Corrected

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- Introduction to AM

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ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Open Book Part

Name :

Wait for the beginning of the test before turning the page. This document is printed double-sided, it contains 15 pages, the last ones can be empty. Do not remove the staple.

- Put your student card on the table.
- Any document is allowed.
- The use of a computer or of a cell phone is prohibited during the test.
- You may use a pencil instead of a pen.
- After the exam, the teachers reserve the right to cancel any questions which they will consider to be ill-posed.
- Write your name on every page

Question	Number of points
1)	
2)	
3)	
4)	
Total	

to be filled by the teacher

Question 1: *This question is scored on 5.5 points*

Fill each of the 22 gaps below with 22 words (or acronyms) giving a sense to each sentence:

- If it uses a shape tool, a process is said to be **replicative** .
- The shape tool is said to be expendable if it has to be **destroyed** to recover the **part** .
- In casting, master models and lost patterns are used to fabricate the **mould** . The difference between a lost pattern and a master model is that the master model can be **used** **many** **time** .
- In **investment** **casting** , parts are made out of a mould obtained by plastering a cluster of **lost** **patterns** made out of wax.
- In **vacuum** **casting** , a part is made out of a mould obtained by immersing a **master** **model** in a silicone bath which is then left to dry. The typical material for the part is **polyurethane** .
- In the **UV-LiGa** process chain, metallic parts are obtained by electro-plating **expendable** counter-parts made by **UV-lithography** .
- The **BPM** process is an additive method that can produce wax lost patterns.
- The DLP process can also be used to produce lost patterns. But, in that case they will be made out of a **thermofusible** **photoresist** .

Question 2: *This question is scored on 15 points*

You develop a new SLA machine with a square platform of side $c = 200\,\text{mm}$.

a) A focusing lens has to be installed between the scan head and the platform in a way that the axis of the lens goes through the center of the platform. To avoid optical aberrations, the *view angle* under which the lens center sees the diagonal of the platform has to be $< 20°$. Use this information to determine the smallest focal length f to be used[1].

Solution

In any SLA station, the focal distance f of the focusing lens corresponds to the distance from the lens to the platform (which is in the focal plane). Therefore the view angle θ under which the lens center C sees the diagonal of the platform is given as (see Fig. 1):

$$\theta = 2\arctan\frac{d}{2f}$$

where $d = 200\sqrt{2} \simeq 282.84\,\text{mm}$ is the length of the diagonal of the platform. If we want θ to be less then $\theta_{\text{max}} = 20°$ we need f to be large enough so that

$$\arctan\frac{d}{2f} < \frac{1}{2}\theta_{\text{max}} \Longrightarrow \frac{d}{2f} < \tan\frac{1}{2}\theta_{\text{max}}$$

because $\tan t$ is an increasing function of t. The conclusion is that

$$f > \frac{d}{2\tan\frac{1}{2}\theta_{\text{max}}}.$$

With the numerical data, we get that

$$f > \frac{282.84}{2 \times \tan 0.5 \times 20°} \simeq 802\,\text{mm} \tag{1}$$

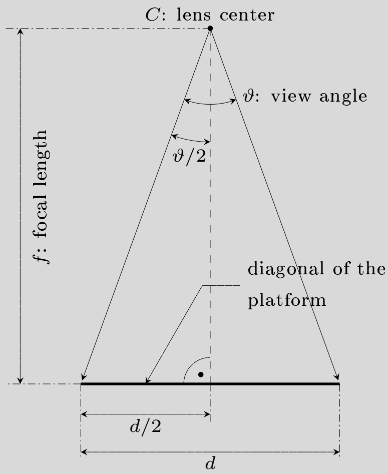

Figure 1: The situation in the plane containing C and the diagonal of the platform

b) The radius of the lens is $R = 6.25\,\text{mm}$ and the laser wavelength is $\lambda = 324\,\text{nm}$. Under this condition compute the smallest focal waist w_f^{min} that you can reach.

Solution

The smallest focal waist is obtained by applying the equation for the *focusing limit* of a lens with radius R and focal distance f for a beam of wavelength λ:

$$w_\text{f}^{\text{min}} = \lambda\frac{f}{2\pi R}.$$

The smaller the focal distance, the smallest the waist. Therefore w_f^{min} is obtained with the smallest accepted value for f: $f \simeq 802\,\text{mm}$. The conclusion is that

$$w_\text{f}^{\text{min}} \simeq 0.324 \times \frac{802}{2 \times 3.14 \times 6.25} \simeq 6.62\,\mu\text{m}. \tag{2}$$

[1] If you cannot answer this question, take the value $f = 800\,\text{mm}$ to continue the exercise.

c) What is the necessary condition to impose on the laser beam if we want to achieve the particular value w_f^{min} for the focal waist?

Solution

The laser beam has to be a **perfect gaussian** wave.

d) On any laser data sheet, an optical characteristic indicates how far we are from the necessary condition mentioned in the previous item. What is the name and the symbol of this quantity and which value does it take when the smallest focal waist can be achieved?

Solution

The **beam quality** M^2 measured the deviation of the actual laser beam intensity and the intensity of a gaussian wave. We always have $M^2 \geq 1$ and the limit value $M^2 = 1$ corresponds to a perfectly gaussian beam.

e) You select a laser such that the smallest possible focal waist can be achieved. The power of this laser is $P_0 = 0.005\,\text{W}$. You consider to work with a photoresist and with a laser scan-peed such that the curing reaction will be limited to regions receiving **at least** a threshold intensity of $I_{\text{th}} = 3.5\,\text{W}/\text{mm}^2$.

1) The laser travels over the photoresist from point A to point B over a distance $l = 0.05\,\text{mm}$ (see Fig. 2). It provokes the curing of a certain volume V_c. You are asked to hatch on Fig. 2 the shape of the upper surface of this volume V_c. You are also asked to compute the precise dimensions of this shape (length L and width w).

Solution

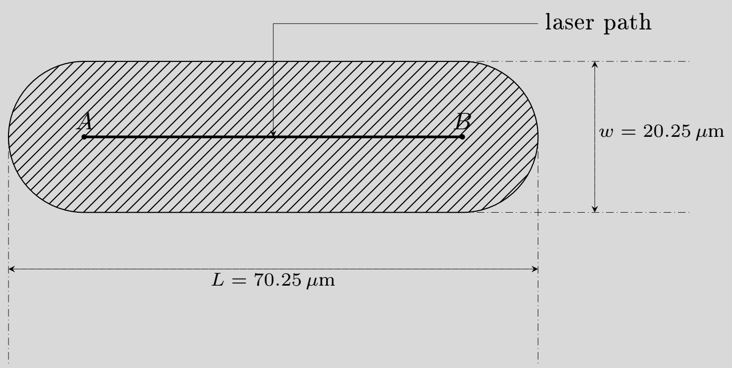

Figure 2: Laser path and cured surface

At a fixed time, in the surface of the photoresist, the intensity distribution is gaussian. At a distance r from the center of the beam the value of the intensity is given by (see the formula given in the lecture):

$$I(r) = \frac{P_0}{\pi w_{\mathrm{f}}^2} e^{-\left(\frac{r}{w_{\mathrm{f}}}\right)^2} \tag{3}$$

In particular the iso-lines of the intensity are **circles** and the trace of V_{c} in the surface will be a **rounded rectangle** of axis AB (see Fig. 2) and with a half-width $\frac{w}{2}$ corresponding to the radius of the threshold iso-line:

$$I(\frac{w}{2}) = I_{\mathrm{th}}.$$

Substituting $I(r)$ by its value (3), we get an equation for w

$$\frac{P_0}{\pi w_{\mathrm{f}}^2} e^{-\frac{w^2}{4w_{\mathrm{f}}^2}} = I_{\mathrm{th}}$$

that we can solve:

$$w = 2w_{\mathrm{f}} \sqrt{\ln \frac{P_0}{\pi w_{\mathrm{f}}^2 I_{\mathrm{th}}}}.$$

With the numerical value for P_0, I_{th} and for w_{f} (see (2)), we get

$$w \simeq 2 \times 6.62 \times \sqrt{\ln \frac{0.005}{3.14 \times 0.00662^2 \times 3.5}} \simeq 20.25\,\mu\mathrm{m}. \tag{4}$$

2) We assume that the scan head in your system ensures a positioning accuracy of $\delta = \pm 5\,\mu$m for the laser beam on the photoresist surface. Give an estimate of the process resolution in the $xy-$plane. Justify your answer[2].

Solution

According to the theory given in the lecture, the process resolution δ_{res} in the $xy-$plane is limited by tow factors

1) the positioning accuracy δ of the consolidation tool,
2) the lateral dimension $\frac{w}{2}$ (see Fig. 2) of the smallest volume that can be consolidated.

Since they are independent, these two contributions just sum up. We get that

$$\delta_{\mathrm{res}} = \delta + \frac{w}{2} \simeq 5 + 0.5 \times 20.25 \simeq 15.125\,\mu\mathrm{m}. \tag{5}$$

3) How many Digital Mirror Devices (DMD's) do you approximately need to obtain a comparable accuracy over the **same** build table in the DLP process?

Solution

A lateral resolution δ_{res} of about $15\,\mu$m is obtained with a pixel size

$$\delta_{\mathrm{res}}^2 \simeq 225\,\mu\mathrm{m}^2 = 2.25 \cdot 10^{-4}\,\mathrm{mm}^2.$$

[2] If you cannot answer this question, take any **reasonable** value for the process resolution and continue the exercise.

To cover the complete platform of area $A = c^2 = 4 \cdot 10^4\,\mathrm{mm}^2$ a total number of

$$N_{\mathrm{pix}} = \frac{A}{\delta_{\mathrm{res}}^2} \simeq \frac{4 \cdot 10^4}{2.25 \cdot 10^{-4}} \simeq 1.78 \cdot 10^8$$

pixels is needed. Since it has been said in the lecture that a single DMD provides about $n_{\mathrm{pix}} \simeq 10^6$ pixels (1 Mpixel), we need approximately

$$n_{\mathrm{DMD}} \simeq \frac{1.78 \cdot 10^8}{10^6} \simeq 178$$

DMD's to get the same accuracy with the DLP process.

Question 3: *This question is scored on 9 points*

a) What is the minimum number n of triangular facets required to describe **exactly** the shape of the house of Fig. 3 in an .stl file?

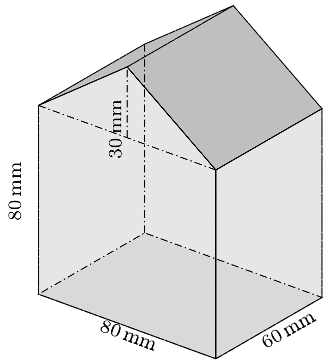

Figure 3: The part to be printed

Solution
$n = 16$

b) Can a higher number of facets increase the precision of the design? Explain.

Solution
No

c) This part of the .stl file describes one bottom facet of the house:

`beginfacet`			
`facet normal`	0	0	-1
`outer loop`			
`vertex`	40	-30	0
`vertex`	-40	-30	0
`vertex`	40	30	0
`endloop`			
`endfacet`			

Table 1: A part of the .stl file

Fill the empty boxes below in order to define the other bottom facet and all roof facets:

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```
beginfacet
  facet normal
      0      0      −1
  outer loop
    vertex
      −40    30     0
    vertex
      40     30     0
    vertex
      −40    −30    0
  endloop
endfacet
```

```
beginfacet
  facet normal
      −0.6   0      0.8
  outer loop
    vertex
      −40    −30    80
    vertex
      −40    30     80
    vertex
      0      −30    110
  endloop
endfacet
```

```
beginfacet
  facet normal
      0.6    0      0.8
  outer loop
    vertex
      40     −30    80
    vertex
      0      −30    110
    vertex
      40     30     80
  endloop
endfacet
```

```
beginfacet
  facet normal
      −0.6   0.0    0.8
  outer loop
    vertex
      0      30     110
    vertex
      0      −30    110
    vertex
      −40    30     80
  endloop
endfacet
```

```
beginfacet
  facet normal
      0.6    0.0    0.8
  outer loop
    vertex
      0      30     110
    vertex
      40     30     80
    vertex
      0      −30    110
  endloop
endfacet
```

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Question 4: *This question is scored on 27 points*

You need to produce two polymer parts, part A and part B in small series. You will apply an additive process for that. You ask an offer to your provider. He proposed to apply a process called process 1 and another process referred to as process 2. The offer you get depends on the number of items you order. It is summarized in Tab. 2:

Process	**Part**	**nr. of ordered items**							
		1	2	5	10	25	100	104	1000
1	**A**	22 Frs	44 Frs	110 Frs	220 Frs	550 Frs	2′200 Frs	2′288 Frs	22′000 Frs
	B	11 Frs	22 Frs	55 Frs	110 Frs	275 Frs	1′100 Frs	1′144 Frs	11′000 Frs
2	**A**	110 Frs	120 Frs	150 Frs	200 Frs	350 Frs	1′400 Frs	1′540 Frs	14′000 Frs
	B	96 Frs	101 Frs	116 Frs	141 Frs	216 Frs	864 Frs	975 Frs	8′640 Frs

Table 2: The price for the **whole** series as a function of its size

a) For one of the two processes, the items are built by batches of $n_b > 1$ items (**same** batch size for part A an B). Moreover for each batch a non-zero set-up price p_b is charged (**same** price for each part). From the price list given in Tab. 2, it is obvious whether this process is process 1 or process 2. You are asked to check the right affirmation and to justify your answer[1]:

■ The process with $n_b > 1$ and $p_b > 0$ is process 2
□ The process with $n_b > 1$ and $p_b > 0$ is process 1

Solution

The price $p(N)$ of a series of size N is given by the basic formula:

$$p(N) = aN + bE\left(\frac{N}{n_b}\right) \tag{6}$$

where n_b is the batch size, a the price par item, b the price per batch and where $E(x)$ is the smallest integer larger or equal to x:

$$E(x) = \min\{n \in \mathbb{N} \mid n \geq x\} \tag{7}$$

so that $E\left(\frac{N}{n_b}\right)$ exactly represents the number of batches needed to produce the series. Observe that b is always more then the set-up price:

$$b > p_b.$$

If $n_b > 1$ and $b > p_b > 0$, then the function $N \to p(N)$ given in (6) **cannot be linear**. Since $p(N) = Np(1)$ for process 1 (see Tab. 2), we have proved ab absurdo that the process we are speaking is 2.

b) You get an extra information. One of the processes proposed by your provider is FDM, the other is L-PBF. Use the results obtained above to guess which process is probably named 1 by your provider and which one is probably named 2. You are asked to check the right affirmation and to **justify** your answer[1]:

[1] no points will be attributed if the justification is wrong or missing

- [x] process 2 is certainly L-PBF
- [ ] process 2 is certainly FDM

Solution

In the L-PBF process, the items are generally built in batches while FDM process build them sequentially ($n_\mathrm{b} = 1$). The L-PBF process is obviously the process with $n_\mathrm{b} > 1$ (=process 2) and the process 1, for which the price $p(N)$ increases linearly with N, is certainly FDM

c) In process 2, both parts are made of the **same** material and with the **same** process parameters. However, Tab. 2 indicates that, for the same numbers N of ordered items, the manufacturing costs are different for part A and part B: $p_A(N) \neq p_B(N)$. What does it mean? Check the correct answer (no justifications are needed in that case)

- [ ] The cheap part has probably a simple geometry while the expensive one is more intricated
- [ ] The average radius of curvature R_A, R_B of the parts are **necessarily** different. If they would be the same, $R_A = R_B$, then we would have $p_A(N) = p_B(N)$
- [ ] The external surface S_A, S_B of the parts are necessarily **different**. If they would be the same, $S_A = S_B$, then we would have $p_A(N) = p_B(N)$
- [x] The volumes V_A, V_B of the parts and/or their heights in the build direction H_A, H_B are different. If they would be the same, $V_A = V_B$ and $H_A = H_B$, then we would have $p_A(N) = p_B(N)$

d) In case of process 2, observe that the difference between the prices for two and one items is not the same for each parts:

$$p_B(2) - p_B(1) \neq p_A(2) - p_A(1).$$

What does it mean? Check the correct affirmation and **justify** your answer[2]:

- [ ] The heights of the parts in the build direction H_A, H_B are different. If they would be the same, $H_A = H_B$, then, we would have $p_B(2) - p_B(1) = p_A(2) - p_A(1)$.
- [x] The volumes V_A, V_B of the parts are different. If they would be the same, $V_A = V_B$ then, we would have $p_B(2) - p_B(1) = p_A(2) - p_A(1)$.

Solution

It results from the basic formula (6) that

$$p(1) = a + bE\left(\frac{1}{n_\mathrm{b}}\right) \quad \text{and} \quad p(2) = 2a + bE\left(\frac{2}{n_\mathrm{b}}\right) \tag{8}$$

From a), we know that the batch size n_b is > 1 for process 2. But, if $n_\mathrm{b} > 1$, the two first items are built in the **same** batch:

$$E\left(\frac{1}{n_\mathrm{b}}\right) = E\left(\frac{2}{n_\mathrm{b}}\right) = 1.$$

It means that (8) reduces to

$$p(1) = a + b \quad \text{and} \quad p(2) = 2a + b. \tag{9}$$

[2] no points will be attributed if the justification is wrong or missing

As a consequence the difference between the prices for two and is simply the **coefficient** a:

$$p(2) - p(1) = 2a + b - (a + b) = a. \tag{10}$$

From what we said at Qu.c), a depends **a priori** on the volume V of the piece and of its height H in the build direction. But all the costs linked to the height H are integrated in **coefficient** b (see formula (6)) because they correspond to the **layering time** and can be **mutualized** between all the n_b items fabricated in the same batch. The final conclusion is that $p(2) - p(1)$ which is equal to a (10) only depends on the **volume** V of the part.

e) The volume of part A is $V_A = 50'000\,\text{mm}^3$. Use the price list for process 2 (see Tab. 2) to determine the volume V_B of part B.

Solution

At Qu.d) we justified that the difference $p(2) - p(1)$ between the prices for two and one items only depends on the volume of the part. But this dependancy is actually **proportional** to the volume. The proportionality coefficient involves the material consolidation rate MCR and the hourly rate R of the station which are assumed to be the **same** for both parts (see Qu.c). Therefore we have

$$\frac{V_A}{V_B} = \frac{p_A(2) - p_A(1)}{p_B(2) - p_B(1)}.$$

Using the numerical values we get

$$\frac{V_A}{V_B} = \frac{120 - 110}{101 - 96} = \frac{10}{5} = 2.$$

The conclusion is that

$$V_B = \frac{V_A}{2} = \frac{50'000}{2} = 25'000\,\text{mm}^3.$$

f) The hourly rate of the machine used in process 2 (including depreciation, manpower and energy consumption etc...) is $R = 72\,\text{Frs/hour}$ and the material costs is so low that it can be neglected. Use these information as well as the price list in Tab. 2 to evaluate the material consolidation rate MCR of this process.

Solution

If the material costs are neglected, then the volume dependent coefficient a (price per part) involved in the basic formula (6) only reflects the machine usage time. It can be computed as the product of the consolidation time $\frac{V}{\text{MCR}}$ by the hourly rate R:

$$a = R\frac{V}{\text{MCR}}. \tag{11}$$

The value of coefficient a can be obtained by using the price list: $a = p(2) - p(1)$ (10). Therefore we have

$$p(2) - p(1) = R\frac{V}{\text{MCR}}$$

Solving this equation for MCR, we get

$$\text{MCR} = \frac{RV}{p(2) - p(1)}.$$

We know that

$$R = 72\,\text{Frs/hour} = 0.02\,\text{Frs/sec}$$

and, considering part A, that $V = 50'000\,\text{mm}^3$, that $p(2) = 120\,\text{Frs}$ and that $p(1) = 110\,\text{Frs}$. As a conclusion:

$$\text{MCR} = \frac{0.02 \times 50'000}{120 - 110} = 100\,\text{mm}^3/s.$$

g) For process 2, the price for two items, is less than two times the price for one item: $p(2) < 2p(1)$. How can you justify this and what does the difference $2p(1) - p(2) > 0$ represent?

Solution

The difference $2p(1) - p(2)$ corresponds to the cost per batch b (set-up+layering costs): using (9) to express the price for one and two items we get exactly that

$$2p(1) - p(2) = 2(a + b) - (2a + b) = b \tag{12}$$

h) Always for process 2, note that the differences $2p(1) - p(2)$ are not the same if we consider part A and part B. How do you justify this fact? Recall that we have assumed that

- the price p_b for setting up the machine does not depend on the part to be built (see Qu.a)
- both parts are made of the **same** material and with the **same** process parameters (see Qu.c)

Solution

The coefficient b involved in the basic formula (6) is different if we consider part A or part B. According to (12) and of the data in Tab. 2 we have

$$b_A = 2p_A(1) - p_A(2) = 2 \times 110 - 120 = 100\,\text{Frs} \quad \text{for part A,} \tag{13}$$

$$b_B = 2p_B(1) - p_B(2) = 2 \times 96 - 101 = 91\,\text{Frs} \quad \text{for part B.} \tag{14}$$

This difference is due to a difference in the **height** H_A respectively H_B of the parts in the build direction.

Justification. The price per batch b integrates the set-up cost p_b and the layering cost p_l which are mutualized over the n_b parts built in the same batch. Since the set-up cost is assumed to be independent of the part to be built, the difference between b_A and b_B can only be attributed to different layering costs. But the material and the process parameters are also assumed to be the same. Therefore a difference in layering costs can only be due to a difference in the height of the parts in the build direction: $H_A \neq H_B$.

i) Use the data in Tab. 2 to determine the batch size n_b for process 2. Recall that the batch size is the same for both parts A and B (see Qu.a).

Solution

To compute the batch size n_b, it is enough to evaluate the number of batches $E\left(\frac{N}{n_\text{b}}\right)$ needed to produce certain series. The number of batches can be deduced from the price $p(N)$ by solving the basic equation (6). We get that

$$E\left(\frac{N}{n_\text{b}}\right) = \frac{p(N) - aN}{b}.$$

Replacing a by its value $a = p(2) - p(1)$ (see (10)) and b by its value $b = 2p(1) - p(2)$ (see (12)) in that relation, we get that

$$E\left(\frac{N}{n_\text{b}}\right) = \frac{p(N) - (p(2) - p(1))N}{2p(1) - p(2)}. \tag{15}$$

which only involves data from Tab. 2.

- For $N = 25$ and part A we get for instance that

  $$E\left(\frac{25}{n_\text{b}}\right) = \frac{350 - (120 - 110) \times 25}{2 \times 110 - 120} = \frac{350 - 10 \times 25}{100} = 1,$$

  which means that the batch size n_b is not less than 25:

  $$n_\text{b} \geq 25. \tag{16}$$

- For $N = 104$, we get that

  $$E\left(\frac{104}{n_\text{b}}\right) = \frac{1540 - (120 - 110) \times 104}{2 \times 110 - 120} = \frac{1540 - 10 \times 104}{100} = 5.$$

  which means that the batch size n_b is < 26, otherwise we could produce 104 items in only 4 batches:

  $$E\left(\frac{104}{n_\text{b}}\right) = 5 \Longrightarrow \frac{104}{n_\text{b}} > 4 \Longrightarrow \frac{104}{4} > n_\text{b} \Longrightarrow 26 > n_\text{b}. \tag{17}$$

The conclusion we can draw from (16) and (17) is that

$$n_\text{b} = 25. \tag{18}$$

j) Part A is 25% higher in the build direction than part B:

$$H_A = 1.25 H_B. \tag{19}$$

Use this information and the data in Tab. 2 to compute the set-up cost p_b of process 2.

Solution

As it has been said in Qu. h), the coeffcient b in the basic formula 6 integrates both the set-up price p_b and the layering price p_l:

$$b = p_\text{b} + p_\text{l} \tag{20}$$

The set-up cost p_b is independant of the part, but the layering price is proportional to its height. Because of the proportion rule (19), it holds:

$$p_{\mathrm{l;A}} = 1.25 p_{\mathrm{l;B}}$$

and we can deduce from (20) two linear equations:

$$b_A = p_{\mathrm{b}} + 1.25 p_{\mathrm{l;B}} \quad (21)$$

$$b_B = p_{\mathrm{b}} + p_{\mathrm{l;B}} \quad (22)$$

for two unknows: p_{b} and $p_{\mathrm{l;B}}$. The second unknown can be eliminated by multiplying (22) by 1.25 and substracting (21) to the results. This operation yields:

$$1.25 b_B - b_A = 0.25 p_{\mathrm{b}}.$$

We now solve for p_{b}:

$$p_{\mathrm{b}} = \frac{1.25 b_B - b_A}{0.25}$$

and we replace by the numercial values for b_A and b_B obtained in (13)-(14). The result is

$$p_{\mathrm{b}} = \frac{1.25 \times 91 - 100}{0.25} = 55\,\mathrm{Frs}. \quad (23)$$

The machine set-up for process 2 is 55 Frs.

k) The height of part B is $H_B = 36\,\mathrm{mm}$ and the layer thickness is $e = 100\,\mu\mathrm{m}$. Use this information and the results obtained in the previous questions to compute the time needed to prepare a layer (layering time) in process 2.

Solution

The layering time τ_{layer} of process 2 impacts directly the layering price $p_{\mathrm{l;B}}$ of part B. This is the argument we will use to identify τ_{layer}. At first we compute the layering price $p_{\mathrm{l;B}}$. Then we conclude.

Compuation of the layering price

According to (22), the layering price $p_{\mathrm{l;B}}$ is connected to the coefficient b_B in the basic equation (6) and to the set-up price p_{b}:

$$p_{\mathrm{l;B}} = b_B - p_{\mathrm{b}}.$$

Replacing b_B end p_{b} by their values $b_B = 91\,\mathrm{Frs}$ (14) and $p_{\mathrm{b}} = 55\,\mathrm{Frs}$ (23), we get that

$$p_{\mathrm{l;B}} = 91 - 55 = 36\,\mathrm{Frs}. \quad (24)$$

Conclusion

The connection between the layer time of process 2 and the layering cost $p_{\mathrm{l;B}}$ is

$$p_{\mathrm{l;B}} = R N_{\mathrm{l;B}} \tau_{\mathrm{layer}} \quad (25)$$

where R is the hourly rate of the machine ued in process 2 and $N_{\mathrm{l;B}}$ is the number of layers needed to build part B. Obviously we have

$$N_{\mathrm{l;B}} = \frac{H_B}{e}$$

with H_B the height of part B in the build direction and e the layer thickness. Substituting this relation in (25) we get

$$p_{\mathrm{l;B}} = \frac{RH_B}{e} \tau_{\mathrm{layer}}$$

ane we can solve for τ_{layer}:

$$\tau_{\mathrm{layer}} = \frac{p_{\mathrm{l;B}} e}{RH_B}.$$

The last step is to replace the quantities in the right hand side by their known values given in the **right unit**: $p_{\mathrm{l;B}} = 36\,\mathrm{Frs}$ (24), $R = 0.02\,\mathrm{Frs/s}$, $e = 0.1\,\mathrm{mm}$ and $H_B = 36\,\mathrm{mm}$. We get

$$\tau_{\mathrm{layer}} = \frac{36 \times 0.1}{0.02 \times 36} = 5\,\mathrm{s}. \tag{26}$$

The layering time of process 2 is 5 s.