n\right].\phi.dS + q_s k_f = f \end{cases} \Rightarrow \begin{cases} \phi_u.n = \phi.n \\ -\ddot{q}_s\left[M\displaystyle\int_I \phi_p n.\phi.dS\right] + q_s k_f = f \end{cases}$$

Dimensionless equations of fluid and solid motions

- ***1st term (f_1) in the boundary conditions***

$$\boldsymbol{f_1 = -\ddot{q}_s \left[M \int_I \phi_p n \,.\, \phi .\, dS \right] = -m_A \ddot{q}_s}$$

→ The fluid force f_1 proportional to $\ddot{q}_s$ → f_1 is an inertia force

$$\boldsymbol{m_A = M \int_I \phi_p n \,.\, \phi .\, dS}$$ ***: Added Mass***

m_A is constant for a given mode shape

Small Reduced Velocity: Added mass, added stiffness

Dimensionless equations of fluid and solid motions

- ***Summary of fluid-solid coupling at low reduced velocity:***

• ***Initial dimensionless parameters:***	D	F_R	R_E	C_Y	U_R
• ***Alternate choice*** $(c \rightarrow U_0)$:	D	F_D	S_T	M	U_R
• ***Small reduced velocity*** $(UR \ll 1)$:	D	F_D	S_T	M	
• ***Small displacement*** $(D \ll 1)$:		F_D	S_T	M	
• ***Small viscous force***		F_D		M	

→ ***The coupling of small displacement of a solid in an incompressible and inviscid fluid is equivalent to an increase of its mass and stiffness:***

$$f = -\underbrace{k_f q_s}_{\text{Added Stiffness}} - \underbrace{m_A \ddot{q}_s}_{\text{Added Mass}}$$

Evaluation of Added Mass

$$f_1 = -m_A \ddot{q}_s = -\ddot{q}_s \left[M \int_I -\phi_p n \,.\, \phi \,.\, dS \right]$$

$$\begin{cases} \nabla\phi_u = 0 \\ \phi_u = -\nabla\phi_p \end{cases}$$

Interface: $\phi_u . n = \phi . n$

$\Rightarrow$

$\Delta\phi_p = 0$ $\left(\textit{since } \nabla(\nabla\phi_p) = \Delta\phi_p \right)$

Interface: $-\nabla\phi_p . n = \phi . n$

- ***To evaluate m_A, we must solve the Laplace equation: $\Delta\phi_p = 0$ with the boundary condition at the interface: $-\nabla\phi_p . n = \phi . n$***

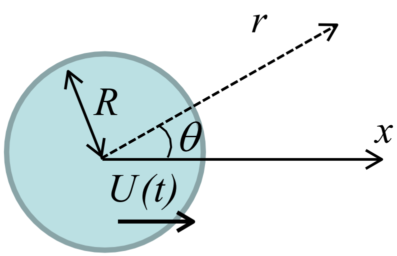
- ***Analytical solutions are available for many simple geometries***

Evaluation of Added Mass

- ***Example : A cylinder vibrating horizontally in a still liquid***

$$\Delta\phi_p = 0$$

Interface: $-\nabla\phi_p . n = \phi . n$



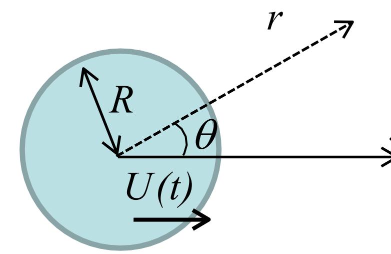
$$\phi . n = cos\theta \quad \Rightarrow \quad \phi_p(r,\theta) = \frac{cos\theta}{r}$$

We verify easily that the Laplace Eq. is fulfilled

$$\Delta\phi_p = \frac{\partial^2\phi_p}{\partial r^2} + \frac{1}{r}\frac{\partial\phi_p}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi_p}{\partial\theta^2} + \frac{\partial^2\phi_p}{\partial z^2} = 0$$

Evaluation of Added Mass

- ***Example : A cylinder vibrating horizontally in a still liquid***



- ***→ The velocity shape reads:***

$$\boldsymbol{\phi_u = -\nabla\phi_p = -\left(\frac{\partial\phi_p}{\partial r}e_r + \frac{1}{r}\frac{\partial\phi_p}{\partial\theta}e_\theta + \frac{\partial\phi_p}{\partial z}e_z\right)}$$

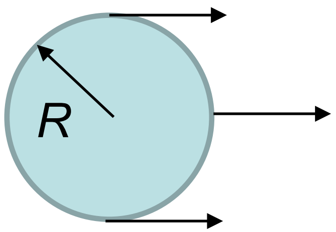
$$\Rightarrow \qquad \boldsymbol{\phi_u = \frac{1}{r^2}[(cos\theta)e_r + (sin\theta)e_\theta]}$$

- ***Finally the dimensionless added mass m_A is:***

$$\boldsymbol{m_A = M\int_I -\phi_p n\,.\,\phi.\,dS = M\pi}$$

Evaluation of Added Mass

- ***Example : A cylinder vibrating horizontally in a still liquid***



  - ***Dimensional added mass M_A:***

$$\begin{cases} m_A = \dfrac{M_A}{M_s} \\ M = \dfrac{\rho_f}{\rho_s} = \dfrac{M_f}{M_s} = \dfrac{\rho_f R^2}{M_s} \end{cases} \Rightarrow \quad M_A = \pi M M_s = \pi M_f = \rho_f \, \pi R^2$$

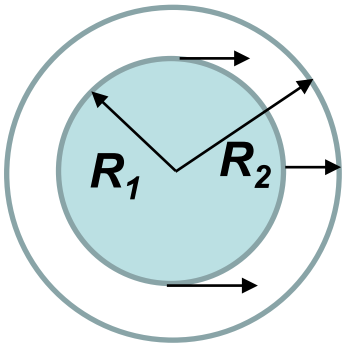
Mass per unit length of the cylinder

→ Here, the added mass is equal to the mass of displaced liquid
!! Not always true

If M ~ 1 ($\rho_f \approx \rho_s$), the "apparent" mass of the cylinder is the double of its mass in vacuum

Evaluation of Added Mass

- ***Example : Case of a cylinder vibrating in a liquid confined between 2 cylinders of radii R_1 and R_2 ($a=R_2/R_1$)***



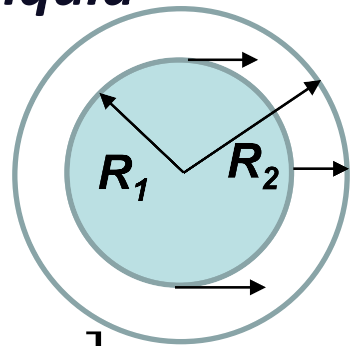
  - ***There is a solution of the Laplace equation $\Delta\phi_p=0$ with zero velocity condition on fluid boundaries***
  - ***The added mass reads:***

$$m_A = M\pi\frac{a^2+1}{a^2-1}$$

  - ***This result is surprising since:*** $\lim_{a\to 1} m_A = \infty$

Evaluation of Added Mass

- ***Example : Case of a cylinder vibrating in a confined liquid***
  - ***Let's consider the kinetic energy of the fluid:***



$$Ec = \int_{fluid\ bulk} \frac{1}{2} M u^2 dV$$

$$\begin{cases} u_f(x,t) = \dot{q}_s(t)\phi_u(x) \\ \phi_u = -\nabla\phi_p \\ \text{Interface: } -\nabla\phi_p . n = \phi . n \end{cases} \Rightarrow Ec = \frac{1}{2}\left[M \int_{Interface} -\phi_p n . \phi dS \right] \dot{q}^2$$

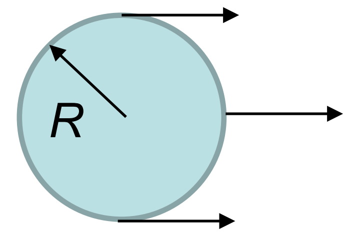
$$Ec = \frac{1}{2} m_A \dot{q}^2$$

→ The added mass is the mass which leads to the kinetic energy of the fluid when associated with the velocity of the solid

→ The added mass increases with the confinement because the kinetic energy required to displace the fluid grows without upper limit.

Evaluation of Added Mass

- *Example : A sphere vibrating horizontally in a still liquid*



$$\Delta\phi_p = 0$$

Interface: $-\nabla\phi_p . n = \phi . n$

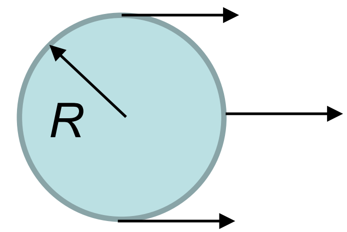
$$\phi . n = cos\theta \quad \Rightarrow \quad \phi_p(r, \theta, \varphi) = \frac{r . cos\theta}{2}$$

We verify easily that the Laplace Eq. is fulfilled

$$\Delta\phi_p = \frac{\partial^2\phi_p}{\partial r^2} + \frac{2}{r}\frac{\partial\phi_p}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi_p}{\partial\theta^2} + \frac{1}{r^2 tan(\theta)}\frac{\partial\phi_p}{\partial\theta} + \frac{1}{r^2 sin^2(\theta)}\frac{\partial^2\phi_p}{\partial\varphi^2} = 0$$

Evaluation of Added Mass

- *Example : A sphere vibrating horizontally in a still liquid*
  - *The dimensionless added mass m_A is:*



$$m_A = M\int_I -\phi_p n\,.\,\phi.\,dS$$

$$n.\,\phi.\,dS = cos(\theta).\,2\pi sin(\theta)d\theta \;\Rightarrow\quad m_A = \frac{2}{3}M\pi$$

Dimensional added mass M_A:

$$\begin{cases} m_A = \dfrac{M_A}{M_s} \\ M = \dfrac{\rho_f}{\rho_s} = \dfrac{M_f}{M_s} = \dfrac{\rho_f R^3}{M_s} \end{cases} \Rightarrow \quad M_A = \frac{2}{3}\pi M M_s = \frac{2}{3}\pi M_f = \frac{2}{3}\pi\rho_f R^3$$

→ ***Here, the added mass is equal to half of the mass of displaced liquid***

→ ***If M ~ 1, the apparent mass of the sphere is 1.5 of its mass in vacuum***

Added mass – kinetic energy approach[1]

- **Added mass: inertia added to a system due to the displacement of the fluid surrounding a body when it accelerates.**
- **May be also seen as the mass added to the solid so that the corresponding kinetic energy is the one of the displaced fluid**

$$Ec = \int_{fluid\ bulk} \frac{1}{2} M u^2 dV = \cdots = \frac{1}{2}\left[M \int_{Interface} -\phi_p n.\phi dS \right] \dot{q}^2$$

$$Ec = \frac{1}{2} m_A \dot{q}^2$$

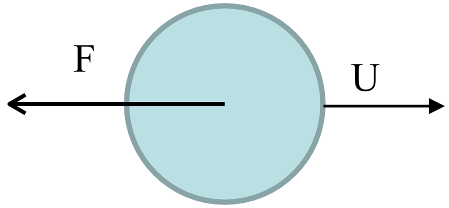
[1]Brennen, C. E. (1982). A review of added mass and fluid inertial forces.

Added mass – kinetic energy approach[1]

- **Kinetic energy associated with fluid motion:**

$$Ec = \frac{\rho}{2}\int_V ({u_1}^2 + {u_2}^2 + {u_3}^2)dV = \frac{\rho}{2}\int_V (u_i u_i)dV$$

- **Body steadily in translation with velocity U:**



$$Ec = \frac{\rho}{2}\mathrm{I}U^2 \qquad where \;\; I = \int_V \left(\frac{u_i u_i}{UU}\right)dV$$

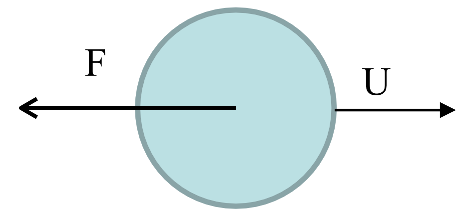
- **We may assume that when U varies, the velocity at each point of the fluid varies in direct proportion to U**

  → **I is then a constant and the kinetic energy is proportional to U^2**

- **This assumption is justified in some cases such as potential flows and low Reynolds number flows.**

[1]*Brennen, C. E. (1982). A review of added mass and fluid inertial forces.*

Added mass – kinetic energy approach[1]



- **When the body accelerates:**
  - Kinetic energy increases → work must be supplied to the fluid by the body
  - This work, which is equal to the change of kinetic energy, is experienced by the body as additional drag force: F

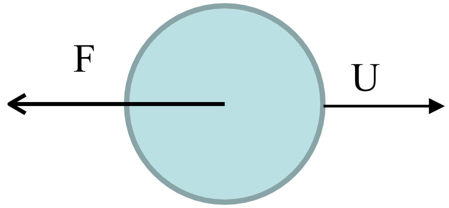
$$\Delta \boldsymbol{Ec} = \boldsymbol{W} \rightarrow \frac{\boldsymbol{dEc}}{\boldsymbol{dt}} = -\boldsymbol{FU}$$

[1]Brennen, C. E. (1982). A review of added mass and fluid inertial forces.

Added mass – kinetic energy approach[1]

- **If the flow pattern does not change, the integral I remains constant, and we have the following:**

$$F = \frac{-1}{U}\frac{dEc}{dt} = -\rho I \frac{dU}{dt} = -M_A \frac{dU}{dt}$$



$$I = \int_V \left(\frac{u_i u_i}{UU}\right) dV$$

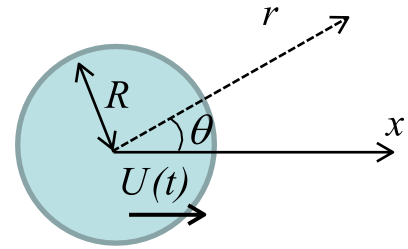
I is an invariant of the problem

«it is often convenient to consider the mass, M=ρI, as an "added mass" which is being accelerated along with the body. Of course, there is no such identifiable fluid mass; rather all the fluid is accelerating to some degree as the total kinetic energy is increasing.»[1]

[1]Brennen, C. E. (1982). A review of added mass and fluid inertial forces.

Added mass – kinetic energy approach[1]

- **Added mass of a cylinder (2D) :**



  - **Velocity potential, φ**

$$\varphi = \frac{-UR^2}{r} cos\theta$$

$$u_r = \frac{\partial \varphi}{\partial r} = \frac{UR^2}{r^2} cos\theta; \quad u_\theta = \frac{1}{r}\frac{\partial \varphi}{\partial \theta} = \frac{UR^2}{r^2} sin\theta$$

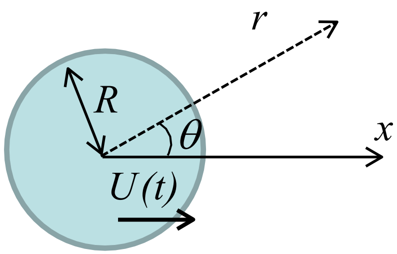
  - **Added Mass:**

$$M_A = \rho \int_R^\infty \int_0^{2\pi} \left[\left(\frac{1}{U}\frac{\partial \varphi}{\partial r} \right)^2 + \left(\frac{1}{U.r}\frac{\partial \varphi}{\partial \theta} \right)^2 \right] r.dr.d\theta = \rho \pi R^2$$

→ ***Added mass = mass of the cylinder filled with the fluid***

Added mass – kinetic energy approach[1]

- **Added mass of a sphere :**



  - **Velocity potential, φ**

$$\varphi = \frac{-UR^3}{2r^2} \cos\theta$$

$$u_r = \frac{\partial \varphi}{\partial r} = \frac{UR^3}{r^3} \cos\theta; \quad u_\theta = \frac{1}{r} \frac{\partial \varphi}{\partial \theta} = \frac{UR^3}{2r^3} \sin\theta$$

  - **Added Mass:**

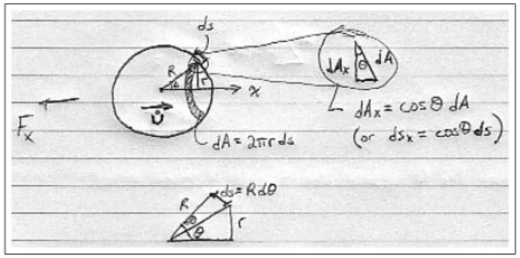
$$M_A = \rho \int_R^\infty \int_0^\pi \left[\left(\frac{1}{U} \frac{\partial \varphi}{\partial r} \right)^2 + \left(\frac{1}{U.r} \frac{\partial \varphi}{\partial \theta} \right)^2 \right] 2\pi r.\sin\theta.r dr.d\theta = \frac{2}{3} \rho \pi R^3$$

→ ***Added mass = half of the mass of the sphere filled with the fluid***

Added mass derivation using unsteady Bernoulli equation

2.016 Hydrodynamics
Professor A.H. Techet
by TA B. P. Epps

0.1 Derivation of Added Mass around a Sphere



When a body moves in a fluid, some amount of fluid must move around it. When the body accelerates, so too must the fluid. Thus, more force is required to accelerate the body in the fluid than in a vacuum. Since force equals mass times acceleration, we can think of the additional force in terms of an imaginary *added mass* of the object in the fluid.

One can derive the added mass of an object by considering the hydrodynamic force acting on it as it accelerates. Consider a sphere of radius, R, accelerating at rate $\partial U/\partial t = \dot{U}$. We find the hydrodynamic force in the x-direction by integrating the pressure over the area projected in the x-direction:

$$\vec{F_x} = \int p\, d\vec{A_x}$$

where

- $d\vec{A_x} = \cos\theta dA, \quad dA = 2\pi r ds, \quad r = R\sin\theta, \quad ds = Rd\theta$
- $p = -\rho\left[\frac{\partial\phi}{\partial t} + \frac{1}{2}|\vec{\nabla}\phi|^2\right]$, by unsteady Bernoulli's equation

  $\phi = U\cos\theta\frac{R^3}{2r^2}$, for axisymmetric flow around a sphere

  $\frac{\partial\phi}{\partial t}|_{r=R} = \dot{U}\cos\theta\frac{R^3}{2r^2} = \dot{U}\cos\theta\frac{R}{2}$

  $\frac{1}{2}|\vec{\nabla}\phi|^2|_{r=R} = \frac{1}{2}|(-U\cos\theta\frac{R^3}{r^3}, -U\sin\theta\frac{R^3}{2r^3})|^2 = \frac{1}{2}[U^2\cos^2\theta + \frac{1}{4}U^2\sin^2\theta]$

so

$$\begin{aligned}
F_x &= \int_0^\pi \left[-\rho\left[\frac{\partial\phi}{\partial t} + \frac{1}{2}|\vec{\nabla}\phi|^2\right]\right]\cos\theta 2\pi R^2\sin\theta d\theta \\
&= \int_0^\pi \left[-\rho\left[\dot{U}\cos\theta\frac{R}{2} + \frac{1}{2}(U^2\cos^2\theta + \frac{1}{4}U^2\sin^2\theta)\right]\right]\cos\theta 2\pi R^2\sin\theta d\theta \\
&= -\rho\cdot 2\pi R^2\cdot\dot{U}\frac{R}{2}\underbrace{\int_0^\pi \sin\theta\cos^2\theta d\theta}_{=2/3} - \rho\cdot 2\pi R^2\cdot\frac{1}{2}U^2\underbrace{\int_0^\pi [\sin\theta\cos^3\theta + \frac{1}{4}\sin^3\theta\cos\theta]d\theta}_{=0} \\
&= -\frac{2}{3}\rho\pi R^3\dot{U}
\end{aligned}$$

where $\dot{U}$ is the acceleration of the body, and the negative sign indicates that the force is in the negative x-direction, opposing the acceleration. Thus, the body must exert this extra force, and the apparent *added mass* is

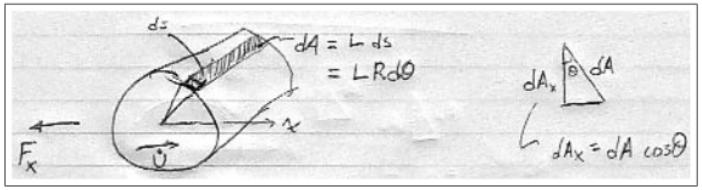
$$m_a = \frac{2}{3}\rho\pi R^3$$

Added mass derivation using unsteady Bernoulli equation

Case of a cylinder

0.2 Derivation of Added Mass around a Cylinder

Similarly, for a cylinder of radius, R, and length, L, accelerating at rate $\dot{U}$. We find the hydrodynamic force in the x-direction by integrating the pressure over the area projected in the x-direction:



$$\vec{F}_x = \int P d\vec{A}_x$$

where

- $d\vec{A}_x = \cos\theta dA$

  $dA = Lds$

  $ds = Rd\theta$

- $P = -\rho\left[\frac{\partial\phi}{\partial t} + \frac{1}{2}|\vec{\nabla}\phi|^2\right]$, by unsteady Bernoulli's equation

  $\phi = U\frac{R^2}{r}\cos\theta$, for flow around a cylinder

  $\frac{\partial\phi}{\partial t}|_{r=R} = \dot{U}\frac{R^2}{r}\cos\theta = \dot{U}R\cos\theta$

  $\frac{1}{2}|\vec{\nabla}\phi|^2|_{r=R} = \frac{1}{2}|(-U\frac{R^2}{r^2}\cos\theta, -U\frac{R^2}{r^2}\sin\theta)|^2 = \frac{1}{2}U^2$

so

$$\begin{aligned}
F_x &= \int_0^{2\pi}\left[-\rho\left[\frac{\partial\phi}{\partial t} + \frac{1}{2}|\vec{\nabla}\phi|^2\right]\right]\cos\theta RLd\theta \\
&= \int_0^{2\pi}\left[-\rho\left[\dot{U}R\cos\theta + \frac{1}{2}U^2)\right]\right]\cos\theta RLd\theta \\
&= -\rho\cdot RL\cdot\dot{U}R\underbrace{\int_0^{2\pi}\cos^2\theta d\theta}_{=\pi} - \rho\cdot RL\cdot\frac{1}{2}U^2\underbrace{\int_0^{2\pi}\cos\theta d\theta}_{=0} \\
&= -\rho\pi R^2 L\dot{U}
\end{aligned}$$

where $a = \dot{U}$ is the acceleration of the body, and the negative sign indicates that the force is in the negative x-direction, opposing the acceleration. Thus, the body must exert this extra force, and the apparent *added mass* is

$$m_a = \rho\pi R^2 L$$

Added mass and added stiffness in real life ?

- ***Analytical solutions: only valid for simple geometries***

- ***Numerical simulations:***
  - ***Possible for simple geometries, not yet reliable for more realistic cases***

- ***Empirical solutions***
  - ***Experimentation:***
    - ***Difficulty to achieve experiments on reduced scale models***

Almost impossible to satisfy similarity rules in both solid and fluid domains

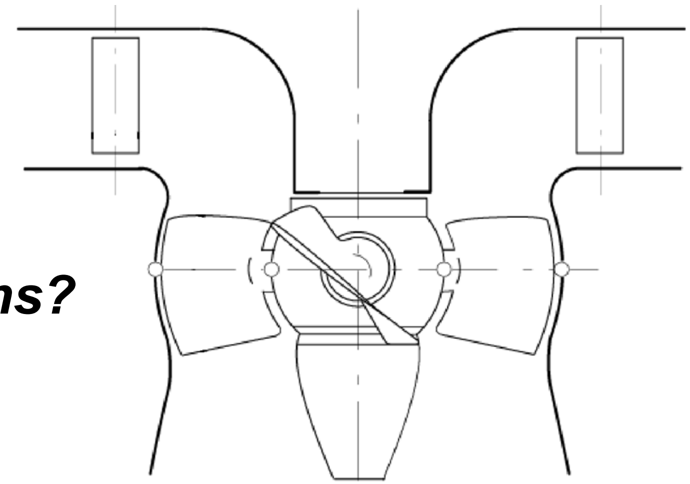
Example : Model tests of hydraulic machines

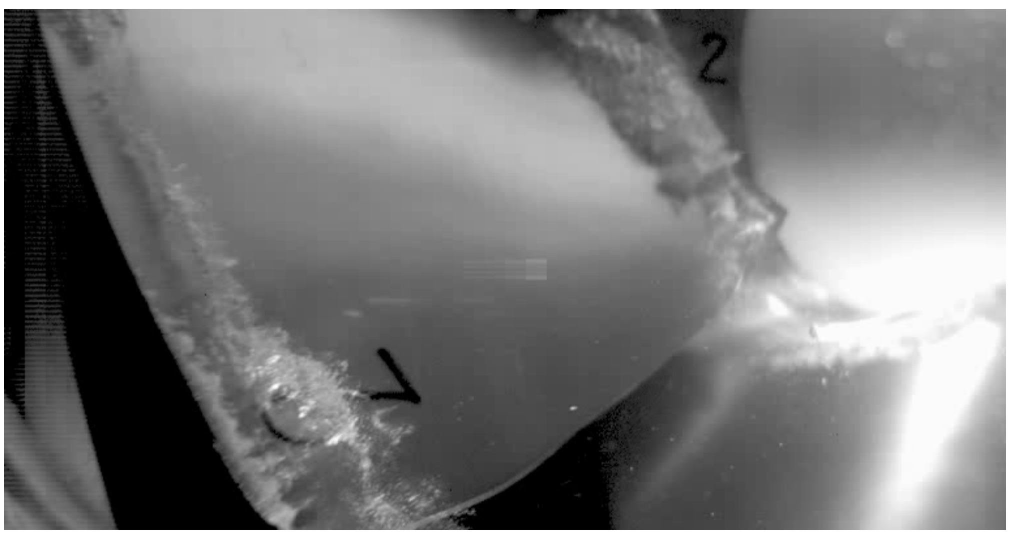
→ Good representation of the flow field (efficiency, stability, cavitation, …)
→ Poor representation of the fluid structure interaction

Small Reduced Velocity: Added mass, added stiffness

Added mass: application to hydraulic turbine

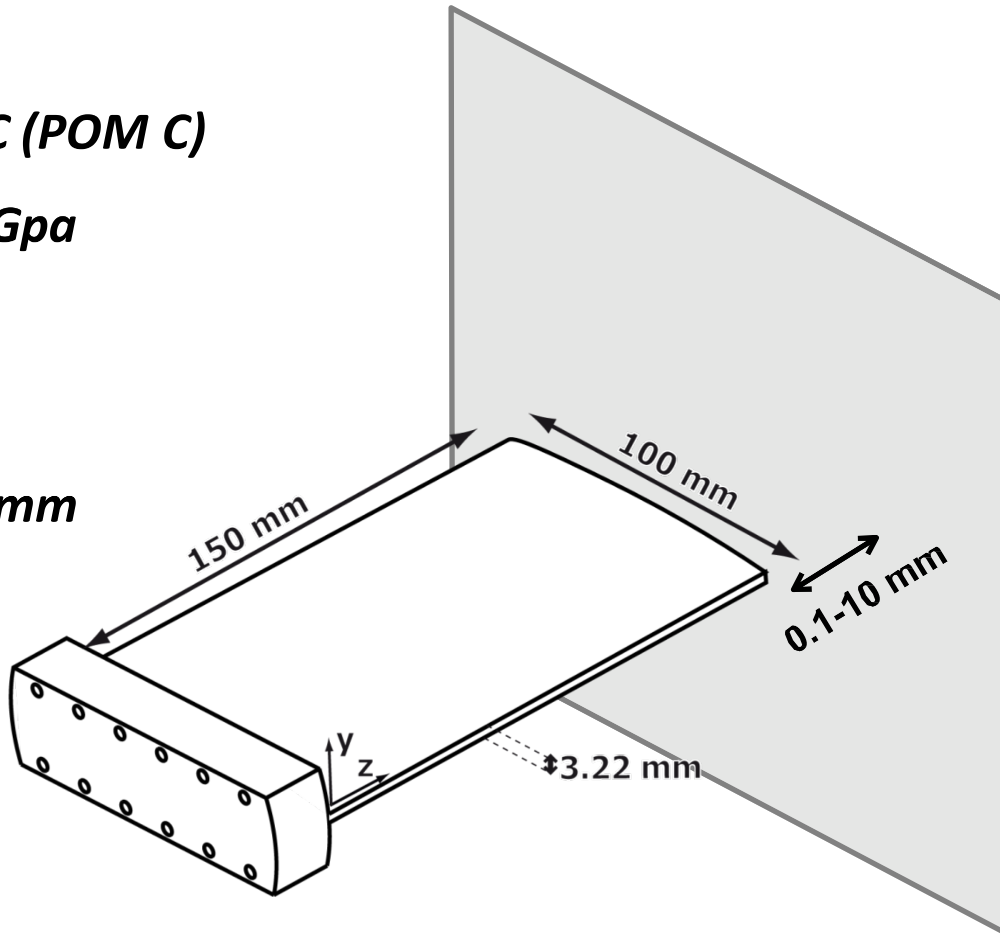
- ***Kaplan turbine***
  - ***Gap between the rotor tip and casing***
  - ***Effect of gap flow on the blade vibrations?***





Added mass of a hydrofoil - case study

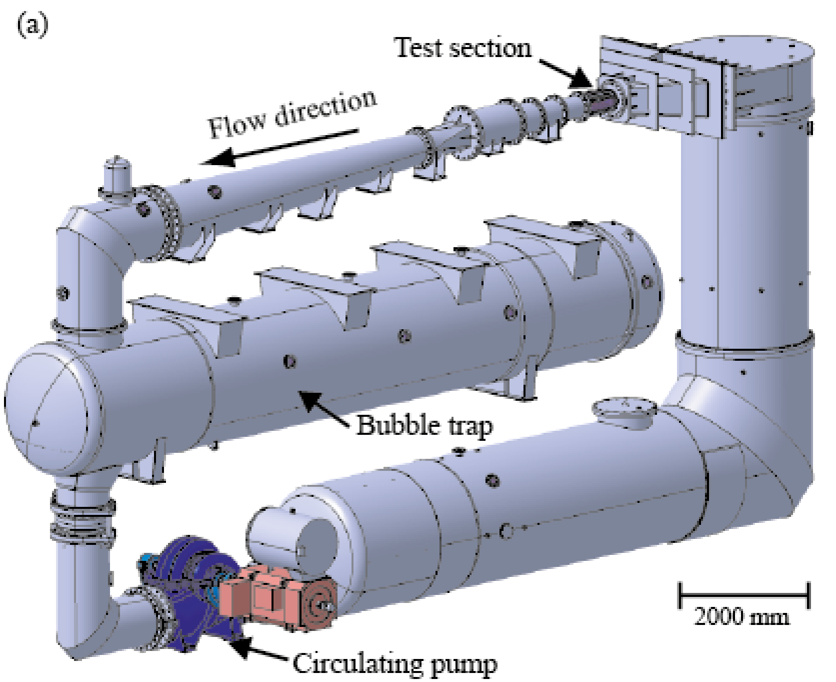
- ***Hydrofoil Naca0009***
    - ***Polyoxymethylene type C (POM C)***
    - ***Elasticity modulus = 3.1 Gpa***
    - ***Span: 0.15 m***
    - ***Chord: 0.1 m***
    - ***Maximum thickness: 10 mm***
- ***Gap variation***
    - ***0.1-10 mm***

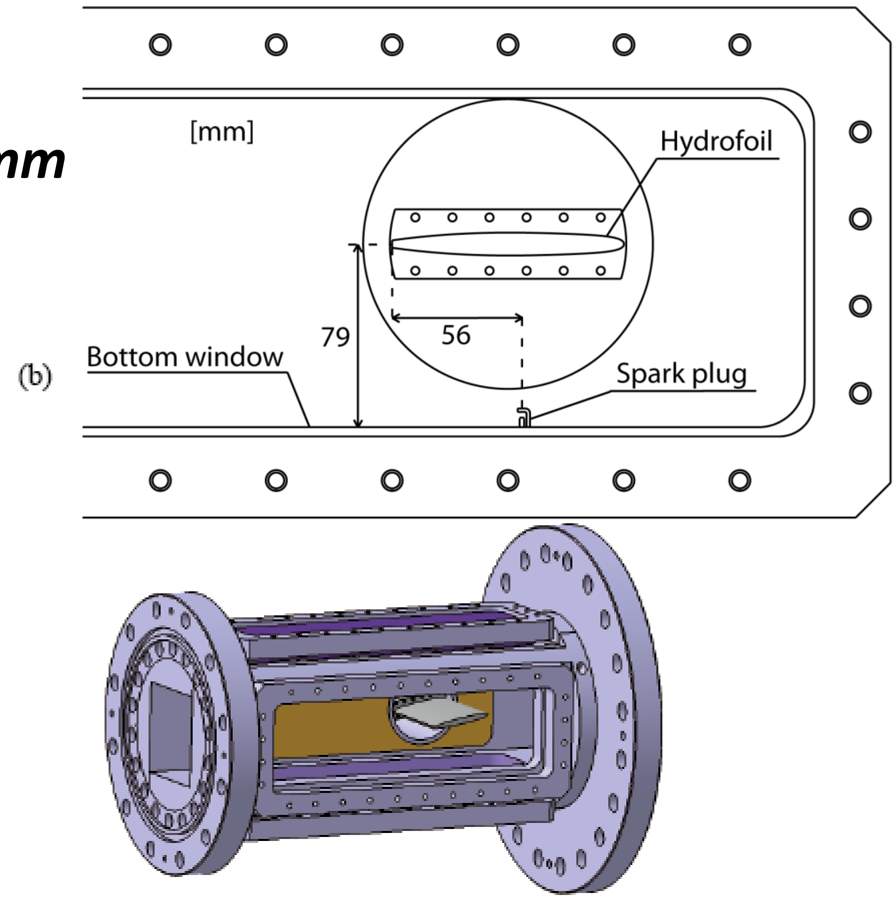


Small Reduced Velocity: Added mass, added stiffness

Added mass of a hydrofoil – Experimental setup

- ***LMH cavitation tunnel***
  - ***Test section 150x150x750mm***





Small Reduced Velocity: Added mass, added stiffness

Reminder

• ***The transfer function, which relates the input and output of a linear time-invariant system, is equal to the response of the system to an impulse***

Dirac function (or unit impulse): $\delta(t)$ *is a "function" which satisfies:*

$$\delta(x \neq 0) = 0, \qquad \delta(0) = +\infty \qquad and \qquad \int_{-\infty}^{+\infty} \delta(t)dt = 1$$

The Dirac function may be constructed as follows:

$$\delta(t) = \lim_{\varepsilon \to 0} \delta_\varepsilon(t) \quad with: \quad \delta_\varepsilon(t = \begin{cases} 0 & -\infty < t < 0 \\ 1/\varepsilon & 0 < t < \varepsilon \\ 0 & \varepsilon < t < +\infty \end{cases} \qquad and \quad \int_{-\infty}^{+\infty} \delta_\varepsilon(t)\, dt = 1$$

Properties :

$$\forall \tau \in \mathbb{R}, \quad (x * \delta)(\tau) = \int_{-\infty}^{+\infty} x(\tau - t)\delta(t)dt = \int_{-\infty}^{+\infty} x(\tau)\delta(t)dt = x(\tau)\int_{-\infty}^{+\infty} \delta(t)dt = x(\tau)$$

$$\Rightarrow \mathcal{F}(x * \delta) = \mathcal{F}(x).\mathcal{F}(\delta) = \mathcal{F}(x) \Rightarrow \mathcal{F}(\delta) = 1$$

Small Reduced Velocity: Added mass, added stiffness

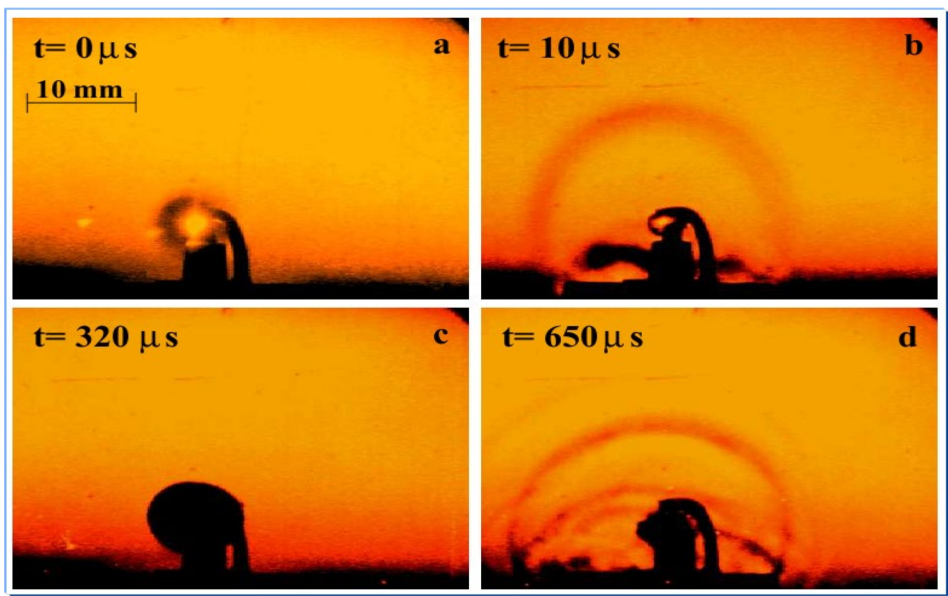
Added mass of a hydrofoil – Experimental setup

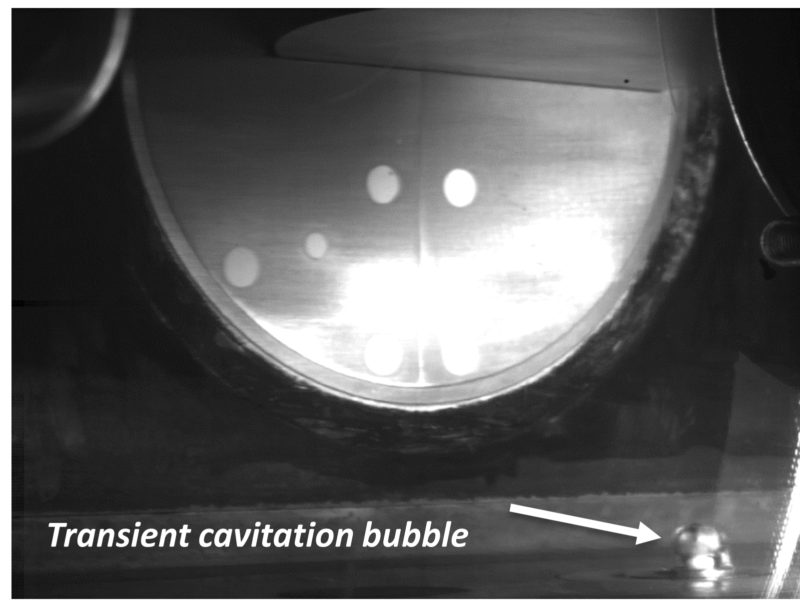
- *EPFL cavitation tunnel*
- *Mechanical Excitation in air: Instrumented hammer*
  - *Impulse response (averaged over several impacts)*

Small Reduced Velocity: Added mass, added stiffness

Added mass of a hydrofoil – Experimental setup

- *LMH cavitation tunnel*
- *Mechanical Excitation in water: Electric discharge in water (Spark)*
    - *Production of a cavitation bubble on the bottom face of the test section*
    - *Strong shockwave → impulse excitation of the hydrofoil*

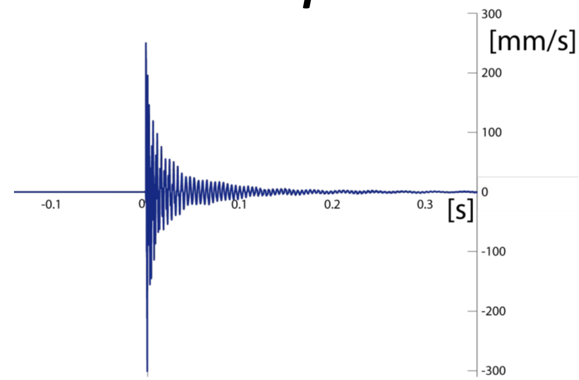


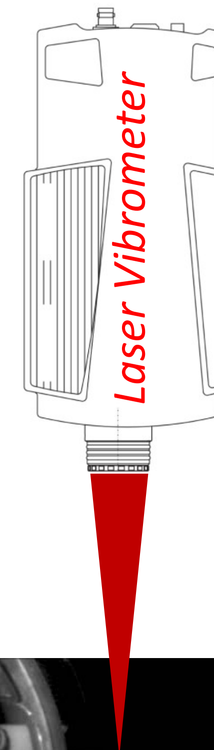


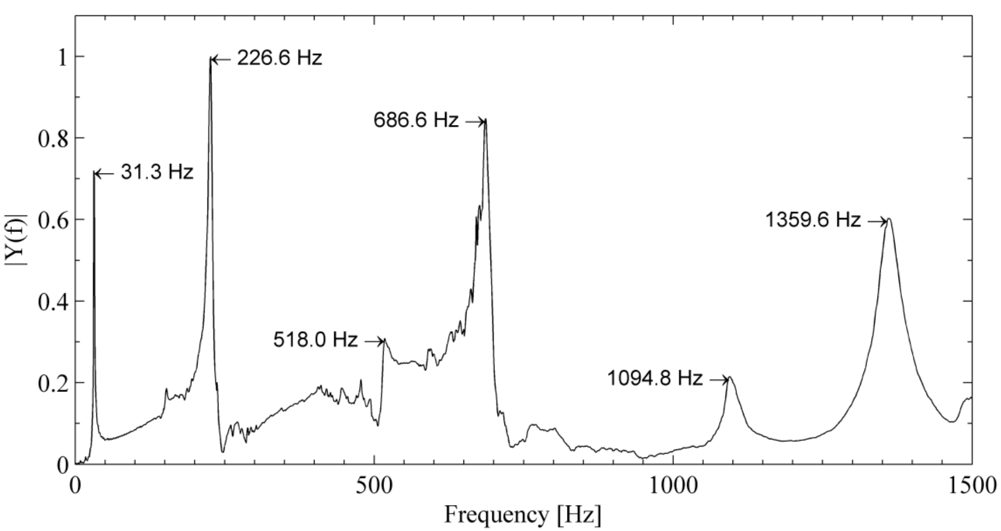
Small Reduced Velocity: Added mass, added stiffness

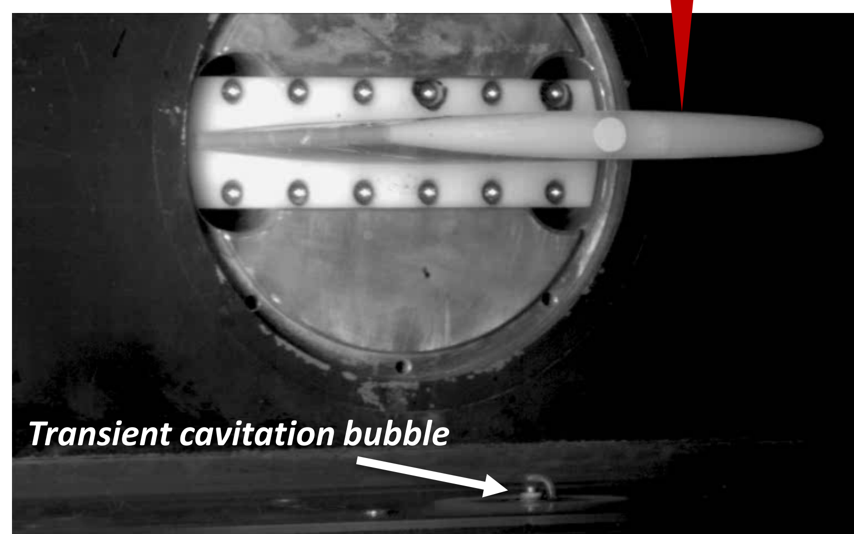
Added mass of a hydrofoil – Experimental setup

- *LMH cavitation tunnel*
- *Spark generated bubble*
- *Laser Vibrometer*
  - *Non-intrusive measurements*
  - *Interferometer: Doppler shift of the reflected laser beam frequency*



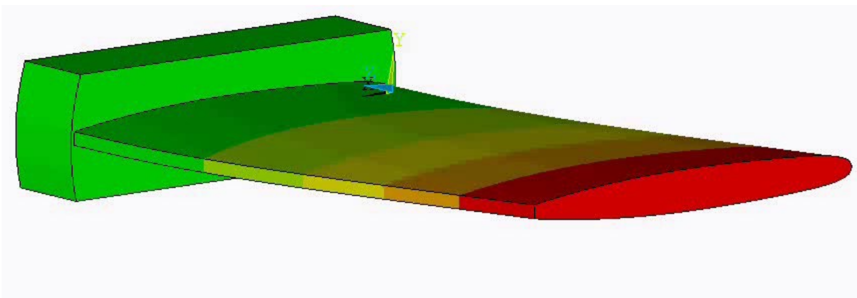




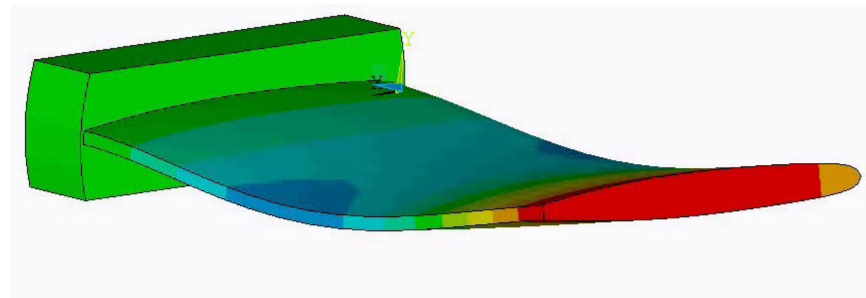


Small Reduced Velocity: Added mass, added stiffness

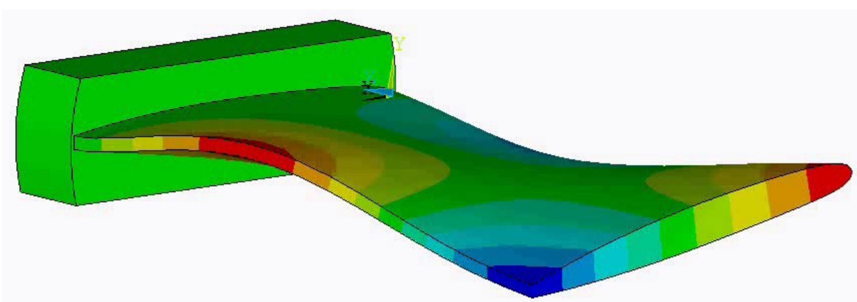
Added mass of a hydrofoil – mode shapes in water



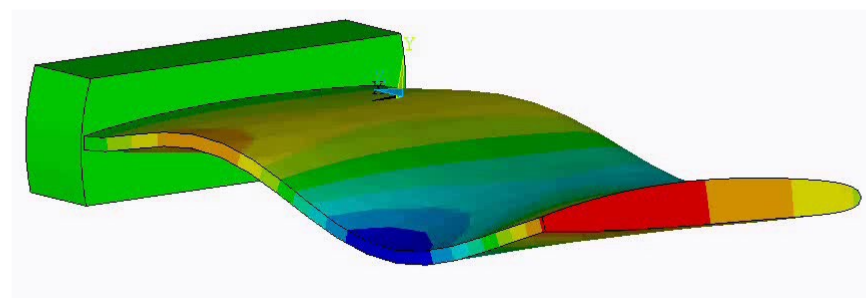
1st bending mode ~32 Hz



2nd bending mode ~228 Hz



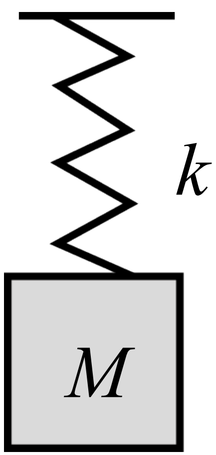
2nd torsion mode ~518 Hz



3rd bending mode ~687 Hz

Added mass determination

- *Single degree of freedom oscillator*
  - *In air (added mass in air neglected)*



Natural frequency:

$$f_{air}^2 = \frac{1}{4\pi^2}\frac{k}{M}$$

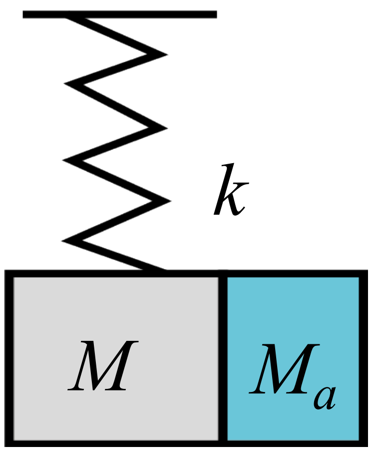
$$M\ddot{x} + kx = 0$$

M *and* ***k*** *are the modal mass and modal stiffness, respectively*

Small Reduced Velocity: Added mass, added stiffness

Added mass determination

- *Single degree of freedom oscillator*
  - *In water*



Natural frequency:

$$f_{water}^2 = \frac{1}{4\pi^2}\frac{k}{M + M_a}$$

$$(M + M_a)\ddot{x} + kx = 0$$

***M** and **k** are the modal mass and modal stiffness, respectively*

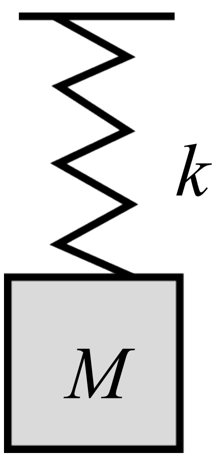
Small Reduced Velocity: Added mass, added stiffness

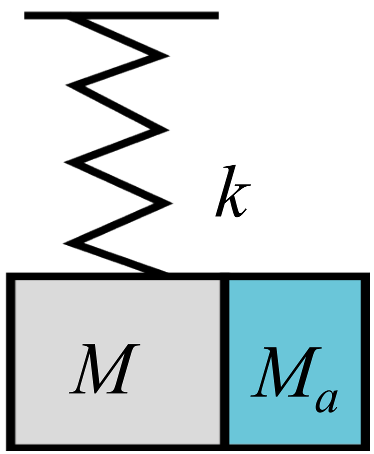
Added mass determination

- *Single degree of freedom oscillator*

$$f_{air}^2 = \frac{1}{4\pi^2}\frac{k}{M}$$

$$f_{water}^2 = \frac{1}{4\pi^2}\frac{k}{M + M_a}$$



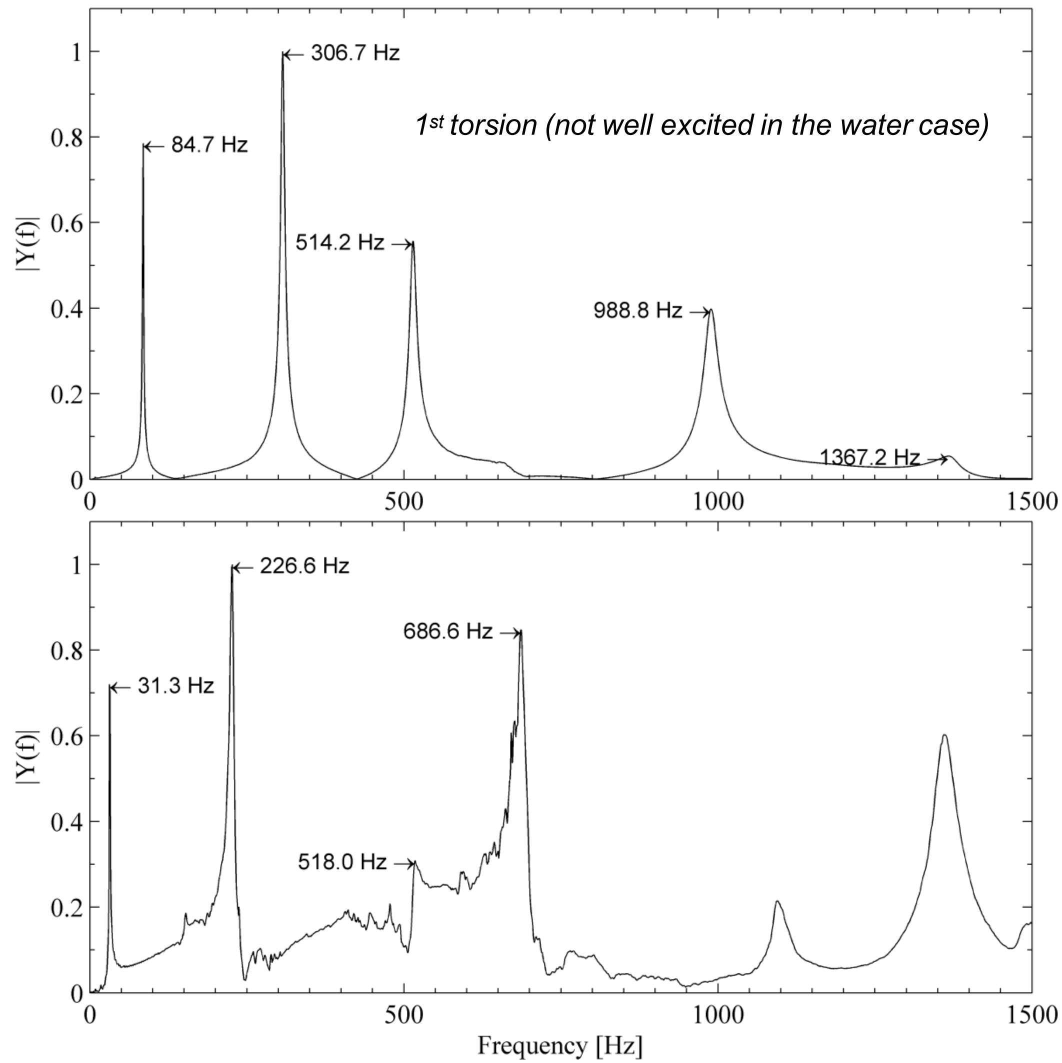


$$\frac{M_a}{M} = \left(\frac{f_{air}}{f_{water}}\right)^2 - 1$$

Small Reduced Velocity: Added mass, added stiffness

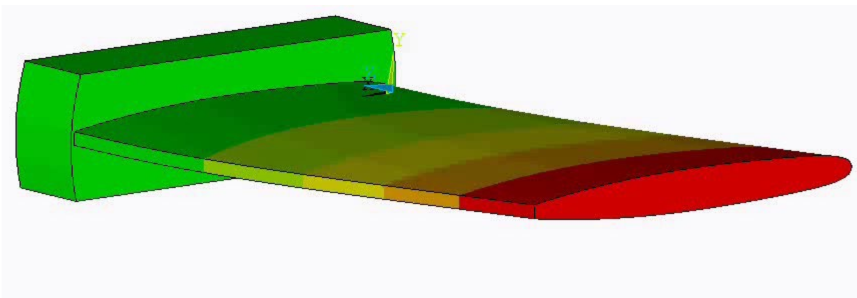
Added mass determination

- ***Impulse response in air***
  - ***Impact hammer excitation***
- ***Impulse response in water***
  - ***Spark generated bubble excitation***

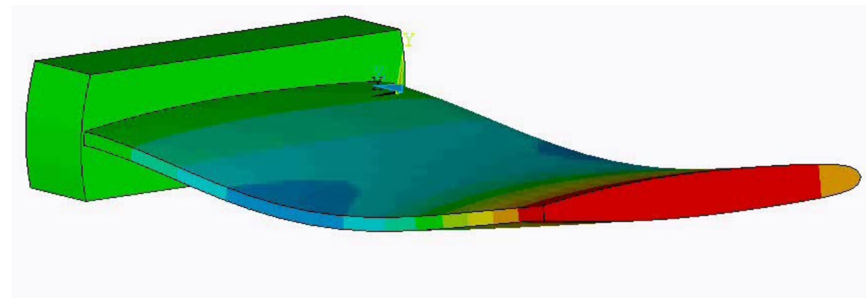


Small Reduced Velocity: Added mass, added stiffness

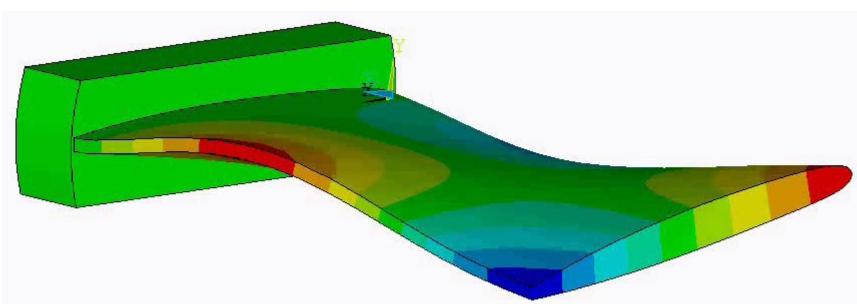
Added mass of a hydrofoil – mode shapes



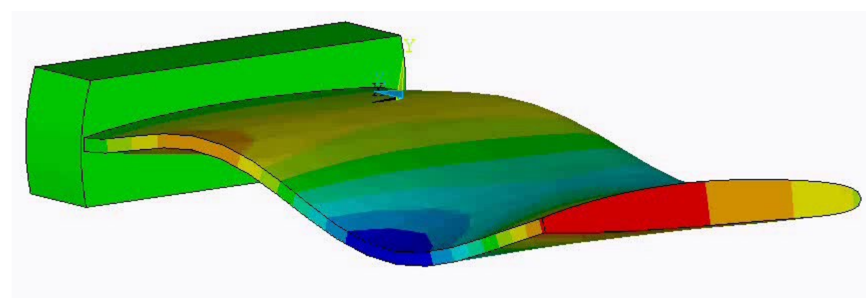
1st bending mode : $M_a/M \approx 6$



2nd bending mode : $M_a/M \approx 3$



2nd torsion mode : $M_a/M \approx 2.8$

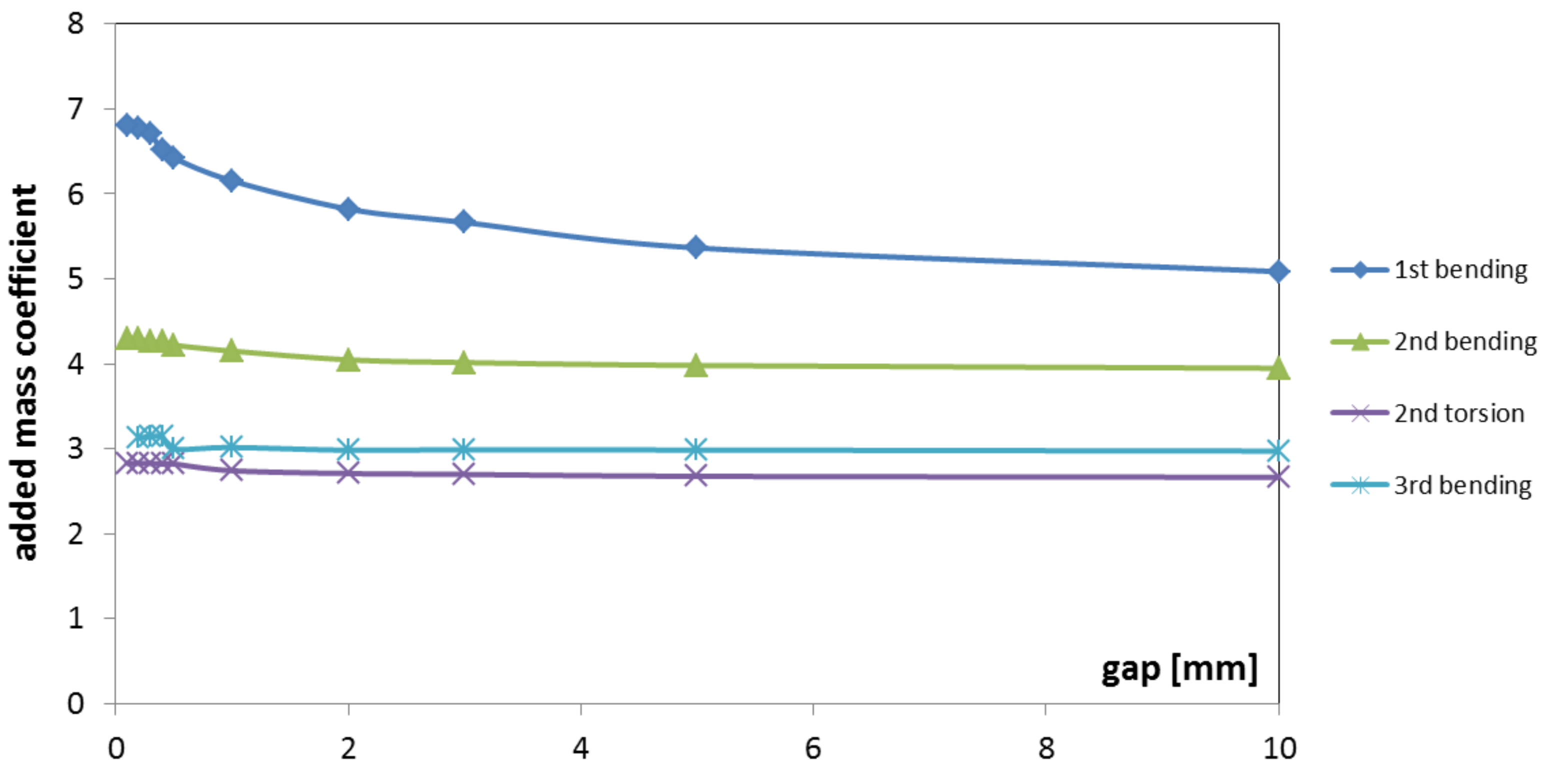


3rd bending mode : $M_a/M \approx 3$

Small Reduced Velocity: Added mass, added stiffness

Added mass of a hydrofoil – variation with gap width

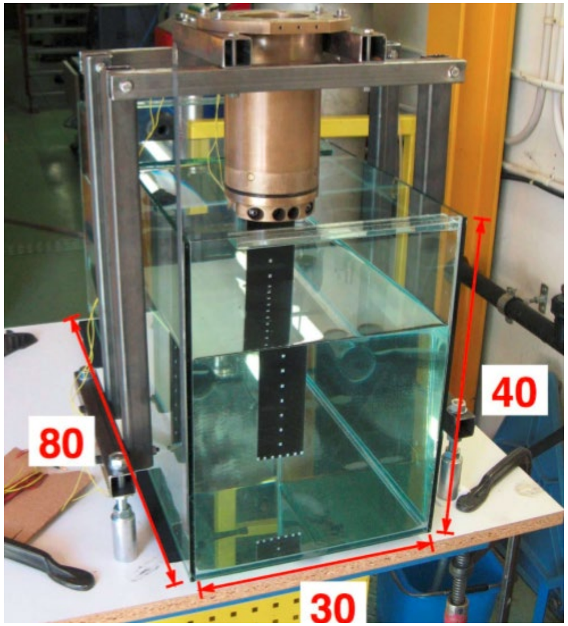
- *The presence of a solid boundary close to the body generally increases the added mass of that body*
- *Highly dependent on the mode shape*

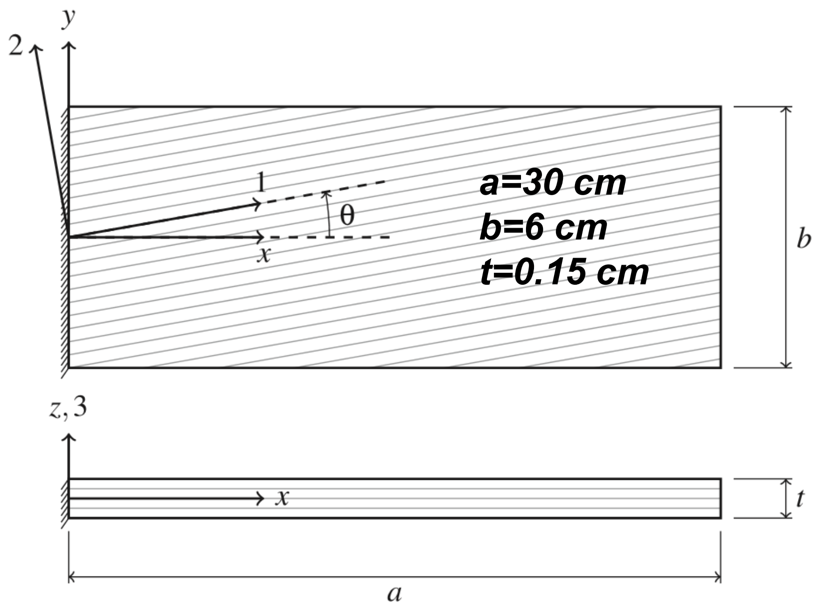


Added mass and added stiffness in real life ?

- ***Mechanical response of a lifting foil partially immersed in water ?***

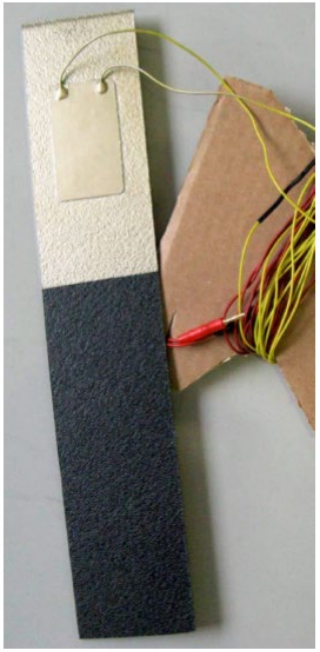
- ***Simplified case study:***
  - ***Rectangular cantilever beam:***
    - ***Stainless steel***
    - ***Composite material with different fiber orientation***

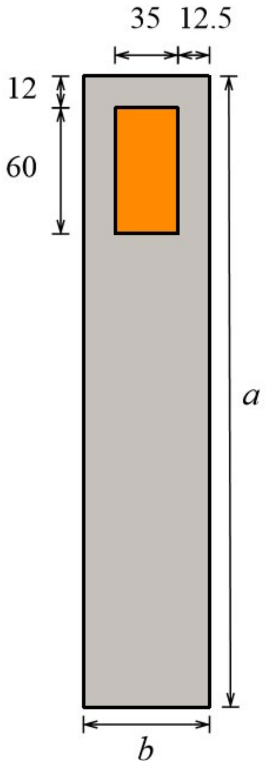


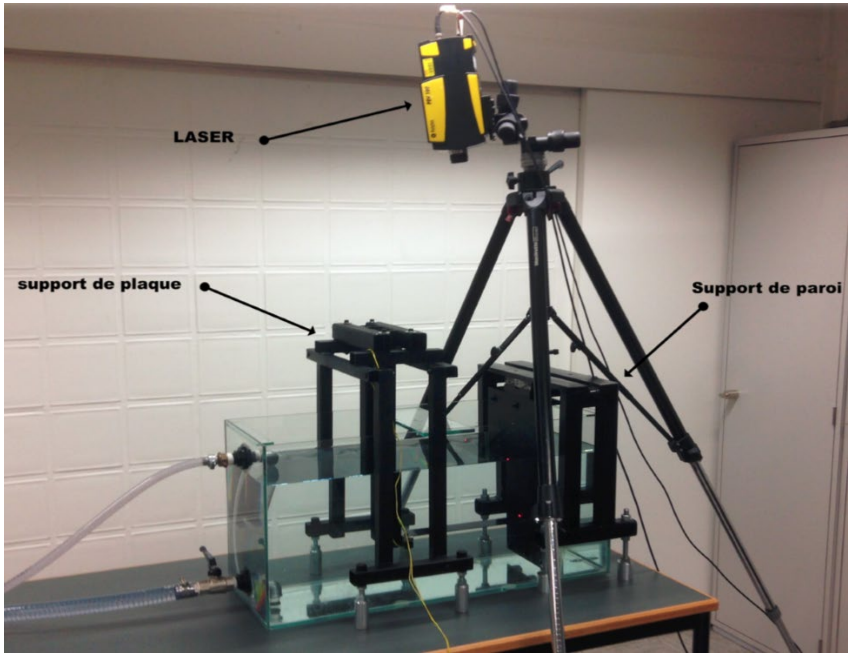


Added mass and added stiffness in real life ?

- ***Mechanical excitation:***
  - ***Piezoelectric patch fitted on the beam***
  - ***Measurement of the vibration response:***
    - ***Laser Doppler Vibrometer (non-intrusive)***

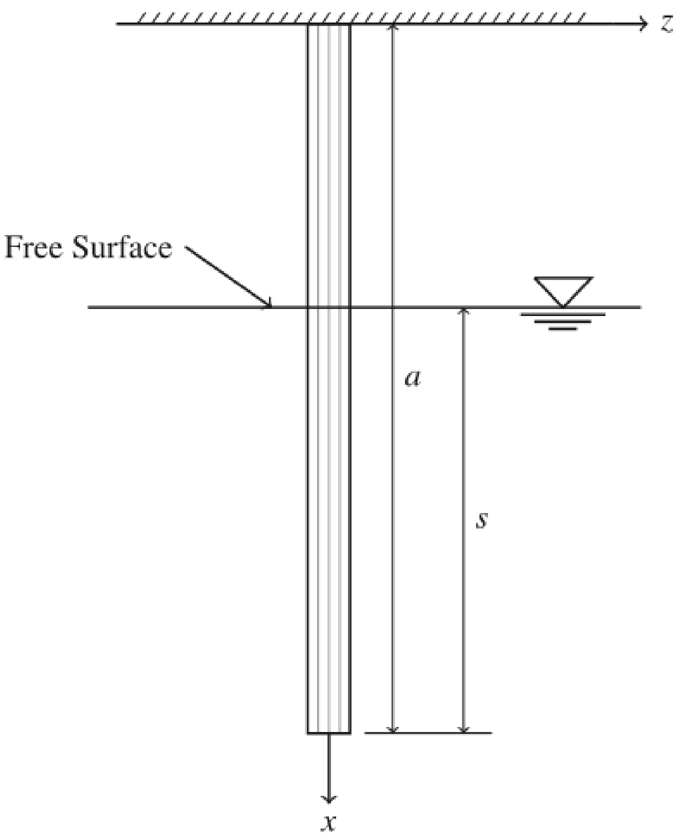




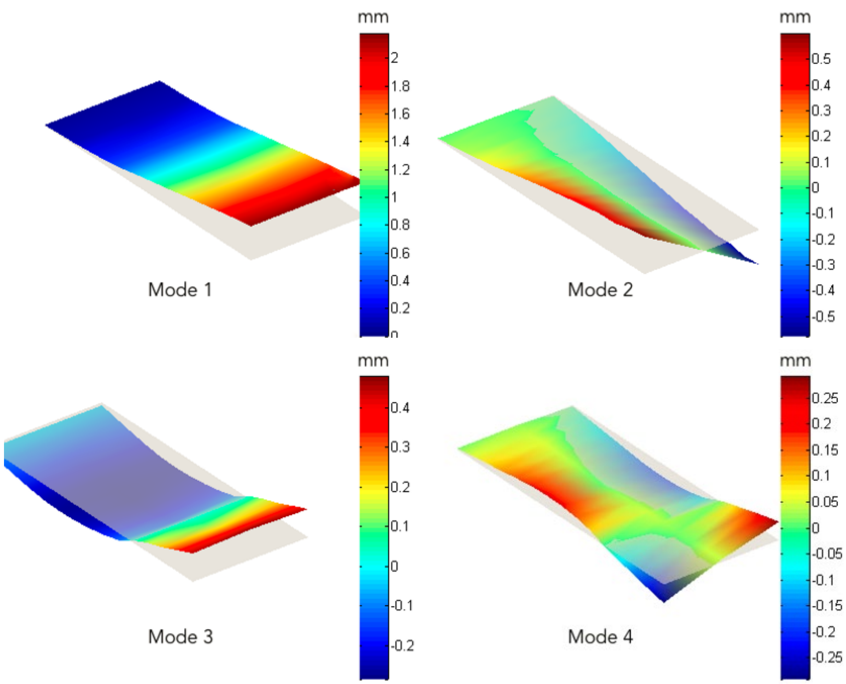


Added mass and added stiffness in real life ?

- ***Rectangular cantilever beam of different materials***
  - ***Effect of the submergence on resonance frequencies***



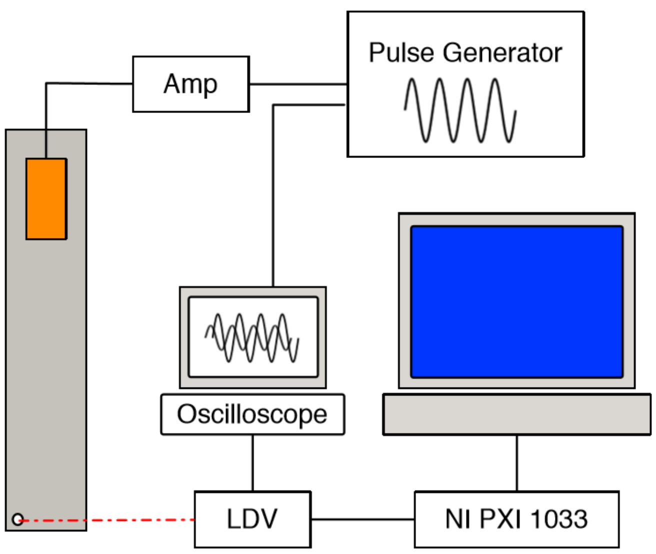
Mode shapes in air (Numerical simulation)

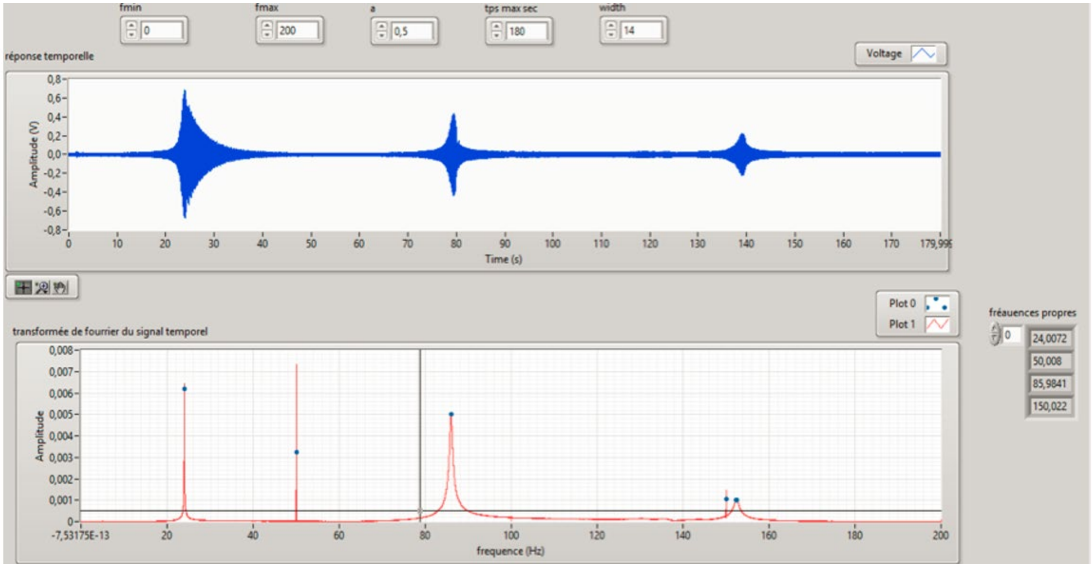


Small Reduced Velocity: Added mass, added stiffness

Added mass and added stiffness in real life ?

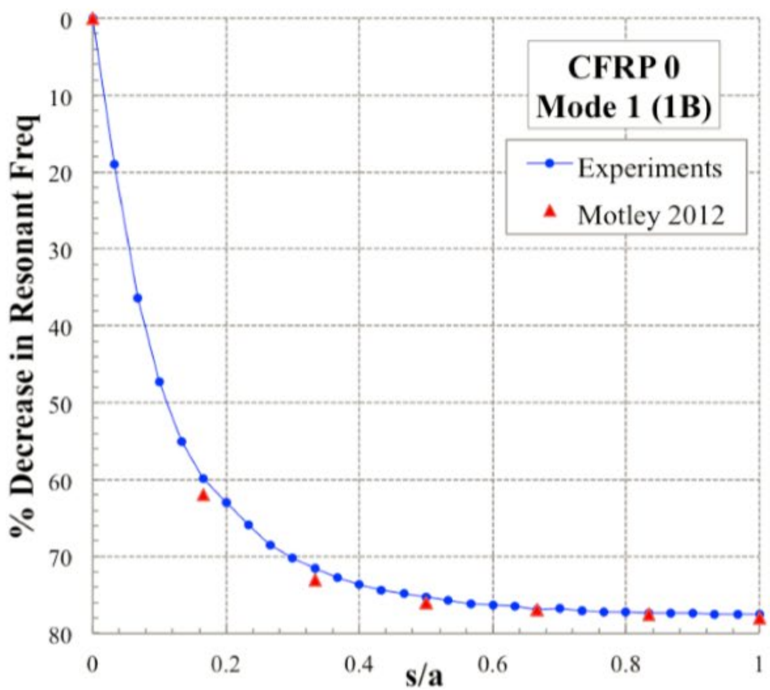
- ***Experimental procedure:***
  - ***Excitation with a frequency sweep***
  - ***Vibration measurements: Laser Doppler Vibrometer***
  - ***Spectral analysis → resonance frequencies vs immersion level***

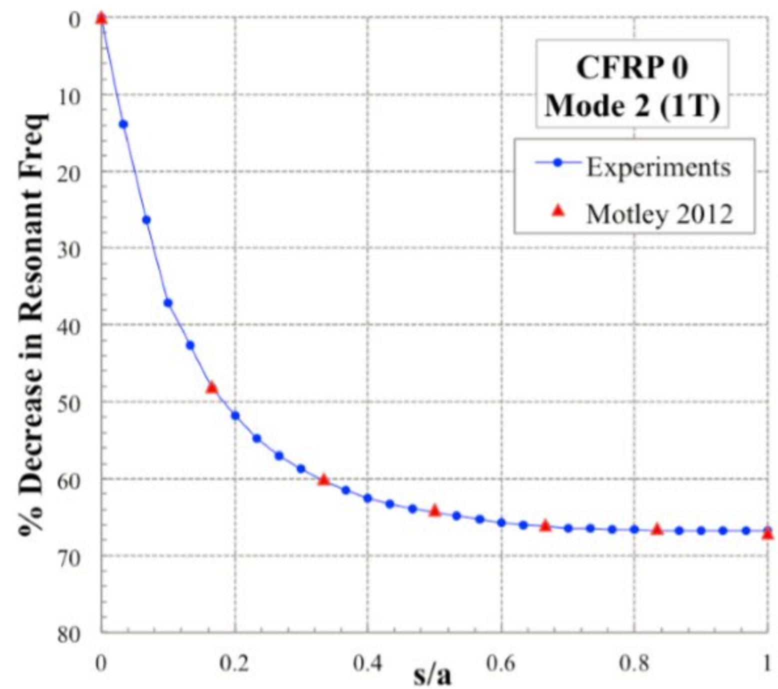




Added mass and added stiffness in real life ?

- ***Result: 1st bending and torsion modes (Exp. vs numerical simulation)***
  - ***Significant decrease of resonance frequencies***
  - ***Strong influence of immersion level***





Added mass and added stiffness in real life ?

- ***Result: 2nd bending and torsion modes (Exp. vs numerical simulations)***
  - ***Significant decrease of resonance frequencies***
  - ***Strong influence of submergence level***
  - ***Effect of the free surface waves (not predicted by the simulations)***