AEROELASTICITY AND FLUID-STRUCTURE INTERACTION

Complement

Sloshing Dynamics

Equivalent Mechanical Model

Sloshing Dynamics – Equivalent Mechanical Model

In certain situations, sloshing can influence the behavior of the structure surrounding the moving liquid → there is a need to incorporate its effects into structural models.

1.

2.

1. ***Coffee mug***
2. ***Cruise ship swimming pool.***
3. ***Rocket liquid O_2 fuel tank***

→ ***Find the first asymmetric natural frequency for case 1 and 2.***

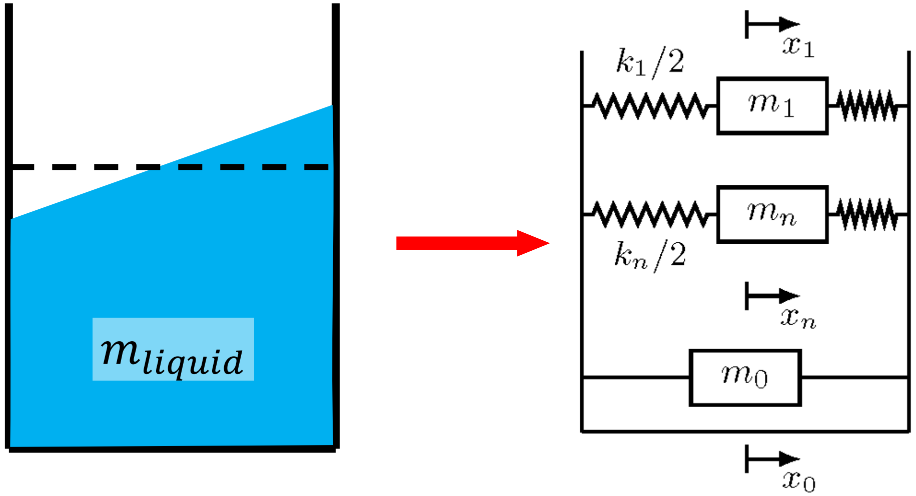
→ ***How to model the fluid-structure interactions?***

3.

Sloshing Dynamics – Equivalent Mechanical Model

To incorporate the effects of sloshing in a dynamical system, it is often convenient to conceptually replace the liquid by an equivalent linear mechanical system:

- ***We will use a linear spring-mass model Each of the n spring-mass corresponds to one of the infinite sloshing modes.***



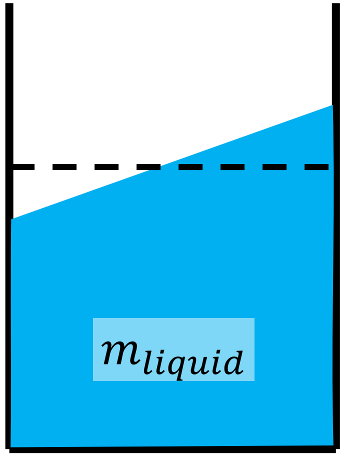
Main benefits:

- ***Easier to include in a system***
- ***Reduced computation cost***
- ***Fluid damping can be easily incorporated in the model by adding linear dashpots***

- ***How to define the parameters*** m_n ***and*** k_n***?***

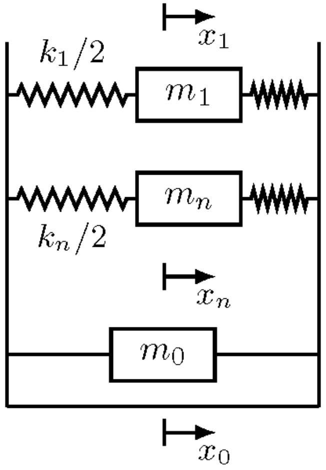
Sloshing Dynamics – Equivalent Mechanical Model

Defining the model parameters for horizontal motion



- ***Static properties:***
  - ***The sum of all the masses must be the same as the liquid mass***

$$m_{liqu} = m_0 + \sum_{n=1}^{\infty} m_n$$



- ***Dynamic properties:***
  - ***The natural frequencies must duplicate the ones of the liquid***

$$\frac{k_n}{m_n} = \omega_n^2$$

where ω_n is the natural frequency of the nth sloshing mode.
(known from potential theory)

Sloshing Dynamics – Equivalent Mechanical Model

Defining the model parameters for horizontal motion

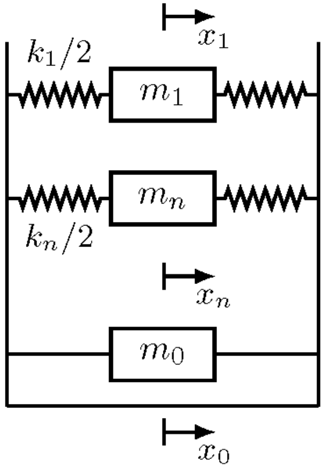
- **Dynamic properties:**
  - The force components exerted on the tank under certain excitation must be equivalent to the one produced by the actual system (= the ones derived from the potential flow theory)

$$-F = m_0\ddot{x}_0 + \sum_{n=1}^{\infty} m_n\left(\ddot{x}_n + \ddot{x}_0\right)$$

  - In the case of pure translational excitation of the tank: $x_0(t) = X_0\sin(\Omega t)$

    From the steady state solution of the undamped equation

    $m_n(\ddot{x}_n + \ddot{x}_0) + k\,\ddot{x}_n = 0$ we obtain $x_n = \frac{\Omega^2}{\omega_n^2-\Omega^2}X_0\sin(\Omega t)$



$$F = X_0\Omega^2\sin(\Omega t)\left[m_0 + \sum_{n=1}^{\infty} m_n\left(\frac{\Omega^2}{\omega_n^2-\Omega^2}+1\right)\right]$$

$$= m_{liqu}X_0\Omega^2\sin(\Omega t)\left[1 + \sum_{n=1}^{\infty}\frac{m_n}{m_{liqu}}\left(\frac{\Omega^2}{\omega_n^2-\Omega^2}\right)\right]$$

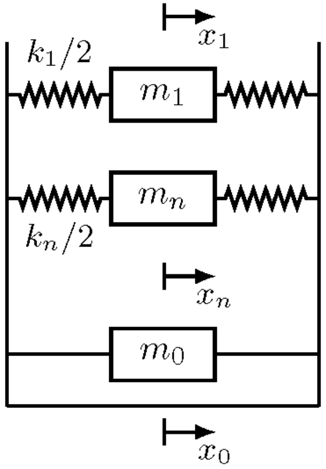
Sloshing Dynamics – Equivalent Mechanical Model

Defining the model parameters for horizontal motion

- ***Dynamic properties:***

$$\boldsymbol{F = X_0\Omega^2\sin(\Omega t)\left[m_0 + \sum_{n=1}^{\infty} m_n\left(\frac{\Omega^2}{\omega_n^2 - \Omega^2} + 1\right)\right]}$$

$$\boldsymbol{= m_{liqu}X_0\Omega^2\sin(\Omega t)\left[1 + \sum_{n=1}^{\infty} \frac{m_n}{m_{liqu}}\left(\frac{\Omega^2}{\omega_n^2 - \Omega^2}\right)\right]}$$



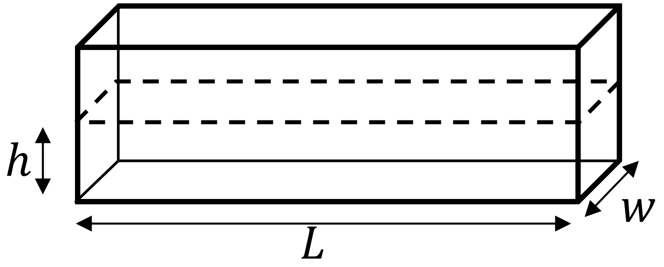
- ***By comparing the force F with the one obtained from potential flow theory for a similar motion, the ratio $\frac{m_n}{m_{liqu}}$ can be evalutated (not demonstrated here)***

Sloshing Dynamics – Equivalent Mechanical Model

Defining the model parameters for horizontal motion

- ***The model parameters depend on the liquid and on the tank shape (not demonstrated here)***
  Rectangular tank with motion along the a-direction

- $m_{liqu} = \rho L b h$
- $\frac{m_n}{m_{liqu}} = 8\left(\frac{L}{h}\right)\frac{\tanh((2n-1)\pi\frac{h}{L})}{(2n-1)^3\pi^3}$
- $\frac{k_n}{m_{liqu}} = 8\left(\frac{g}{h}\right)\frac{\tanh^2((2n-1)\pi\frac{h}{L})}{(2n-1)^2\pi^2}$



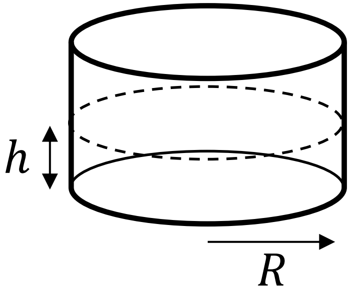
- *One may observe that the masses rapidly decrease for all modes exceeding the first one.*

Sloshing Dynamics – Equivalent Mechanical Model

Defining the model parameters for horizontal motion

- ***The model parameters depend on the liquid and on the tank shape (not demonstrated here)***
  Cylindrical tank with motion along the R-direction (linear sloshing)

  - $m_{liqu} = \rho \pi R^2 h$
  - $\frac{m_n}{m_{liqu}} = \left(\frac{2R}{\varepsilon_{1n} h (\varepsilon_{1n}^2 - 1)}\right) \tanh\left(\frac{\varepsilon_{1n} h}{R}\right)$
  - $\frac{k_n}{m_{liqu}} = \left(\frac{2g}{h (\varepsilon_{1n}^2 - 1)}\right) \tanh^2\left(\frac{\varepsilon_{1n} h}{R}\right)$



  - *ε_{1n} corresponds to the roots of the derivatives of the Bessel function of the first kind ($\varepsilon_{11} = 1.841, \varepsilon_{12} = 5.331, \varepsilon_{13} = 8.536, \ldots$)*
  - *Since ε_{1n} increases with n, the size of the masses also decreases for all modes exceeding the first.*

Sloshing Dynamics – Equivalent Mechanical Model

- ***From potential theory we can model the free-response motion.***

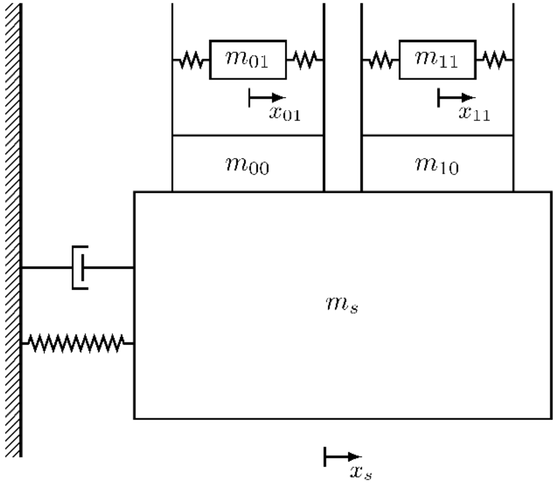
$$\ddot{x} + \omega_1^2 x = 0$$

$$\omega_1 \approx \frac{2\pi g}{L}, 1.841\,\frac{g}{R}$$ ***(h>>2L, respectively Rect. Cyl.)***

→ ***We can now model dynamic interaction with any force as a function of time***

$$\ddot{x} + \frac{k_1}{m_1} x = F(t)$$

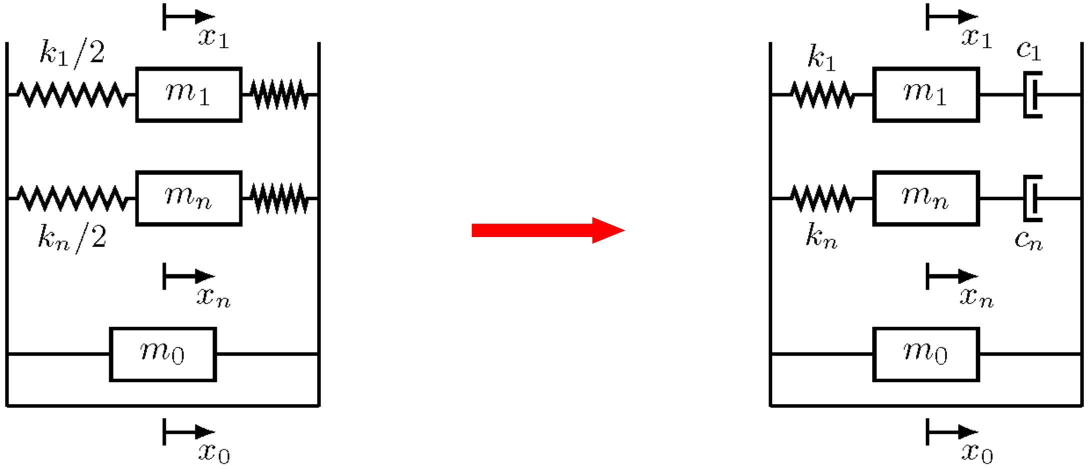
→ ***Or model coupled dynamical systems***



Sloshing Dynamics – Equivalent Mechanical Model

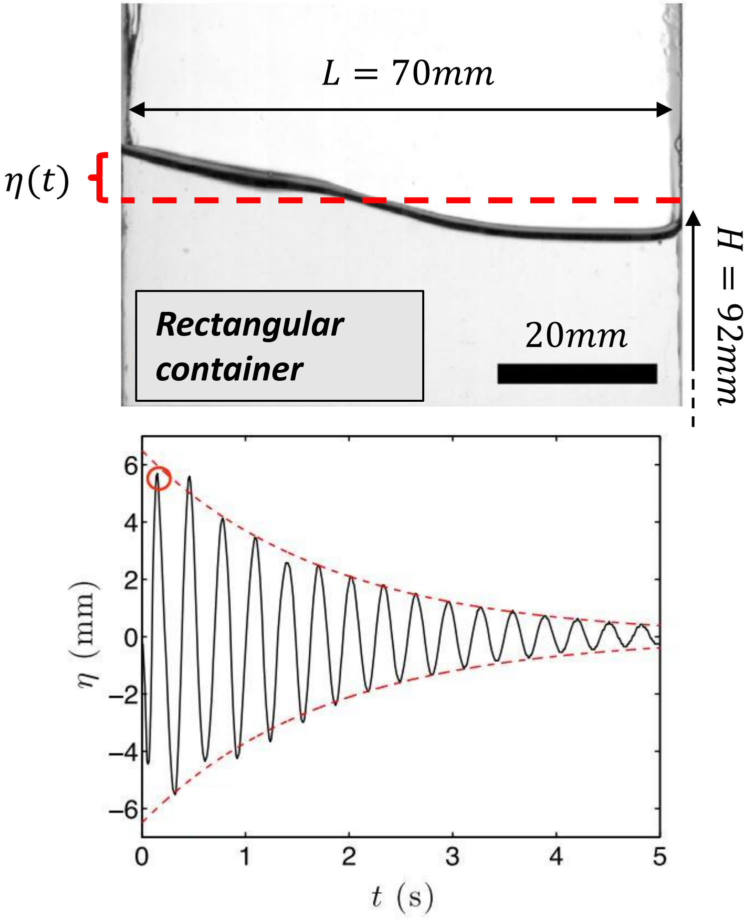
- ***Isn't there something missing to model our experimental observations?***

  → ***Viscous damping !***



Sloshing Dynamics – Equivalent Mechanical Model

Modeling the liquid damping – linear dashpots



Damping of liquid sloshing by foams: from everyday observations to liquid transport, Capello et al., 2015

- ***Experiments show that the free-surface elevation, after an impulse motion, follows a damped harmonic motion:***

$$\eta(t) = \eta_0 e^{(-t/\tau)} cos(2\pi f t)$$

With $f_1 = \frac{\omega_1}{2\pi} = \frac{1}{2\pi}\sqrt{g k_1 \tanh(k_1 H)} = 3.34\ Hz$

$\rightarrow f_{exp} = 3.22\ \pm 0.11\ Hz$

- ***In the linear framework, the effect of damping can thus be modeled by a set of linear dashpots.***

Sloshing Dynamics – Equivalent Mechanical Model

Modeling the liquid damping – linear dashpots

- ***In real liquids, energy dissipation occurs at the tank walls and free surface due to the viscous boundary layer and within the liquid because of viscous stresses.***
- ***For small tanks, the boundary layer dissipation dominates, while for large tanks, the dissipation in the liquid interior may be the larger contribution.***
- ***Most results for the damping ratio have been obtained experimentally (first sloshing mode):***
    - ***In cylindrical tanks (Stephens et al. 1962):***

$$\zeta_1 = 0.83\sqrt{\frac{\nu}{g^{1/2}R^{3/2}}}\left[\tanh\left(\frac{\varepsilon_{11}h}{R}\right)\left(1+2\,\frac{1-\frac{h}{R}}{\cosh\left(\frac{\varepsilon_{11}h}{R}\right)}\right)\right]$$

    - ***In rectangular tanks (Sun, 1991):***

$$\zeta_1 = \frac{1}{2h}\sqrt{\frac{2\nu}{\omega_1}}\left(1+\frac{h}{w}\right)$$

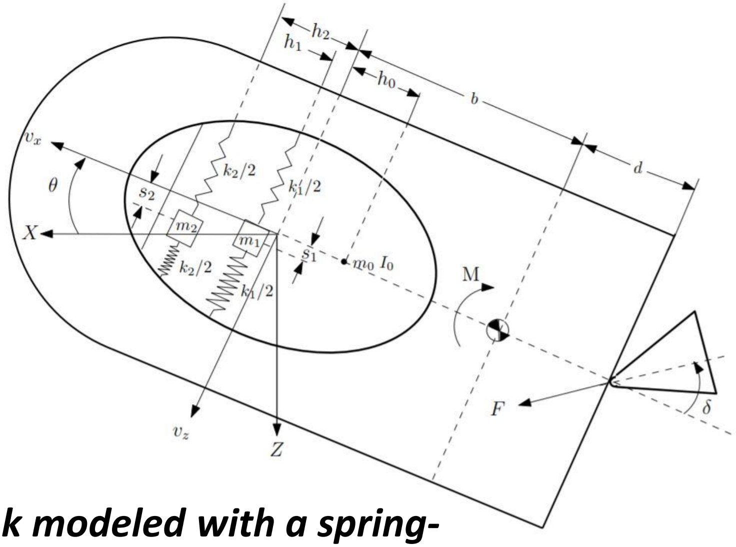
ν is the liquid kinematic viscosity

Sloshing Dynamics – Equivalent Mechanical Model

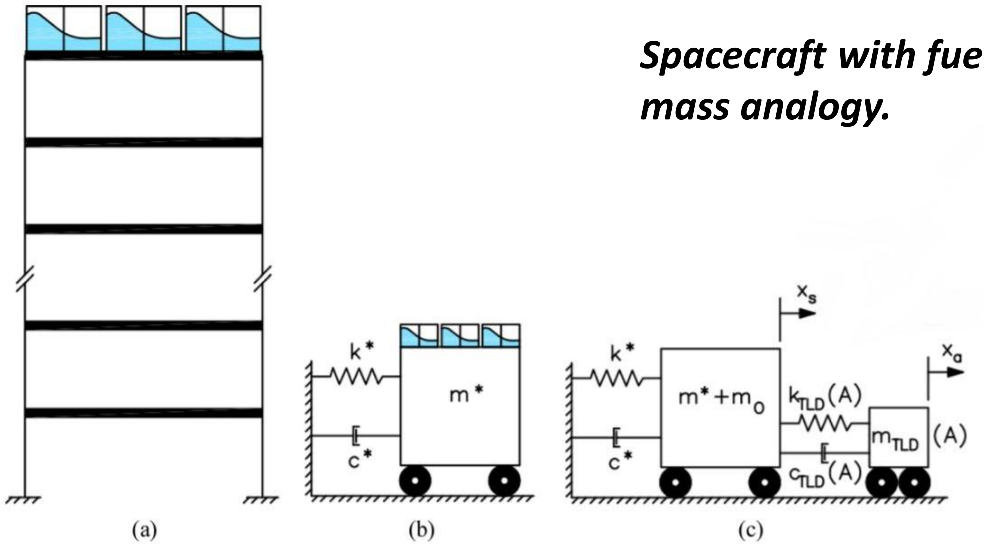
Examples of application:

- ***Control of spacecraft with liquid fuel tanks***
- ***Vibration absorbtion (Tuned Liquid Dampers or TLD)***
- ***Modeling of fuel tanks in aircraft wings , in oil tankers or in trucks***
- ***…***

Dynamics and Control of Higher-Order Nonholonomic Systems, Jaime Rubio Hervas, PhD Thesis, 2013



Spacecraft with fuel tank modeled with a spring-mass analogy.



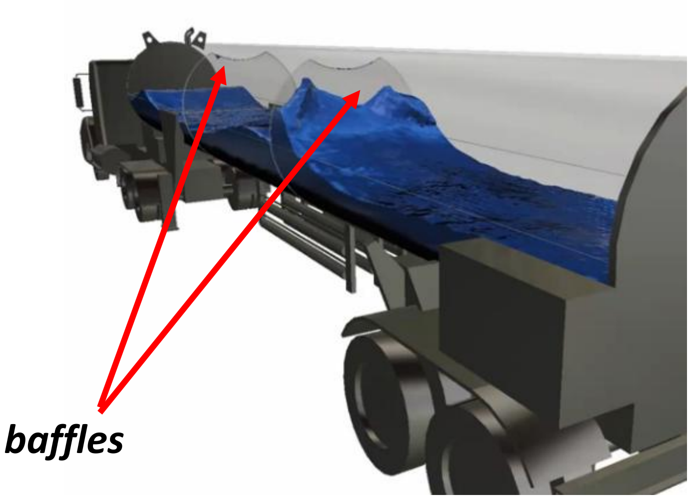
The evolution of (a) a structure-TLD system into, (b) a generalized structural system with TLDs and then into, (c) a system with equivalent Tuned Mass Damper (TMD) representation

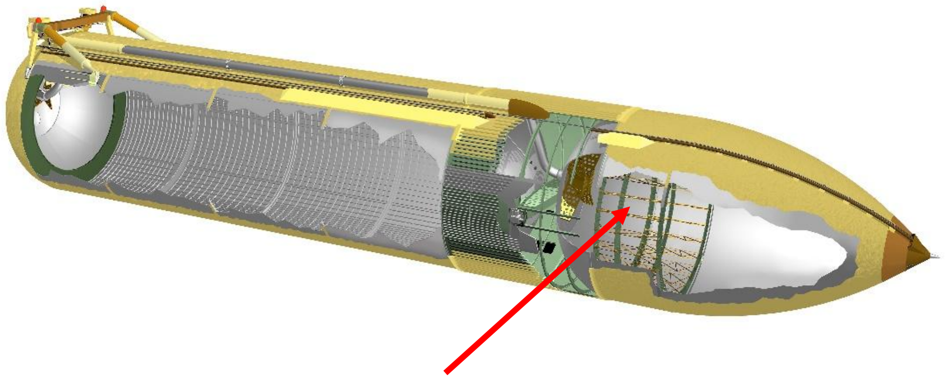
Development and Validation of Finite Element Structure-Tuned Liquid Damper System Models, Soliman et al., 2015

Sloshing Dynamics – Equivalent Mechanical Model

Sloshing mitigation techniques

- ***Liquid containers: Sub-division with baffles or bulkheads are widely used***
  - ***Reduces wave amplitude and wall pressure, increases the liquid damping***
  - ***Ongoing research to define the optimal shape, number and locations of the baffles***

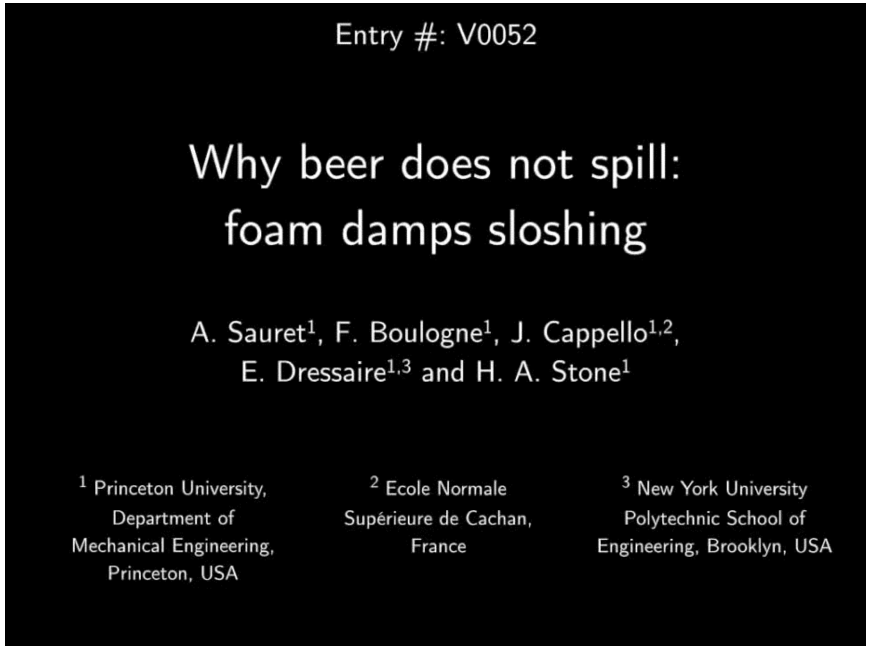



Space shuttle liquid oxygen tank anti-slosh baffles

Sloshing Dynamics – Equivalent Mechanical Model

Sloshing mitigation techniques – the effect of a foam layer

- ***The addition of foam of the free surface damps the sloshing of the liquid.***



Entry #: V0052

Why beer does not spill:
foam damps sloshing

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[2] Ecole Normale Supérieure de Cachan, France

[3] New York University Polytechnic School of Engineering, Brooklyn, USA

- *Foam generates an additionnal friction force that adds damping to the fluid oscillations.*

- *This additionnal force scales as:*

$$f_{foam} \sim K\gamma Ca^{2/3}$$

where γ is the foam surface tension, K represents geometrical properties of the foam layer and $Ca = \frac{\mu \dot{x}}{\gamma}$ is the capillary number → the foam friction force in non-linear.

Sloshing Dynamics – Equivalent Mechanical Model

Exercise 1 – Tuned Liquid damper.

Consider a tall building represented by a cylinder with an equivalent diameter, $D_s = 50\ m$ *and a total height* $H_s = 300\ m$ *as depicted on figure 1. The structure is subjected to strong winds, vortices are periodically shed from its surface thus forcing the structure to oscillate. To dampen those oscillations, a tuned liquid damper (TLD), in the form of rectangular tank partially filled with water, is installed within the building. The tank measures* $L = 10\ m$ *by* $w = 6\ m$*.*