Correction Series 3.

Exercice 1

a) The five major classes of additive processes with which it is possible to make **direct** metal parts are:

name	**acronym**
the *laser metal wire deposition* process	LMWD
the *metal laminated tooling* process	MeLaTo
laser powder bed fusion	L-PBF
the *direct metal deposition* process	DMD

Table 1: The five major classes of the ”direct metal” additive processes

b) The following observations can be made:

- The raw material comes
  - as a powder (L-PBF, DMD),
  - as a wire (LMWD),
  - as a plate (MeLaTo).
- Before consolidation the powder is
  - either deposited in a bed (L-PBF),
  - either jetted by a nozzle (DMD).
- The consolidation principle is
  - liquide phase sintering (L-(P)PBF),
  - total fusion (L-(M)PBF).

This leads to the following classification:

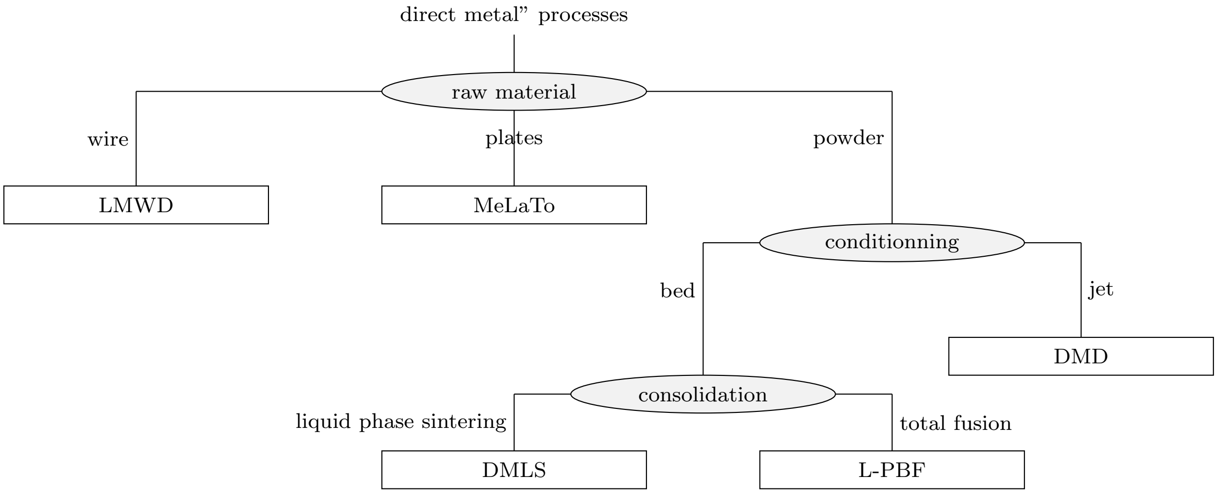

Figure 1: Classification of the "direct metal" additive processes

Exercice 2

a) The specific energy E used to melt a unit volume of material is the sum of two contributions:

(i) the energy needed to reach the melting point from ambient condition ($T_{\text{amb}} \simeq 20°\text{C}$)

(ii) the energy needed to transform the solid to liquid (latent heat).

With the numerical data we have:

$$E = \rho(C_{\text{p}}(T_{\text{melt}} - T_{\text{amb}}) + L) \simeq 8.04 \times 10^{-3} \times (0.4 \times (1413 - 20) + 305) \simeq 6.932\,\text{J/mm}^3.$$

The power P_{abs} the powder is able to absorb is a percentage of the output laser power:

$$P_{\text{abs}} = AP_0.$$

The percentage A is the absorption coefficient given in Tab. 1. Since $P_0 = 500\,\text{W}$, we have

$$P_{\text{abs}} = 0.4 \times 500 = 200\,\text{W}.$$

If we assume that there are no thermal losses inside the powder and between the powder and its surroundings, the volume of material we can melt per unit of time is

$$\text{MCR}_{\text{opt}} = \frac{P_{\text{abs}}}{E} \simeq \frac{200}{6.932} \simeq 28.85\,\text{mm}^3/\text{s}.$$

b) Mass conservation implies a relationship between the thickness e_{cons} of the consolidated layers and the thickness e of the deposited powder bed. Assuming shrinkage in the $z-$direction only we can write the conservation equation

$$\rho e_{\text{cons}} = \rho_{\text{powder}} e.$$

It leads to:

$$e_{\text{cons}} = \frac{\rho_{\text{powder}}}{\rho} e.$$

We know that the ratio between the powder and the bulk densities is 73%. If $e = 50\,\mu$m, we get

$$e_{\text{cons}} \simeq 0.73 \times 50 = 36.5\,\mu\text{m}.$$

The number N_{dep} of successive powder bed depositions needed to build a part of height H is the closest integer to the ratio H/e_{cons}:

$$N_{\text{dep}} = E\left(\frac{H}{e_{\text{cons}}}\right) = E\left(\frac{50}{0.0365}\right) = 1'370.$$

c) The price of the part is computed in three steps:

(i) At first, we have to compute the fabrication time T_{fab}. It consists of two contributions, the layering time T_{dep} proportional to the number N_{dep} of needed depositions (see point (b)) and the consolidation time T_{consol} proportional to the part volume:

$$T_{\text{fab}} = T_{\text{dep}} + T_{\text{consol}}$$

with

$$T_{\text{dep}} = N_{\text{dep}} \tau_{\text{dep}} \quad \text{and} \quad T_{\text{consol}} = \frac{V}{\text{MCR}}.$$

The deposition time τ_{dep} and the volume of the part are given in Tab. 2. If the material consolidation rate MCR is assumed to be half of the optimal value MCR_{opt} obtained at point (a) we get that

$$T_{\text{dep}} \simeq 1'370 \times 6 = 8'220\,\text{s} \simeq 2.29\,\text{h},$$
$$T_{\text{consol}} \simeq \frac{125 \times 10^3}{0.5 \times 28.85} \simeq 8'666\,\text{s} \simeq 2.41\,\text{h}$$

and

$$T_{\text{fab}} \simeq 2.29 + 2.41 = 4.7\,\text{h}$$

(ii) In the second step, we compute the hourly rate M_{mach} of the machine. With an average occupation of 6 hours par day, the machine will be operated during:

$$T = 3 \times 250 \times 6 = 4'500\,\text{h}$$

before the end of its depreciation time of three years. Since the gain G we expect is 1.3 times the price of the machine (machine depreciation plus a ROI of 10% per year), the hourly rate will be

$$M_{\text{mach}} = \frac{G}{T} \simeq \frac{1.3 \times 520'000}{4'500} \simeq 150.25\,\text{Frs/h}.$$

(iii) The price p_{part} of the part is now obtained by adding four contributions, the price of the material, the depreciation of the machine, the wages and the laser energy consumption[1]:

$$p_{\text{part}} = \rho V p + (M_{\text{mach}} + W) T_{\text{fab}} + \frac{P_0}{\eta} c T_{\text{consol}}.$$

---

[1] The energy spent in the powder distribution is neglected.

The parameters ρ (material density) is given in Tab. 1 while p (material price) and η (laser efficiency) can be found in Tab. 2. Recall also that W represents the expected hourly wage ($W = 100\,\mathrm{Frs/h}$), c is the electrical energy cost ($c = 0.25\,\mathrm{Frs/kWh}$) and P_0 is the laser output power ($P_0 = 500\,\mathrm{W}$) which is switched off during deposition. The conclusion is

$$p_{\text{part}} = 8.04 \times 10^{-6} \times 125 \times 10^3 \times 45 + (150.25 + 100) \times 4.7 + \frac{500 \times 10^{-3}}{0.1} \times 0.25 \times 2.41 \simeq 1'225.00\,\mathrm{Frs}.$$

Exercice 3

a) We proceed in three steps.

First step. The consolidated volume V after completion of the scanning strategy shown in Fig. 1 is a parallelipipedon of thickness e_{cons}, and horizontal dimensions l and L:

$$V = Lle_{\text{cons}}.$$

The material consolidation rate MCR can be obtained as the ratio of the volume V and the scan time τ_{scan}, connected to the scan length d_{scan} and to the scan speed v by:

$$\tau_{\text{scan}} = \frac{d_{\text{scan}}}{v}.$$

Therefore

$$\mathrm{MCR} = \frac{V}{\tau_{\text{scan}}} = \frac{Lle_{\text{cons}}}{\frac{d_{\text{scan}}}{v}}$$

and, after simplification:

$$\mathrm{MCR} = \frac{Ll}{d_{\text{scan}}} ve_{\text{cons}}. \tag{1}$$

Second step. The next step is to compute the scan length d_{scan}. From Fig. 1, we see that d_{scan} is proportional to the height of the scanned surface:

$$d_{\text{scan}} = N_{\text{scan}} L$$

where N_{scan} is the total number of scans.

The total number of scans, N_{scan}, depends itself on the hatching distance h and is proportional to the width l of the scanned area. It is clear from Fig. 1 that

$$N_{\text{scan}} \simeq \frac{l}{h}$$

therefore

$$d_{\text{scan}} \simeq \frac{Ll}{h}.$$

Substituting this information into the expression (1) of the material consolidation rate, we find:

$$\mathrm{MCR} \simeq \frac{Llve_{\text{cons}}}{\frac{Ll}{h}}$$

which simplifies to

$$\mathrm{MCR} \simeq v h e_{\mathrm{cons}}. \tag{2}$$

Third step. The conclusion is obtained by putting the consolidated thickness e_{cons} in relation with the powder bed thickness e. Assuming shrinkage in the $z-$direction only and taking mass conservation into account, we can write

$$\rho e_{\mathrm{cons}} = \rho_{\mathrm{powder}} e$$

where ρ is the bulk density and ρ_{powder} the powder density. Introducing the powder relative density r:

$$r = \frac{\rho_{\mathrm{powder}}}{\rho}$$

we conclude that

$$e_{\mathrm{cons}} = re$$

and the final expression of the material consolidation rate is

$$\mathrm{MCR} \simeq vhre. \tag{3}$$

b) The quantity

$$E = \rho(C_{\mathrm{p}}(T_{\mathrm{melt}} - T_{\mathrm{amb}}) + L)$$

represents the energy to bring a unit volume of gold to the melting point from ambient condition ($T_{\mathrm{amb}} \simeq 20°\mathrm{C}$) and to transform it to liquid by providing the latent heat L. With the numerical data of Tab. 1 we have:

$$E \simeq 19.32 \times 10^{-3} \times (0.13 \times (1069 - 20) + 66.2) \simeq 3.913\,\mathrm{J/mm^3}.$$

The power P_{abs} the powder is able to absorb is obtained as the product of the absorption coefficient A and of the net output laser power $P_0 = 400\,\mathrm{W}$:

$$P_{\mathrm{abs}} = AP_0 = 0.1 \times 400 = 40\,\mathrm{W}.$$

If we assume that there are no thermal losses inside the powder and between the powder and its surroundings, the volume of material we can melt per unit of time is

$$\mathrm{MCR_{opt}} = \frac{P_{\mathrm{abs}}}{E} \simeq \frac{40}{3.913} \simeq 10.22\,\mathrm{mm^3/s}.$$

c) We assume that the hatching distance h is adapted to the beam radius w:

$$h = w. \tag{4}$$

Under this condition, the fact that we want to reach the optimal material consolidation rate $\mathrm{MCR_{opt}}$ constrains the product of the unknown scan speed v and beam radius. Because of equations (3) and (4) we must have:

$$vw = \frac{\mathrm{MCR_{opt}}}{re} = \frac{10.22}{0.74 \times 0.05}$$

which means

$$vw = 276.22\,\mathrm{mm}^2/\mathrm{s}. \tag{5}$$

Unfortunately, any pair (v, w) satisfying (5) is not eligible. We still have to ensure that the maximal possible temperature elevation $\Delta T_{\max}$ under the laser beam corresponds at least to the difference between the fusion temperature T_{melt} and the ambient temperature T_{amb}:

$$\Delta T_{\max} \geq T_{\text{melt}} - T_{\text{amb}}. \tag{6}$$

We have seen in the lecture that $\Delta T_{\max}$ can be expressed in term of the operating parameters v and w as:

$$\Delta T_{\max} = C_{\text{b}} \frac{A I_0^{\max} w}{k} H(\frac{\rho C_{\text{p}}}{2k} vw). \tag{7}$$

In this formula, ρ, k, C_{p} and A refer to the material properties given in Tab. 1, $I_0^{\max}$ is the peak intensity of the beam, while and for a gaussian beam, $C_{\text{b}} = \frac{\sqrt{\pi}}{2}$ and

$$H(s) = \frac{1}{\pi} \int_0^{\pi} \operatorname{erfcx}(s \frac{1 + \cos\vartheta}{2})\, \mathrm{d}\vartheta \tag{8}$$

The function $H(s)$ is represented in Fig. 2.

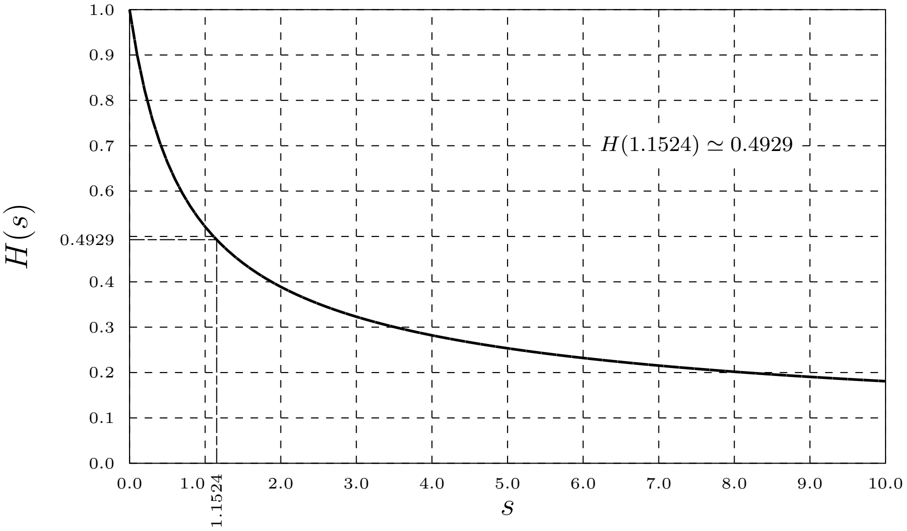

Figure 2: Graphic of the function $H(s)$ (see (8))

The peak intensity is itself connected to the laser power P_0 and to the beam radius w by a formula given in the course book:

$$I_0^{\max} = \frac{P_0}{\pi w^2}.$$

With this and (7) the melting condition (6) is:

$$C_{\text{b}} \frac{A P_0}{k \pi w} H(\frac{\rho C_{\text{p}}}{2k} vw) \geq T_{\text{melt}} - T_{\text{amb}}$$

or, if we solve for w:

$$w \leq C_{\text{b}} \frac{A P_0}{k \pi (T_{\text{melt}} - T_{\text{amb}})} H(\frac{\rho C_{\text{p}}}{2k} vw). \tag{9}$$

Using the numerical value given in Tab. 1 and taking into account that $P_0 = 400\,\mathrm{W}$, $T_{\mathrm{amb}} = 20°\mathrm{C}$ and that the value of the product vw is known (5), we get

$$w \leq \frac{\sqrt{3.14}}{2} \times \frac{0.1 \times 400}{0.301 \times 3.14 \times (1069 - 20)} \times H\left(\frac{19.32 \times 10^{-3} \times 0.13}{2 \times 0.301} \times 276.22\right)$$

i.e

$$w \leq 0.03574 \times H(1.1524). \tag{10}$$

We read the value of $H(1.1524)$ in Fig. 2:

$$H(1.1524) \simeq 0.4929$$

and we conclude that the beam width of our laser has to be adjusted to a sufficiently small value:

$$w \leq 17.6\,\mu\mathrm{m}. \tag{11}$$

On the other side, the scan velocity v, needs to be adjusted to a sufficiently large value in order to respect (5)

$$v = \frac{276.22}{w} \geq \frac{276.22}{17.6 \times 10^{-3}} \simeq 15'700\,\mathrm{mm/s}. \tag{12}$$

d) The limit (11) for the beam width is relatively severe. With standard high power lasers (providing more than 100 W) and standard optics, w can hardly be less that $50 - 80\,\mu$m. Nevertheless, it is known that those systems (e.g. Concept Laser$^{\mathrm{TM}}$-M1 system or EOS$^{\mathrm{TM}}$-M080 system) enable to manufacture parts out of gold powder.

One possible explanation would be that these machines use scan speeds much smaller then those in (12). But this explanation is not valid. Even if the scan speed goes to zero then function H goes to 1 (see Fig. 2) and we still get from (9) a limitation (cf. also (10)):

$$w \leq C_{\mathrm{b}} \frac{AP_0}{k\pi(T_{\mathrm{melt}} - T_{\mathrm{amb}})} \simeq \frac{\sqrt{3.14}}{2} \times \frac{0.1 \times 400}{0.301 \times 3.14 \times (1069 - 20)} = 0.03574\,\mathrm{mm} = 35.74\,\mu\mathrm{m} \tag{13}$$

not fulfilled in practice. The reason why standard systems still can work with gold is to be found elsewhere.

Our theory is that the numerical value we have used for the thermal conductivity k in formula (9) or (13) was the thermal conductivity k of the bulk. This is clearly a very pessimistic guess. In reality, an intermediate value between the bulk and the powder thermal conductivity should be used[1]. With this new value, equations (9) or (13) certainly lead to less strict limitations for w.

Anyway, this reasoning points out an important fact. Usually, high thermal conductivity of the material makes the L-PBF process more difficult to master from the energetic point of view.

---

[1] The powder thermal conductivity is much less (100 times to 200 times less) than the bulk thermal conductivity.