Discrete-time Linear Switched system

System with a finite set $\mathcal{I} = \{1, \cdots, M\}$ of modes of operation and a switching signal indicating the active mode at each time instant

$$x_{k+1} = A_{\sigma(k)} x_k \quad x_k \in \mathbb{R}^n \quad \sigma(k) \in \mathcal{I} \tag{5}$$

Theorem

If there is $P \in \mathbb{R}^{n \times n}, P = P^T > 0$ such that

$$A_i^T P A_i - P < 0, \quad \forall i \in \mathcal{I},$$

then (5) is exponentially stable

- $V(x) = x^T P x$ is a *common* Lyapunov function for all the modes
- The condition is only sufficient and implies all modes of operation are exponentially stable
- How to find P? By solving a Linear Matrix Inequality (LMI) optimization problem (see next lecture)