
AEROELASTICITY AND FLUID-STRUCTURE INTERACTION

Chapter 6:

Aeroelasticity Pseudo-Static framework

Pseudo-static Aeroelasticity

$$U_R \ll 1$$



The solid evolves in an almost still fluid

- *Flow induced stiffness*
- *Added mass*

$$U_R \gg 1$$



The fluid evolves with an almost fixed solid

- *Static instability (Divergence)*
- *Dynamic Instability (Flutter)*

Case of moderate reduced velocity ?

What if U_R is not very small and not very large ?

$$U_R = \frac{T_{solid}}{T_{fluid}}$$

Pseudo-static Aeroelasticity

Moderate reduced velocity:

- *The velocity of the solid may not be neglected:*
- *Far from the interface: $U_f \sim \mathcal{O}(1)$*
- *At the fluid-solid interface:*

$$U_f \sim \mathcal{O}\left(\frac{\xi_0}{T_{solid}} \frac{1}{U_0}\right) = \mathcal{O}\left(\frac{\xi_0}{T_{solid}} \frac{1}{\frac{L}{T_{fluid}}}\right) = \mathcal{O}\left(\frac{D}{U_R}\right)$$

The non-dimensional displacement of the interface is of the order of D and evolves in a non-dimensional time scale of U_R

- *Acceleration:*

$$t_f = \frac{t}{T_{fluid}} \quad \text{and} \quad t \sim T_{solid} \quad \Rightarrow \quad \frac{\partial U_f}{\partial t_f} \sim \mathcal{O}\left(\frac{D}{U_R^2}\right)$$
$$\Delta t_f = 1 \quad \Rightarrow \quad \Delta U_f = \frac{\partial U_f}{\partial t_f} \Delta t_f \sim \mathcal{O}\left(\frac{D}{U_R^2}\right)$$

Pseudo-static Aeroelasticity

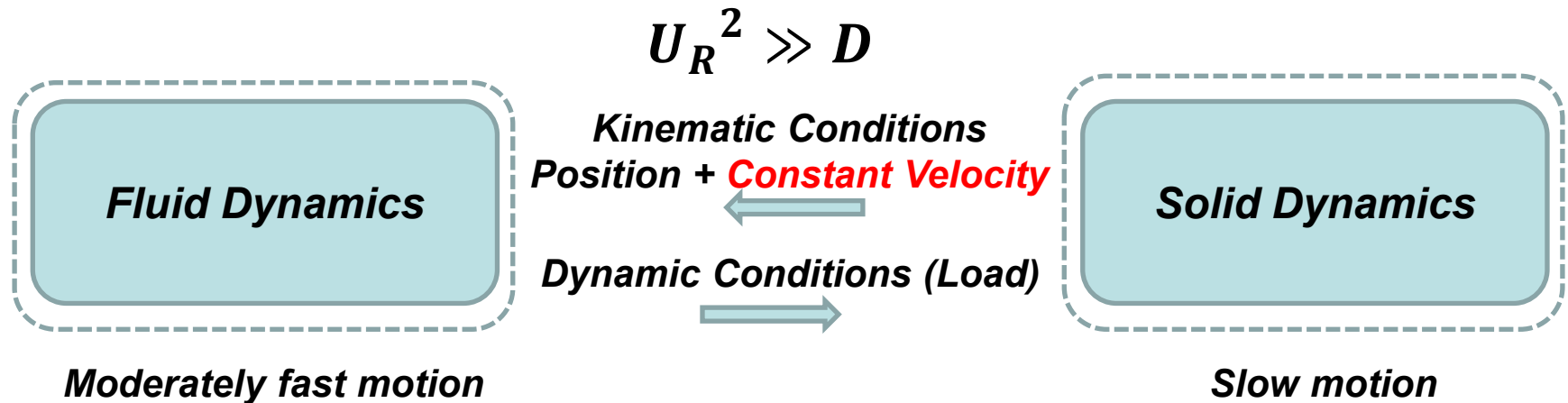
Pseudo-static aeroelasticity:

- **Hypothesis:** $\frac{D}{U_R^2} \ll 1 \Rightarrow U_R^2 \gg D \Rightarrow \frac{\partial U_f}{\partial t_f} \approx 0$ at the interface
- **The velocity of the solid is supposed constant within the time scale of the fluid. Its variations are neglected.**

→ From fluid viewpoint, the solid moves so slowly that its velocity seems constant : **Pseudo-Static Aeroelasticity**
- **Pseudo-static vs quasi-static aeroelasticity:**
The condition $U_R^2 \gg D$ is weaker than in quasi-static aeroelasticity ($U_R \gg 1 \gg D$), where the solid seems frozen from the fluid viewpoint ($U_f = 0$ at the interface)

Pseudo-static Aeroelasticity

Pseudo-Static aeroelasticity approximation



At every time step (on the fluid side):

- **The flow may be solved by considering both the actual position and velocity of the solid frozen in time.**
- **As for quasi-static aeroelasticity approximation, there is no need to solve fluid and solid motions simultaneously.**
→ Iterations : fluid computation ↔ solid computation

Pseudo-static Aeroelasticity

Dimensionless equations of solid motion

- *In the solid side (single mode approximation):*

$$\xi_s(\mathbf{x}_s, t_s) = \mathbf{D}q_s(t_s)\phi(\mathbf{x}_s)$$

$$U_R^2 \frac{d^2 q_s}{dt_f^2} + q_s = f_s$$

- *In the case of stationary flow, we may state that, at the interface and everywhere in the fluid, pressure and velocity depend only on the position and the velocity of the solid:*

$$p_f(\mathbf{D}q_s, \mathbf{D}\dot{q}_s) \quad \text{and} \quad U_f(\mathbf{D}q_s, \mathbf{D}\dot{q}_s)$$

Pseudo-static Aeroelasticity

Dimensionless equations of solid motion

- *The fluid load on the solid (Expansion in D):*

$$F_s = C_y \int_I \left\{ \left[-p_f I + \frac{1}{Re} (\nabla U_f + \nabla^t U_f) \right] \cdot n \right\} \cdot \phi dS$$

$$F_s = C_y F(Re, Dq_s, D\dot{q}_s)$$

$$F_s = F_s^0 + Df_s + \dots \quad f_s : \text{fluctuation of the fluid loading on the solid}$$

F_s^0 only affects the static equilibrium. No influence on the solid dynamics

$$f_s = C_y \left(\frac{\partial F}{\partial Re} \right) Re + C_y \left(\frac{\partial F}{\partial q_s} \right) q_s + C_y \left(\frac{\partial F}{\partial \dot{q}_s} \right) \dot{q}_s \dots$$

Pseudo-static Aeroelasticity

Dimensionless equations of solid motion

- *The fluid load on the solid is the sum of 2 forces:*

$$f_s = C_y \left(\frac{\partial F}{\partial q_s} \right) q_s + C_y \left(\frac{\partial F}{\partial \dot{q}_s} \right) \dot{q}_s \dots$$

- $C_y \left(\frac{\partial F}{\partial q_s} \right) q_s$: *proportional to modal displacement*
→ *fluid induced stiffness force*
(already obtained in quasi-static aeroelasticity approximation)
- $C_y \left(\frac{\partial F}{\partial \dot{q}_s} \right) \dot{q}_s$: *proportional to solid velocity*
→ *fluid induced damping force*

Pseudo-static Aeroelasticity

Dimensionless equations of solid motion

- If T_{solid} is taken as reference time ($t_s = t/T_{solid}$), the modal displacement obeys the following equation:

$$\frac{d^2 q_s}{dt_s^2} + q_s = C_y \left(\frac{\partial F}{\partial q_s} \right) q_s + \frac{C_Y}{U_R} \left(\frac{\partial F}{\partial \dot{q}_s} \right) \dot{q}_s$$

- We define the fluid stiffness k_f and fluid damping c_f as follows:

$$k_f = -C_y \left(\frac{\partial F}{\partial q_s} \right) \quad \text{and} \quad c_f = -\frac{C_Y}{U_R} \left(\frac{\partial F}{\partial \dot{q}_s} \right)$$

$$\Rightarrow \frac{d^2 q_s}{dt_s^2} + c_f \dot{q}_s + (1 + k_f) q_s = 0$$

Here we did not take into account the structural damping

Pseudo-static Aeroelasticity

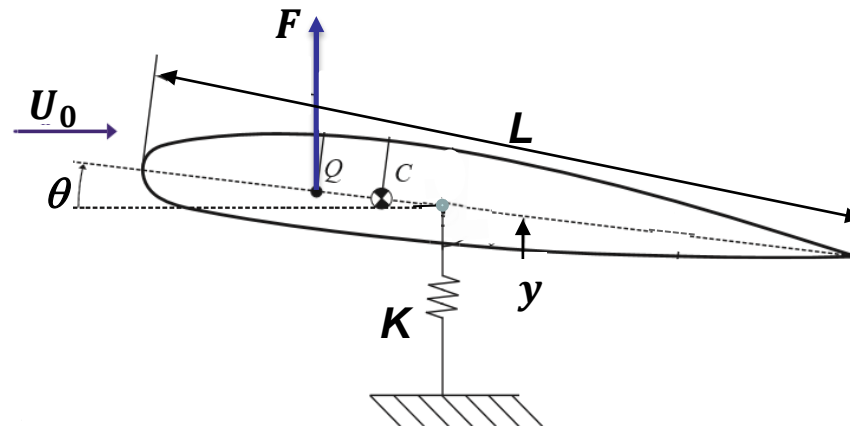
Dimensionless equations of solid motion

- *The fluid damping (as the fluid stiffness) is proportional to Cauchy number $\left(Cy = \frac{\rho U_0^2}{E}\right)$*
- *The fluid damping may be positive or negative:*
 - *$c_f > 0$: The oscillations are damped by the fluid (stable)*
 - *$c_f < 0$: The oscillations increase without limit (unstable)*
- *Comparison with the quasi-static instability:*
 - *Quasi-static instability involves two modes with different frequencies, which come close to each other by the flow*
 - *For pseudo-static instability, any given mode may become unstable without the need of a frequency coincidence with another mode. Such instability is therefore more likely to occur.*

Pseudo-static Aeroelasticity

Case of a flow over an airfoil in plunge mode

- We consider an airfoil of mass M , placed in an air stream of upstream velocity U_0 at a fixed incidence θ , attached to a spring (stiffness K) so that it can move only in the vertical direction
- In quasi-static aeroelasticity approximation ($U_R \gg D$)
 - Equation of the airfoil motion ?
 - Is there any fluid induced stiffness and/or damping ?
 - Stability ?



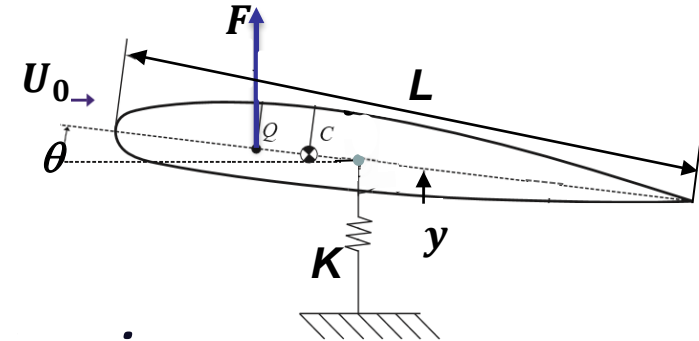
Pseudo-static Aeroelasticity

Case of a flow over an airfoil in plunge mode

- ***In quasi-static aeroelasticity approximation*** ($U_R \gg D$):
 - *As the airfoil moves in the y direction, its incidence angle remains unchanged and so does the lift force.*
 - *The equation of solid motion reads:*

$$M\ddot{y} + Ky = 0$$

(The lift and stiffness forces cancel out)



- *The flow has no effect on the airfoil dynamics (except translational displacement)*
Increase of flow velocity → the spring is more stretched
Dynamically, the airfoil behaves as if it is in still fluid ($U_0 = 0$)
- *Infinite stiffness in torsion → there is no risk of divergence*

Pseudo-static Aeroelasticity

Case of a flow over an airfoil in plunge mode

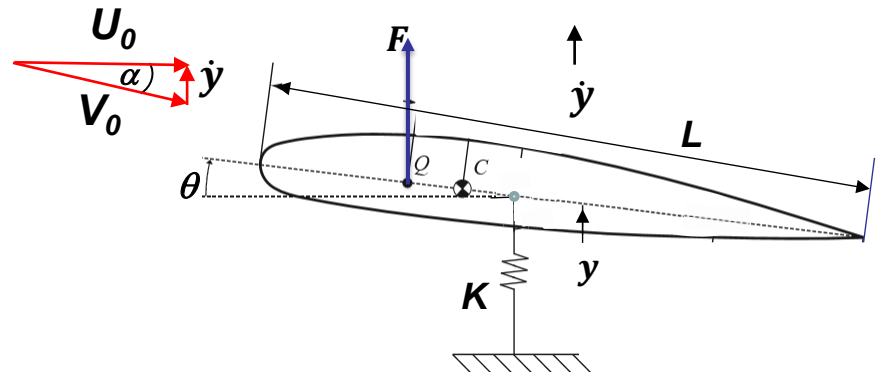
- **In pseudo static aeroelasticity approximation** ($U_R^2 \gg D$)
 - The solid displacement (y) and velocity (\dot{y}) are both frozen
 - In the frame of the airfoil, the upstream velocity is V_0 and the effective incidence angle is θ_{eff} such that:
 - The Lift force (F) depends on both θ and \dot{y}

$$\vec{U}_0 = \vec{V}_0 + \vec{\dot{y}}$$

$$\theta_{eff} = \theta + \alpha$$

$$\tan \alpha = -\frac{\dot{y}}{U_0} \quad \alpha \text{ and } \dot{y} \text{ are of opposite signs}$$

$$V_0^2 = U_0^2 + \dot{y}^2$$



Pseudo-static Aeroelasticity

Case of a flow over an airfoil in plunge mode

- In pseudo static aeroelasticity approximation ($U_R^2 \gg D$)
 - The aerodynamic force (lift):

$$F = \frac{1}{2} \rho V_0^2 L C_L(\theta_{eff}) = \frac{1}{2} \rho V_0^2 L C_L(\theta + \alpha)$$

$$F = \frac{1}{2} \rho U_0^2 (1 + (\tan \alpha)^2) L \left(C_L(\theta) + \frac{\partial C_L}{\partial \theta}(\theta) \cdot \alpha \right)$$

- For small values of α : $\tan \alpha \approx \alpha$ $\alpha^2 \ll 1$

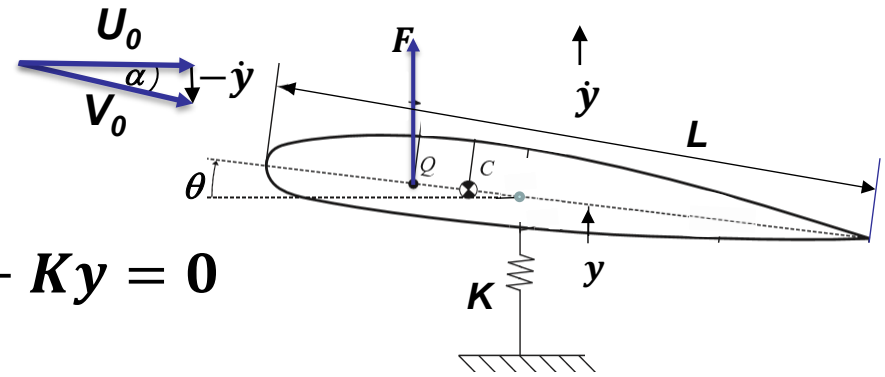
$$\Rightarrow F \approx F_0 + \frac{1}{2} \rho U_0^2 L \frac{\partial C_L}{\partial \theta}(\theta) \cdot \alpha = F_0 - \frac{1}{2} \rho U_0 L \frac{\partial C_L}{\partial \theta}(\theta) \cdot \dot{y}$$

Pseudo-static Aeroelasticity

Case of a flow over an airfoil in plunge mode

- In pseudo static aeroelasticity approximation ($U_R^2 \gg D$)
 - Equation of the airfoil motion:

$$M\ddot{y} + \underbrace{\frac{1}{2} \rho U_0 L \frac{\partial C_L}{\partial \theta}(\theta)}_{\text{Flow induced damping}} \dot{y} + Ky = 0$$

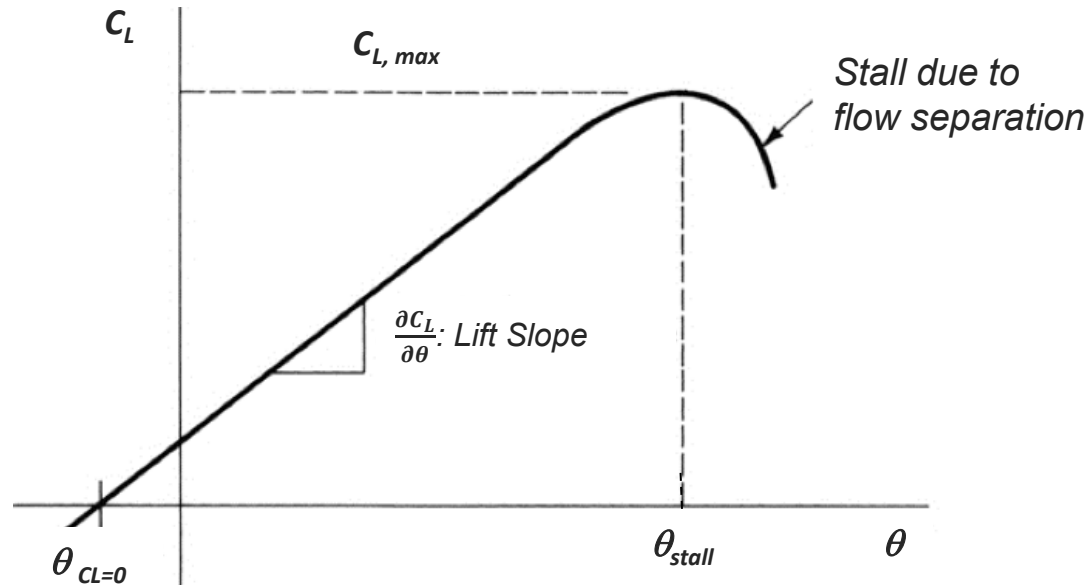


- The sign of lift slope is the key parameter:
 - Positive damping ($\frac{\partial C_L}{\partial \theta} > 0$) \rightarrow Stable
 - Negative damping ($\frac{\partial C_L}{\partial \theta} < 0$) \rightarrow Unstable

Pseudo-static Aeroelasticity

Case of a flow over an airfoil in plunge mode

- In pseudo static aero-elasticity approximation ($U_R^2 \gg D$)
 - Typical curve of the lift coefficient vs angle of attack θ :



$\theta < \theta_{stall} \Rightarrow \frac{\partial C_L}{\partial \theta} > 0$
positive damping \rightarrow Stable

$\theta > \theta_{stall} \Rightarrow \frac{\partial C_L}{\partial \theta} < 0$
Negative damping \rightarrow Unstable \rightarrow Stall flutter

Pseudo-static Aeroelasticity

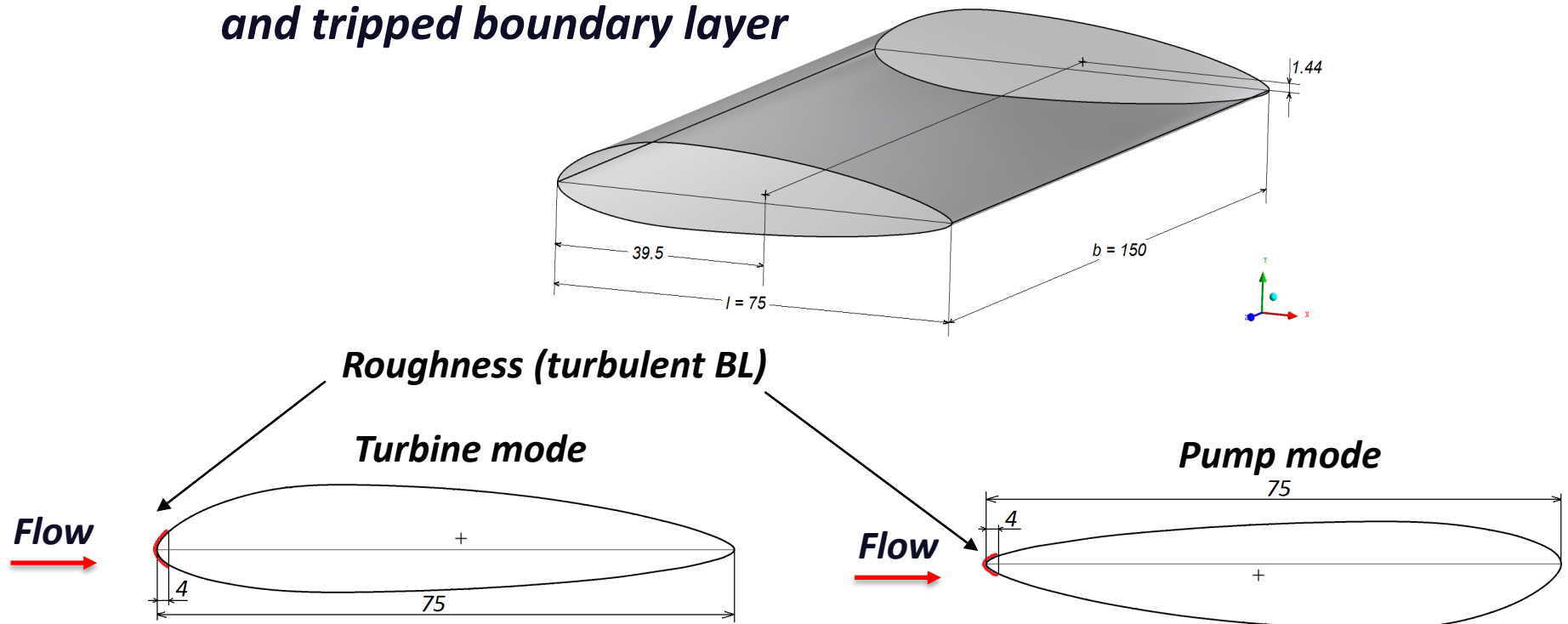
Summary of possible instabilities for a foil

- ***Fast flows ($UR \gg D$) – quasi-static aeroelasticity approximation***
 - ***Divergence: Static instability in torsion mode***
 - ***Flutter: Dynamic instability of the torsion and plunge modes***
- ***Moderate flows ($UR^2 \gg D$) – Pseudo-static aeroelasticity approximation***
 - ***Stall Flutter : Dynamic instability of the plunge mode***
- ***These instabilities may be encountered in any configuration involving lifting surfaces in a flow***

Pseudo-static Aeroelasticity

Case of a flow over a wicket gate of a pump turbine

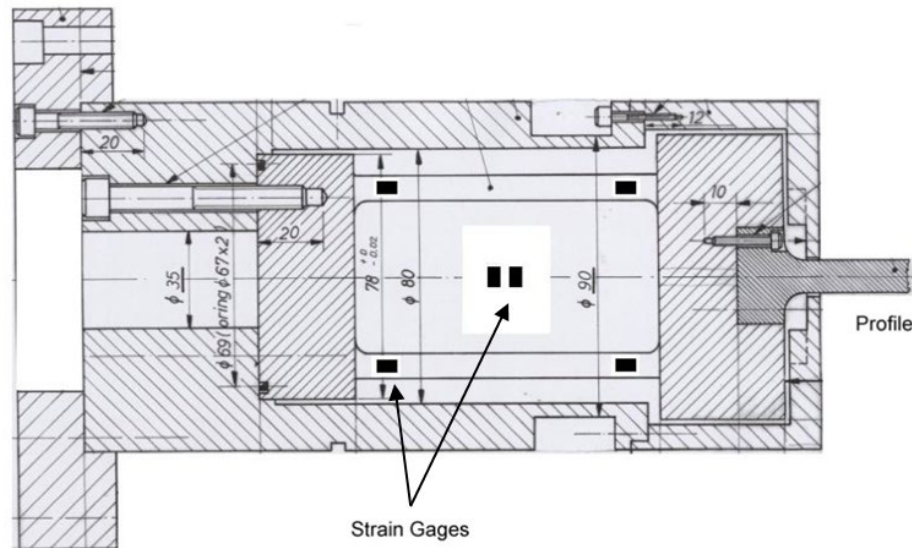
- **Designed to operate in both pumping and generating modes**
 - **Rounded leading and trailing edges**
 - **Tested in EPFL High-Speed Cavitation Tunnel with smooth and tripped boundary layer**



Pseudo-static Aeroelasticity

Case of a flow over a wicket gate of a pump turbine

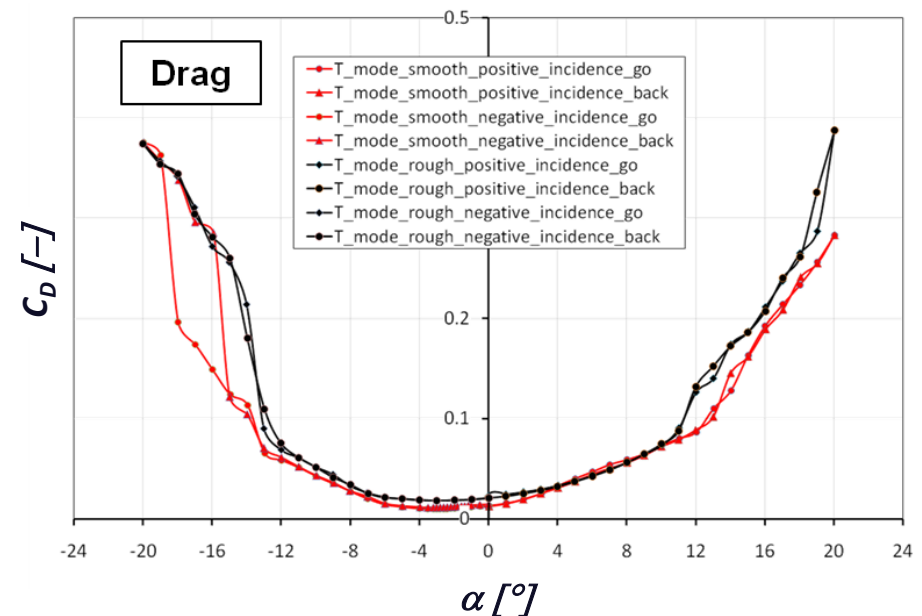
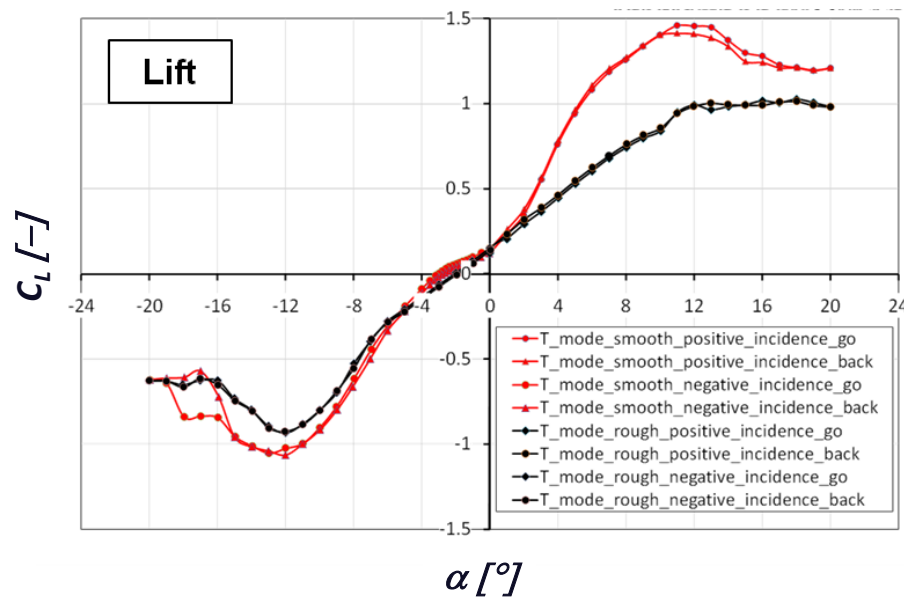
- *Measurement of hydrodynamic forces (lift and drag):*
 - *5-components load cell*



Pseudo-static Aeroelasticity

Case of a flow over a wicket gate of a pump turbine

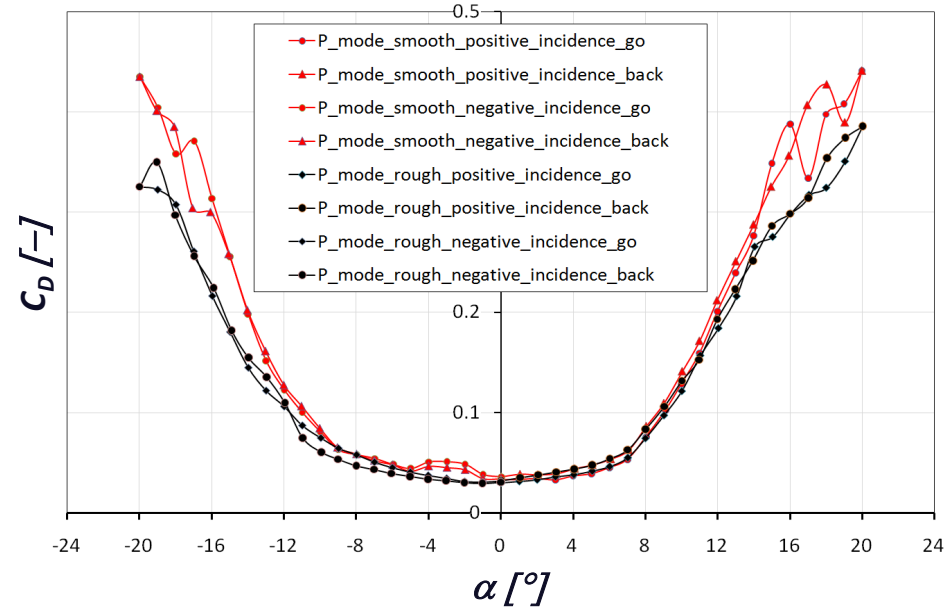
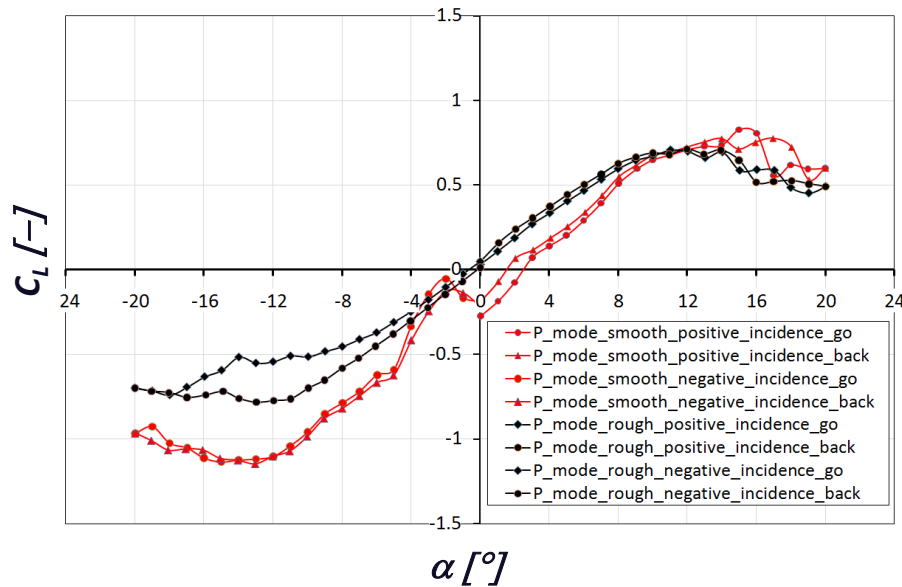
- Lift and drag coefficients vs angle of attack in **turbine mode**:
 - Natural and tripped transition of the boundary layer
 - Stable for $-12^\circ < \alpha < 12^\circ$



Pseudo-static Aeroelasticity

Case of a flow over a wicket gate of a pump turbine

- Lift and drag coefficients vs angle of attack in **pump mode**:
 - Natural and tripped transition of the boundary layer



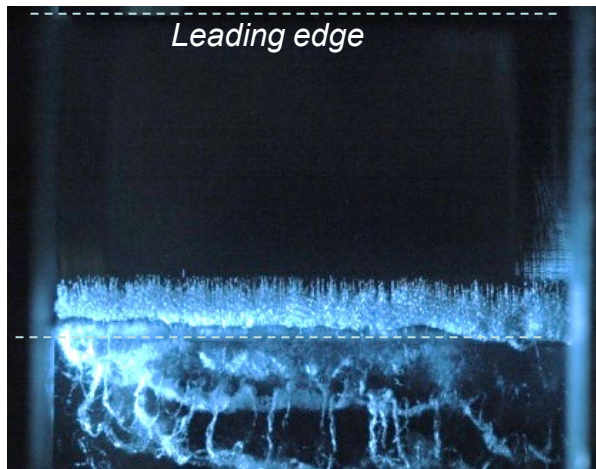
- Natural BL transition: negative lift slope for $-2^\circ < \theta < 0^\circ$
→ Risk of lift galloping due to negative slope of $C_L(\alpha)$
Cause: Boundary layer separation/attachment near 0° incidence

Pseudo-static Aeroelasticity

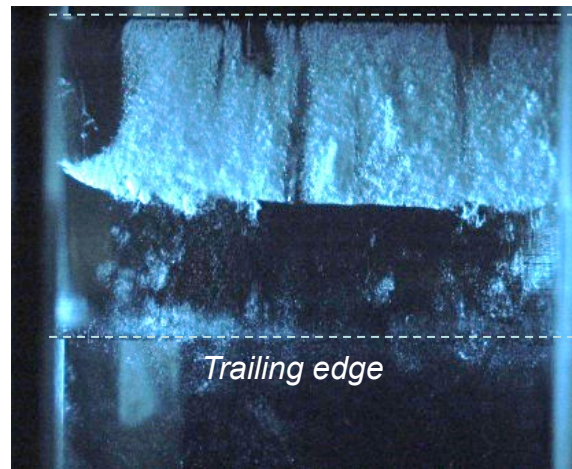
Case of a flow over a wicket gate of a pump turbine

- Flow visualization (top view) in pump mode, using cavitation:

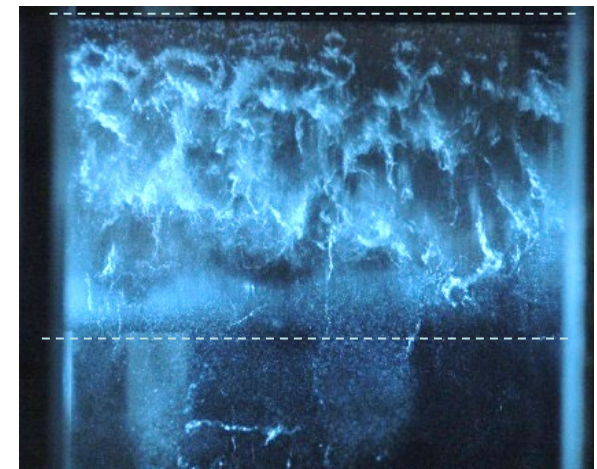
$\alpha = -2^\circ$, Vortex shedding
(Strong vibration)



$\alpha = -8^\circ$, attached flow



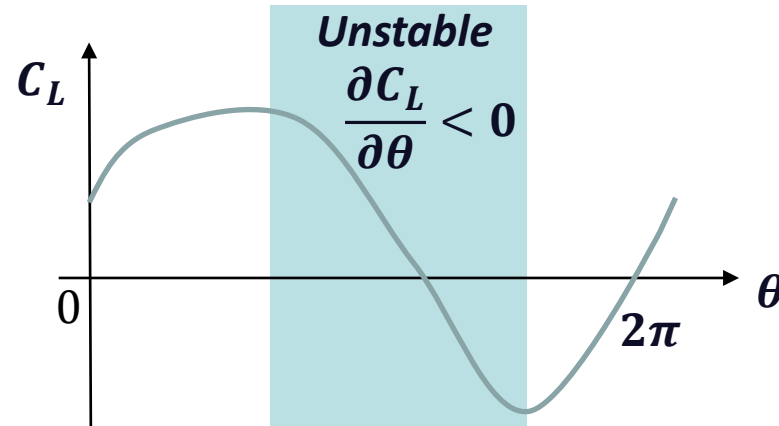
$\alpha = -16^\circ$, BL separation
at leading edge (stall)



Pseudo-static Aeroelasticity

Case of an arbitrary bluff body in plunge mode – Lift Galloping

- In pseudo static aero-elasticity approximation ($U_R^2 \gg D$)
 - The $C_L(\theta)$ curve may be of any shape but must be periodic
 - There must be a portion of the C_L curve where $\frac{\partial C_L}{\partial \theta} < 0$



- Such instability is called “Lift Galloping”.
It is a generalization of the stall flutter

Pseudo-static Aeroelasticity

Example : The case of tall buildings

- *There always exist an interval for wind direction where the lift slope is negative → Risk of lift galloping*
 - *Such a risk is seriously taken into account during the design process (numerical simulation and model testing)*
 - *Sophisticated monitoring of static & dynamic deformations of the building with the help of sensing networks*

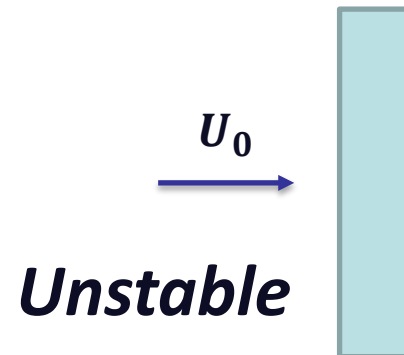
Reduced scale model of Burj Khalifa placed in 2.4 x 2.0 m test section of a wind tunnel



Pseudo-static Aeroelasticity

Lift galloping

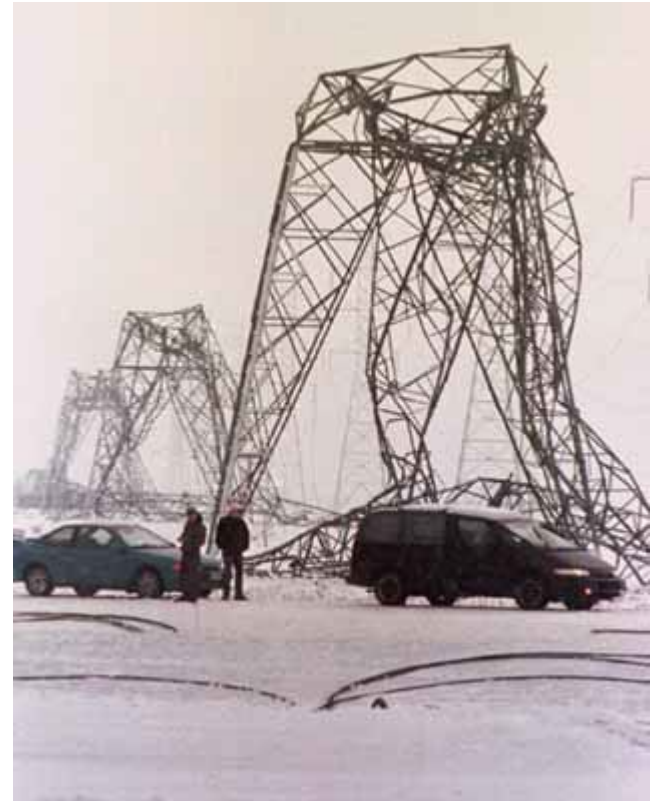
- *It is well known that for small incidence angles:*
 - *Elongated shapes in the streamwise direction are stable*
 - *Elongated shapes in the transverse direction are unstable*



Pseudo-static Aeroelasticity

Lift galloping

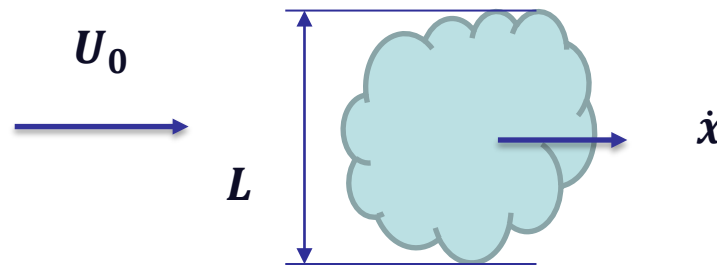
- **Example: Electric lines during ice storms (Quebec blackout, 1998)**
 - **Ice accumulation around the lines**
→ **Increased risk of lift galloping**



Pseudo-static Aeroelasticity

Drag induced instability : Drag Crisis

- **Within the pseudo static aeroelasticity approximation ($U_R^2 \gg D$)**
 - **We consider a bluff body oscillating in the direction of the flow:**

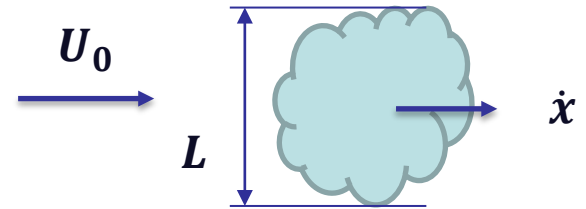


- **The fluid loading (Drag force) ?**
- **Is there any flow induced stiffness or damping ?**
- **Stability in the case of a cylinder ?**

Pseudo-static Aeroelasticity

Drag induced instability : Drag Crisis

- *Within the pseudo static aeroelasticity approximation ($UR^2 \gg D$)*
 - *We consider a bluff body moving the direction of the flow with a speed frozen in time:*



- *The fluid loading (Drag force):*

$$F(U_0 - \dot{x}) = \frac{1}{2} \rho (U_0 - \dot{x})^2 L C_D(R_e) \approx F(U_0) - \dot{x} \frac{\partial F}{\partial \dot{x}} + \dots$$

$$\frac{\partial F}{\partial \dot{x}} = \frac{\partial}{\partial \dot{x}} \left(\frac{1}{2} \rho (U_0 - \dot{x})^2 L C_D(R_e) \right) \quad R_e = \frac{\rho (U_0 - \dot{x}) L}{\mu}$$

$$\frac{\partial F}{\partial \dot{x}} \approx -\rho U_0 L \left(C_D + \frac{1}{2} R_e \frac{\partial C_D}{\partial R_E} \right) \quad \text{assuming } \dot{x} \ll U_0$$

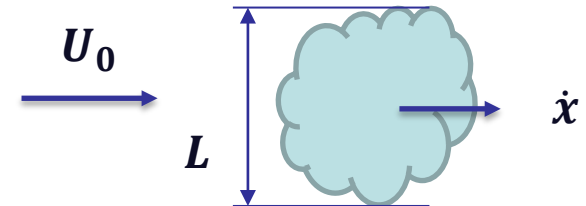
Pseudo-static Aeroelasticity

Drag induced instability : Drag Crisis

- In pseudo static aeroelasticity approximation ($U_R^2 \gg D$)
 - We consider a bluff body oscillating in the direction of the flow:

- The fluid loading (Drag force):

- Damping force



$$F(U_0 - \dot{x}) \approx F(U_0) - \rho U_0 L \dot{x} \left(C_D + \frac{1}{2} R_e \frac{\partial C_D}{\partial R_e} \right)$$

- $C_D + \frac{1}{2} R_e \frac{\partial C_D}{\partial R_e} > 0 \rightarrow$ positive damping \rightarrow Stable
- $C_D + \frac{1}{2} R_e \frac{\partial C_D}{\partial R_e} < 0 \rightarrow$ negative damping \rightarrow Unstable

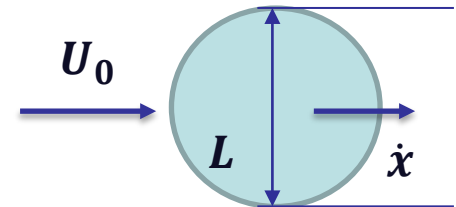
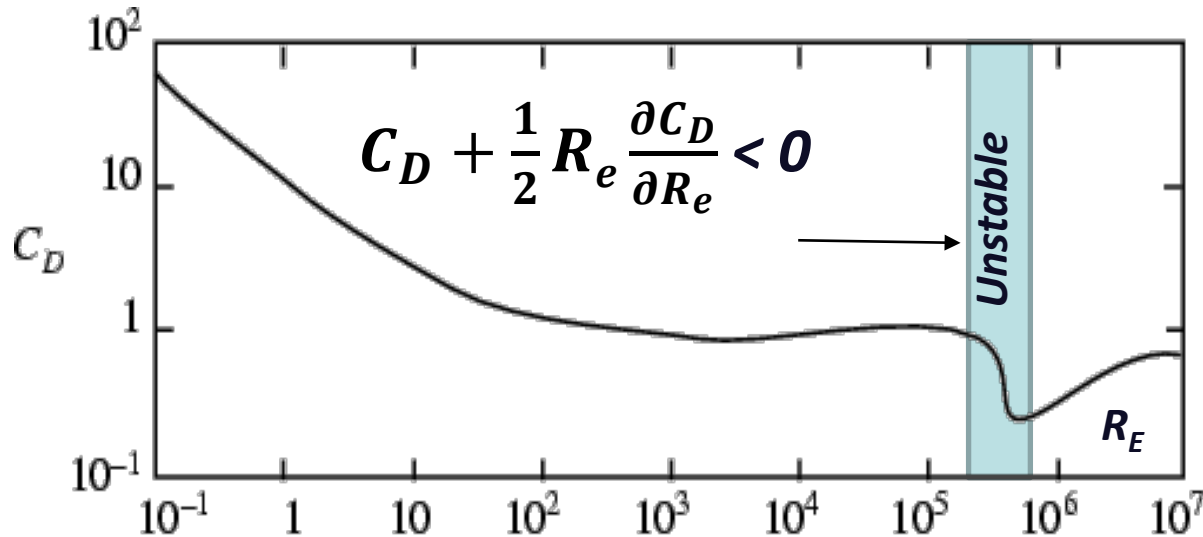
\rightarrow DRAG CRISIS Instability or DRAG Galloping

Pseudo-static Aeroelasticity

Drag induced instability : Drag Crisis

- *Is it possible to have a negative drag-induced damping ?*
- *Example: A cylinder oscillating in the direction of the flow:*

- *The Drag coefficient of a cylinder vs R_e :*

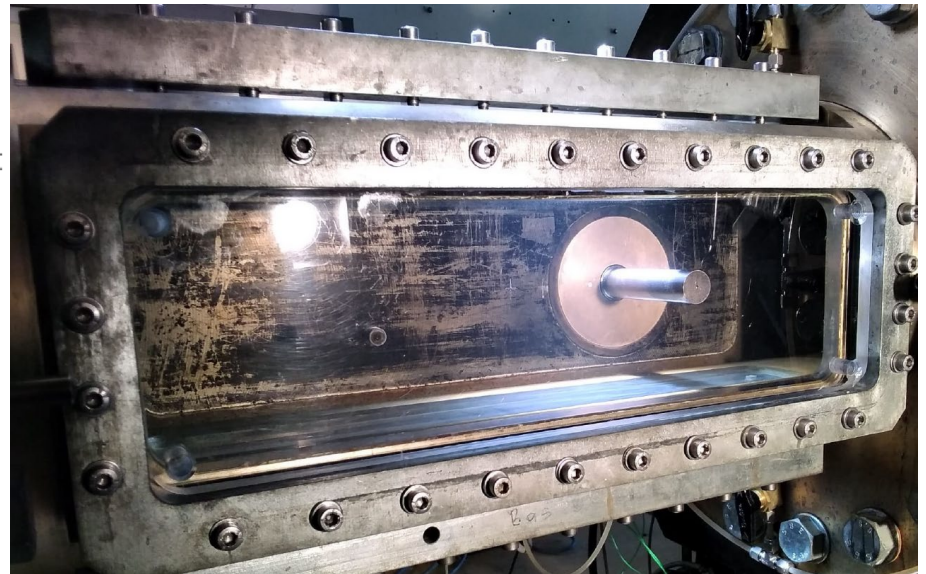
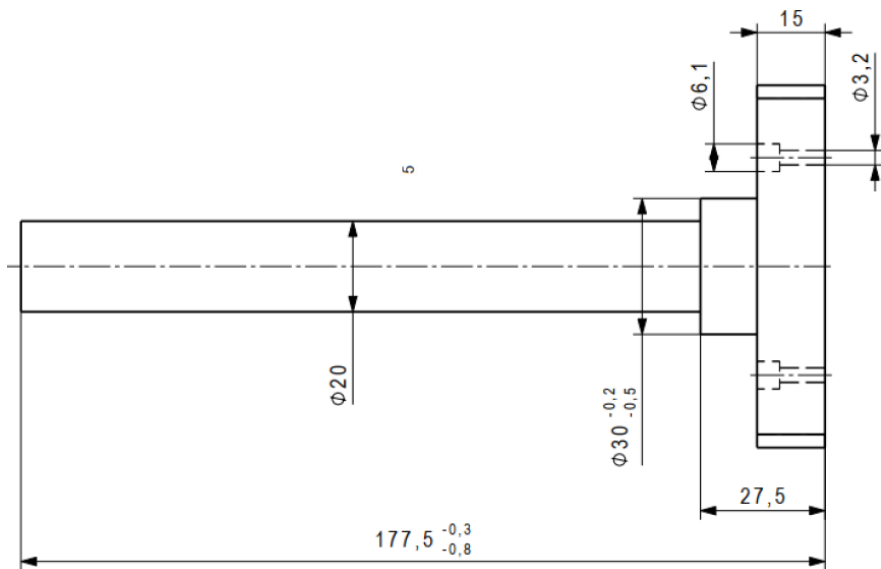


- *A negative damping is the result of a profound change in the flow structure (boundary layer separation delayed)*

Pseudo-static Aeroelasticity

Drag induced instability : Drag Crisis

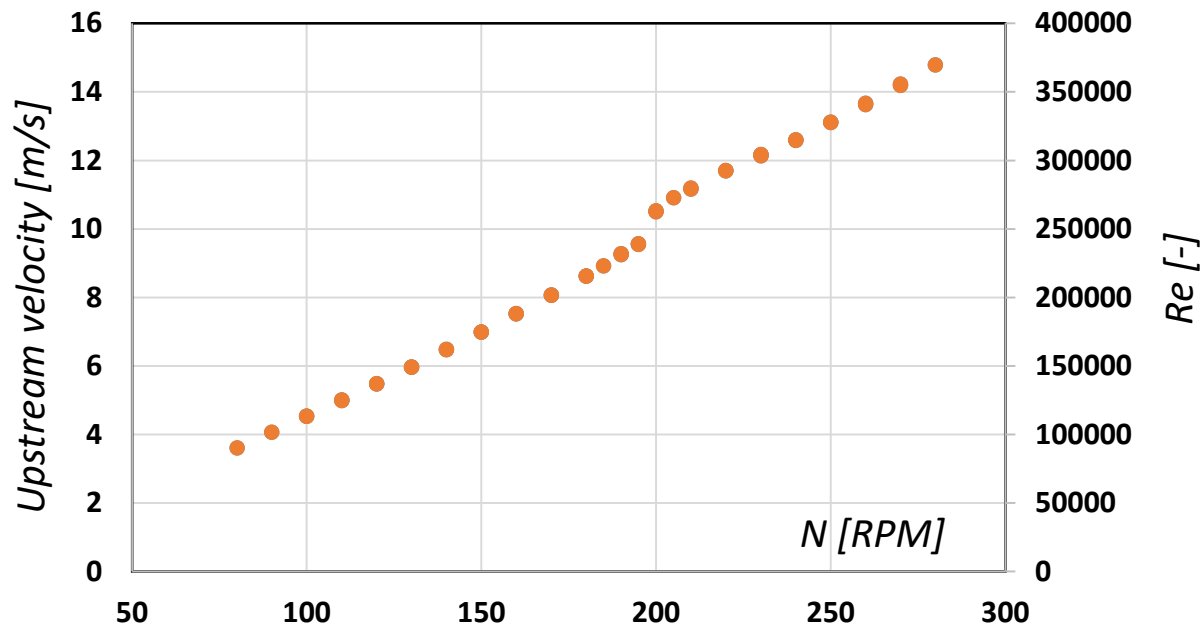
- **Case f a cylinder in EPFL Cavitation Tunnel (semester project 2021)**
 - **Stainless steel, made from one bloc of metal**
 - **Several cylinders made of assembly of 2 parts were destroyed because of too much vibration !!**
 - **25 mm diameter, 150 mm span**
 - **Measurement of vibration, Lift&Drag, High-speed visulization**



Pseudo-static Aeroelasticity

Drag induced instability : Drag Crisis

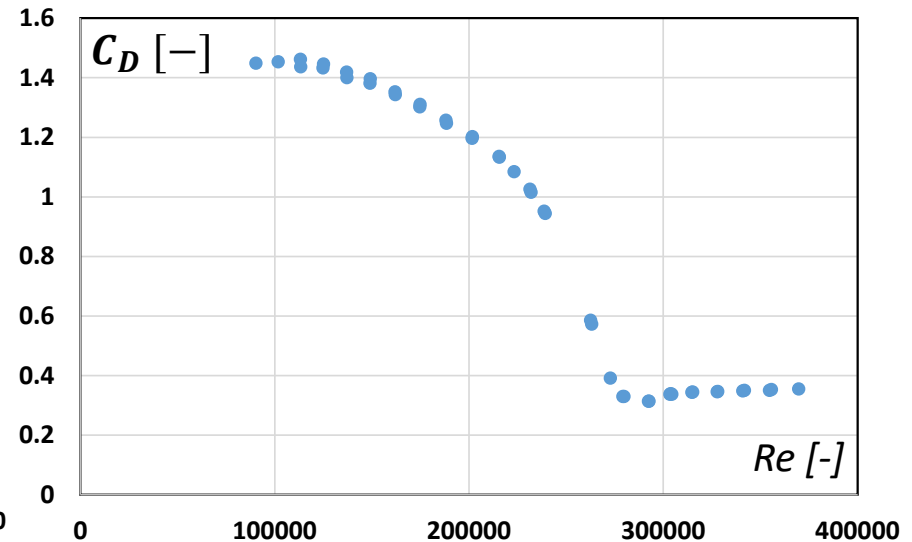
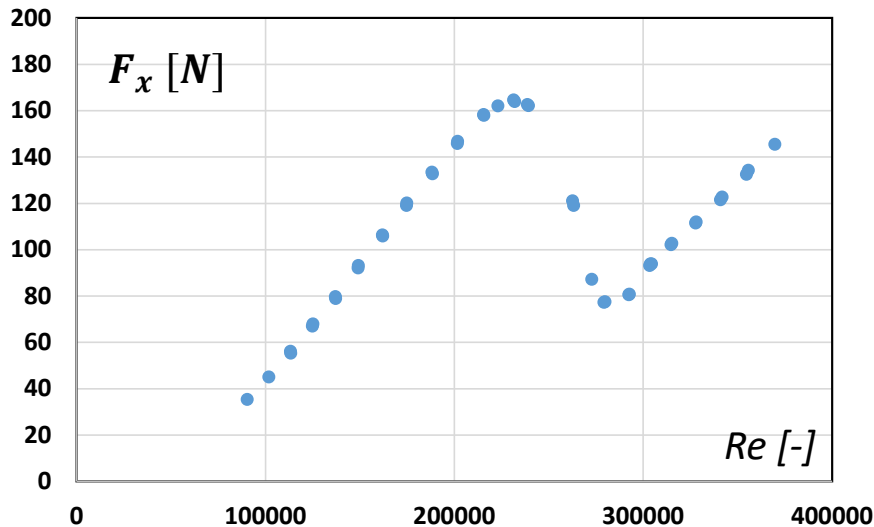
- **Case of a cylinder in EPFL Cavitation Tunnel (semester project 2021)**
 - **Evidence of drag crisis (Cavitation free)**
 - **Upstream velocity vs Rotation speed of the pump**
 - **Constant acceleration of the pump rotation during 3 minutes**
 - **Sudden increase around (200 RPM): $Re=250'000$**
 - **Due to a sudden drop of the drag force**



Pseudo-static Aeroelasticity

Drag induced instability : Drag Crisis

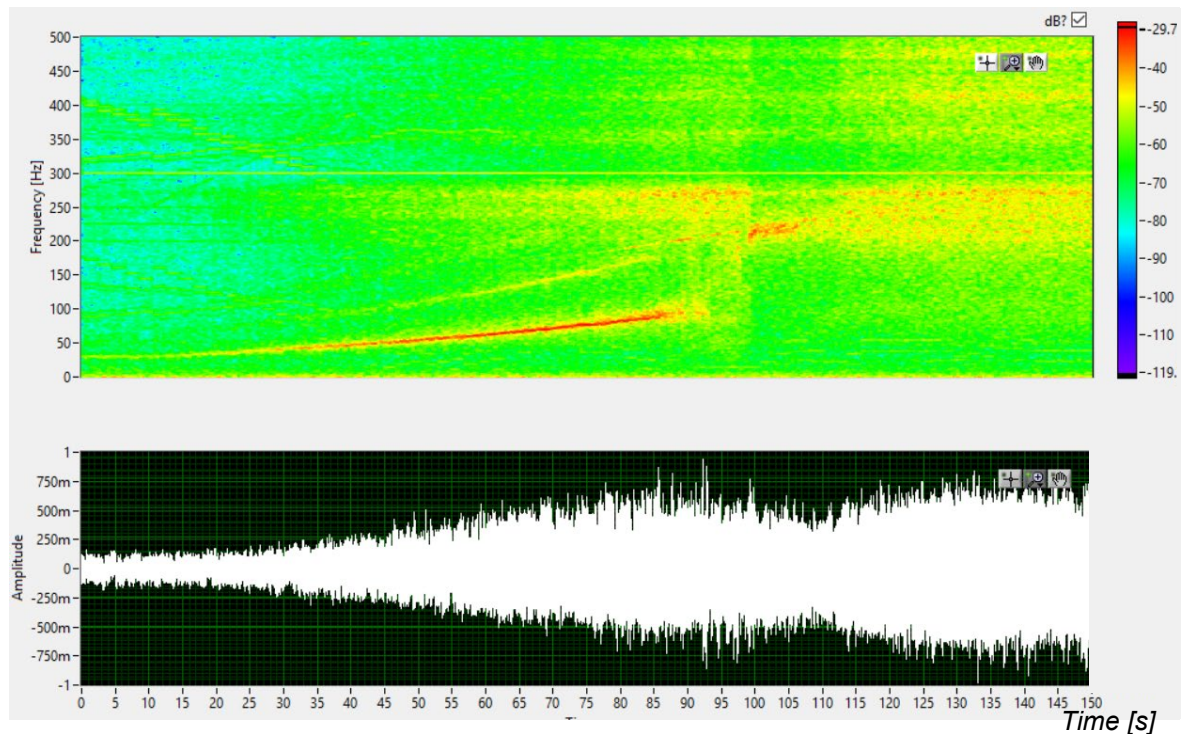
- *Case of a cylinder in EPFL Cavitation Tunnel (semester project 2021)*
 - *Evidence of drag crisis*
 - *Drag force and drag coefficient vs Reynolds number*
 - *Significant decrease of the drag force around $Re=250'000$*
 - *Due to a delay in boundary layer separation*



Pseudo-static Aeroelasticity

Drag induced instability : Drag Crisis

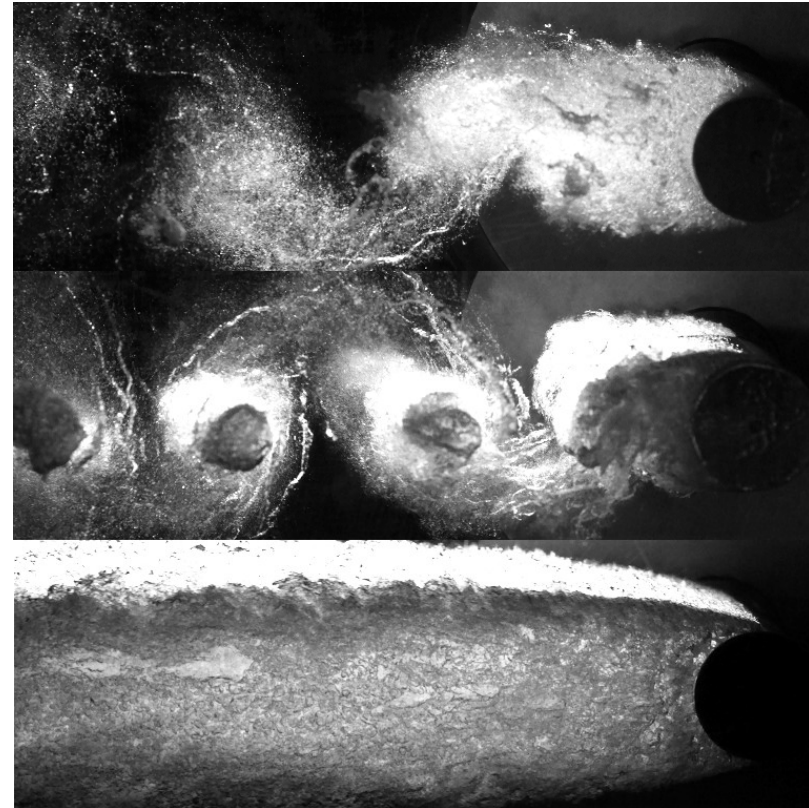
- **Case of a cylinder in EPFL Cavitation Tunnel (semester project 2021)**
 - **Flow induced vibration (upstream velocity 3.6 → 13 m/s)**
 - **No significant increase of vibration at onset or beyond drag crisis**
 - **Strouhal frequency dominant before drag crisis ($St \sim 0.2$)**
 - **Wake less organized beyond drag crisis**



Pseudo-static Aeroelasticity

Drag induced instability : Drag Crisis

- **Case of a cylinder in EPFL Cavitation Tunnel (semester project 2021)**
 - **Effect of cavitation on fluid-structure vibration:**
 - **Cavitation occurrence in the wake**
 - **the vortices are more coherent**
 - **A tremendous increase of vibration**
 - **Supercavitation:**
 - **No vortex shedding**
 - **Minimum vibration**
 - **Minimum drag**
 - **Further research is underway to understand these peculiar effects**



“Lift Crisis”

PRL 117, 234501 (2016)

PHYSICAL REVIEW LETTERS

week ending
2 DECEMBER 2016

Sharp Transition in the Lift Force of a Fluid Flowing Past Nonsymmetrical Obstacles: Evidence for a Lift Crisis in the Drag Crisis Regime

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Bluff bodies moving in a fluid experience a drag force which usually increases with velocity. However in a particular velocity range a *drag crisis* is observed, i.e., a sharp and strong decrease of the drag force. This counterintuitive result is well characterized for a sphere or a cylinder. Here we show that, for an object breaking the up-down symmetry, a *lift crisis* is observed simultaneously to the drag crisis. The term lift crisis refers to the fact that at constant incidence the time-averaged transverse force, which remains small or even negative at low velocity, transitions abruptly to large positive values above a critical flow velocity. This transition is characterized from direct force measurements as well as from change in the velocity field around the obstacle.

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Pseudo-static Aeroelasticity

“Lift Crisis”

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PHYSICAL REVIEW LETTERS

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2 DECEMBER 2016

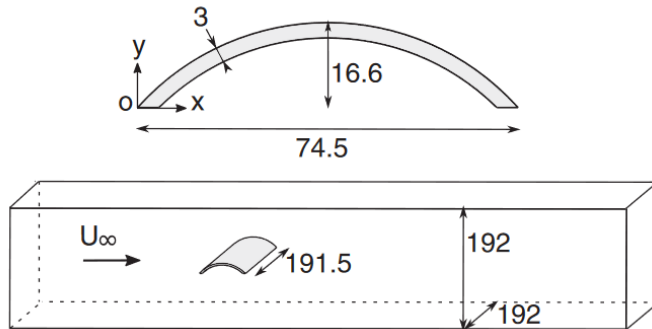
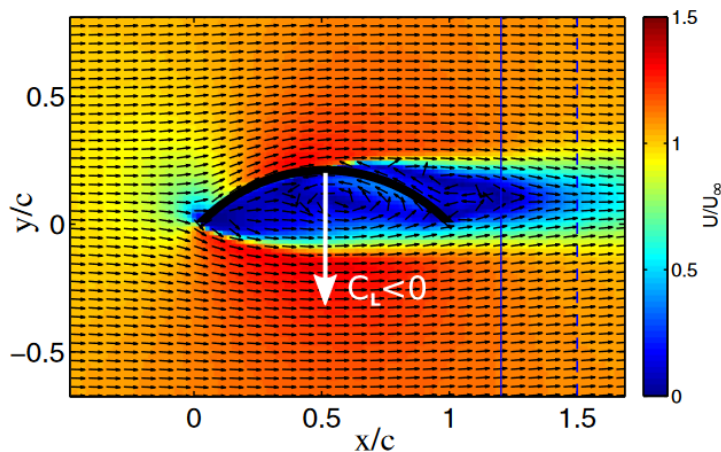
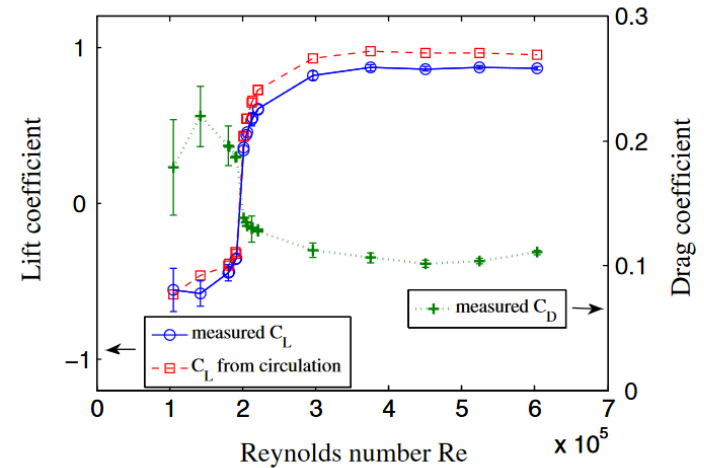
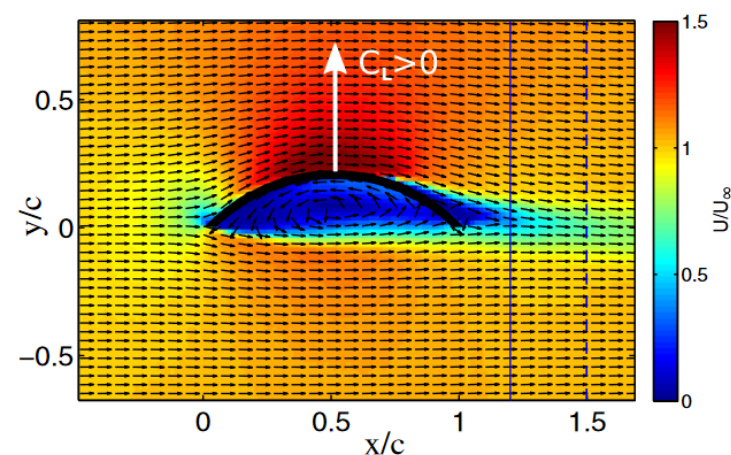


FIG. 1. Curved plate section: chord length $c = 74.5$ mm and camber $t = 16.6$ mm (top) and tunnel test setup (bottom). All dimensions are in mm.

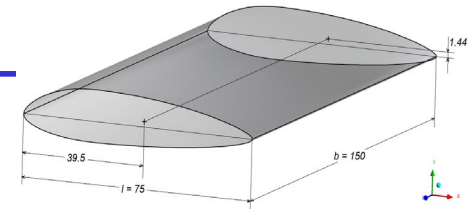


(a) $Re < Re_c$, negative lift



(b) $Re > Re_c$, positive lift

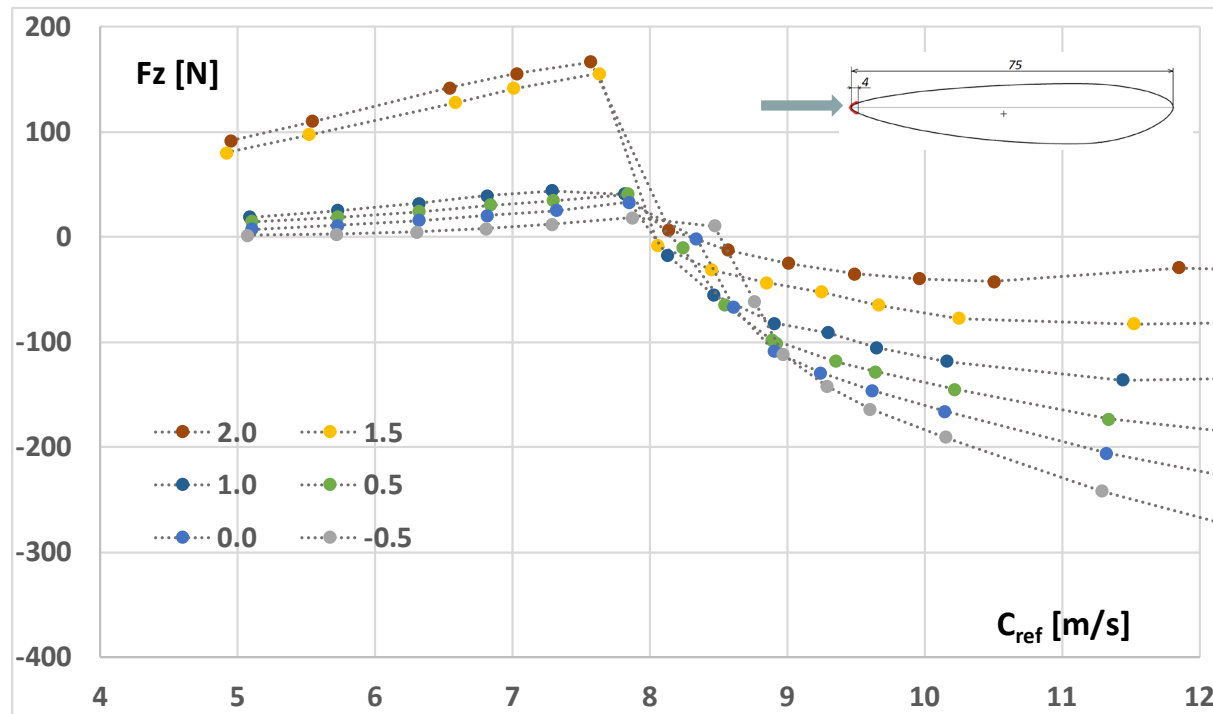
Pseudo-static Aeroelasticity



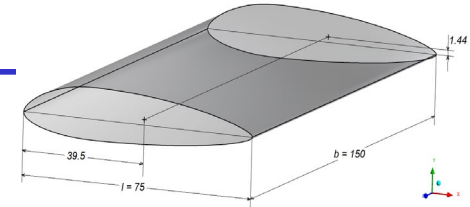
“Lift Crisis”

Case of a flow over a wicket gate of a pump turbine

- **Lift force variation with upstream velocity**
 - Pump mode, smooth leading edge (natural BL transition)



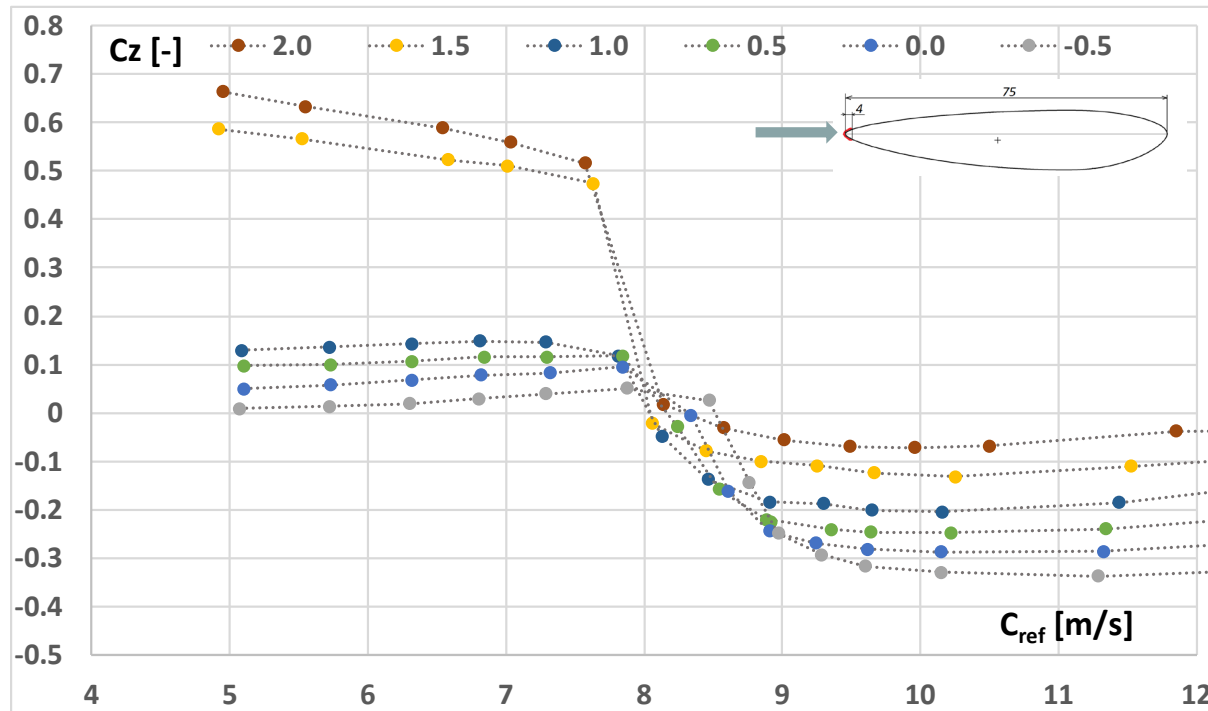
Pseudo-static Aeroelasticity



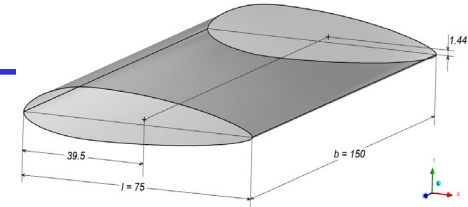
“Lift Crisis”

Case of a flow over a wicket gate of a pump turbine

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Pseudo-static Aeroelasticity



“Lift Crisis”

Case of a flow over a wicket gate of a pump turbine

- **Lift coefficient variation with Reynold Number**
 - **Pump mode, smooth leading edge (natural BL transition)**

