ME-411 Teaching Staff

Lecturer:

Pedro
Reis

Instructors: (Postdocs @ fleXLab)

Ji-Sung
Park

Kanthasamy
Ubamanyu (Uba)

Teaching Assistants: (Ph.D. students @ the fleXLab)

Eduardo
Gutiérrez Prieto

Gilad
Yakir

Javier
Sabater

Danick
Lamoureux

Thanks!

Experts in slender structures:

Basile Audoly
E. Polytechnique
France

Marcelo Dias
Aarhus University
Denmark

Jose Bico
ESPCI
France

Benoit Roman
CNRS/ESPCI
France

John Hutchinson
Harvard
USA

Past members of the fleXLab:

Paul Grandgeorge

Tomohiko Sano

Samuel Poincloux

Tim Chen

Dong Yan

Matteo Pezzulla

Logistics:

	Monday	Tuesday	Wednesday	Thursday	Friday	Monday	Tuesday
09h15				Live Lecture *n*			
10h15				Live Lecture *n*			
11h15				Live Lecture *n*			
14h15		Studio *n-1*					Studio *n*
15h15		Studio *n-1*					Studio *n*
16h15							

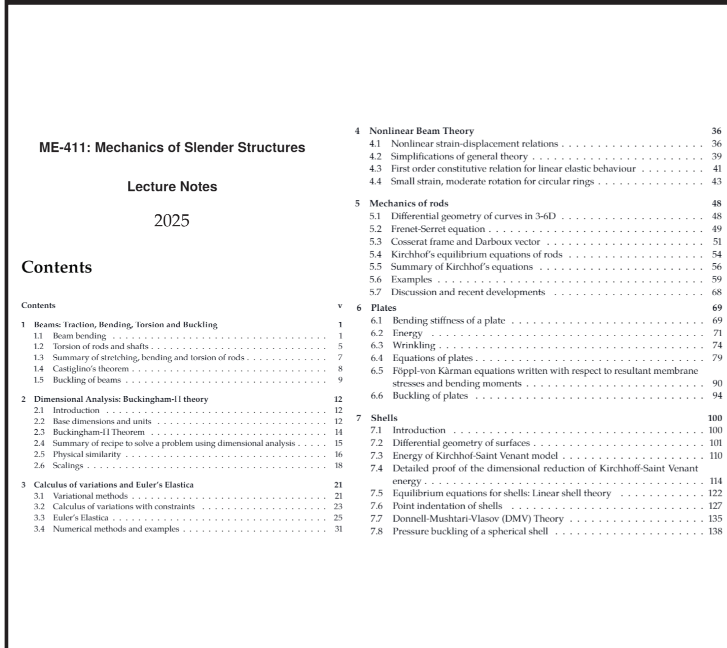
Live Lecture	On-campus: In Room CE1103 & Zoom	Main content. Demos. Q&A.
Lecture recording	Posted ~1 day after Lecture, link on Moodle	
Pre-studio video	Posted Thursdays, link on Moodle	Watch pre-studio video (posted on Monday).
Studio	On-campus: In Room MED 2 2423	Try solving problem on your own, first. Come to Studio with Questions.

- **Zoom link (for lectures):** https://epfl.zoom.us/j/64469475902?pwd=oF4Jjp7Ak86bmNVa9rSwYBDMf86sTw.1
- **Office hours:** (informal) e.g. during the breaks, after lecture, or by appointment.
- **Assessment method:** Final exam (100%).
- **Language of instruction:** English [But, feel free to also ask questions in French (or Portuguese, or Spanish…].

Post-Videos of lectures

Posted on Mediaspace channel (~ 1 day after Lecture)

Lecture Notes


ME-411: Mechanics of Slender Structures

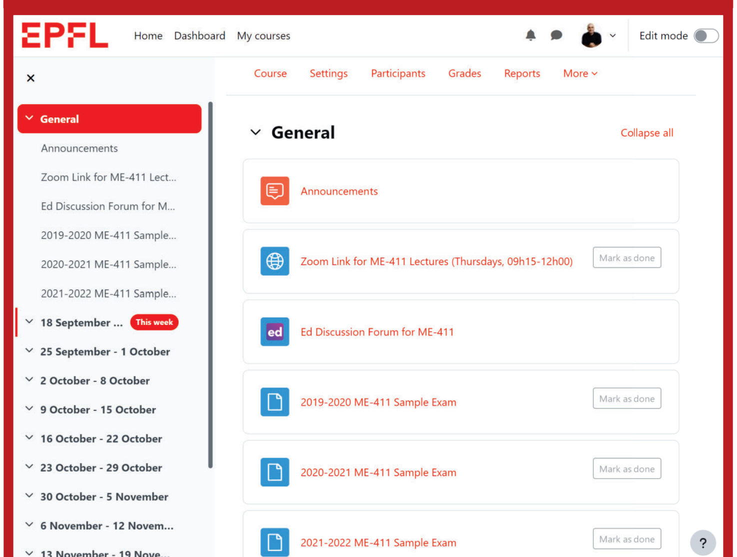
Lecture Notes

2025

Contents

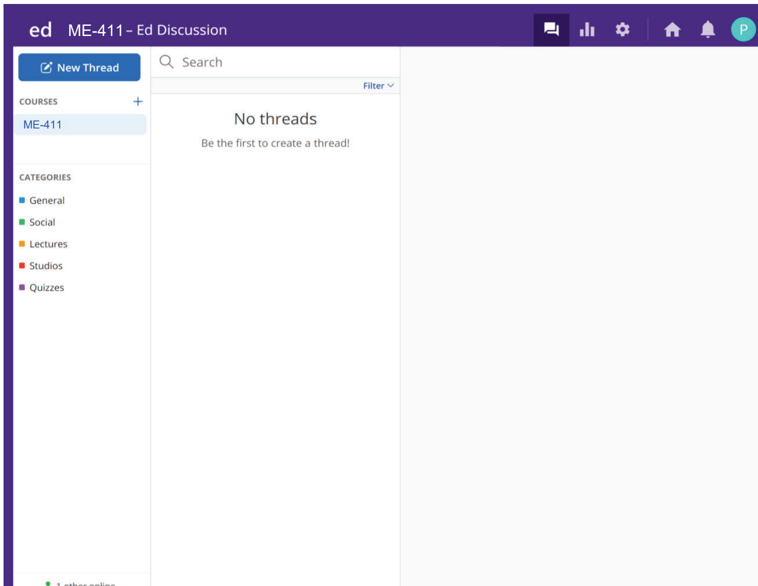
Posted on MOODLE (by Thursday morning)

https://moodle.epfl.ch/course/view.php?id=15853



Official portal to make announcements and post materials such as lecture notes, exercises, practice problems, etc.

https://edstem.org/eu/courses/2504/



General student forum for ME-411. The questions you post will be answered both by lecturers/instructors as well as by other students.

Mechanics of Slender Structures - Topics

Part I

Basics:
Scalings, Calculus of Vatiations,
Euler's *Elastica*, buckling, strings
Nonlinear curved beams

Part II

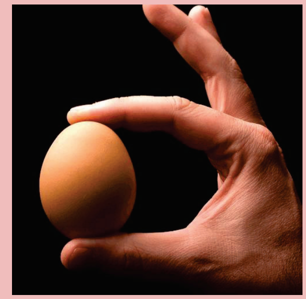
Rods

Part III

https://www.pickpik.com/

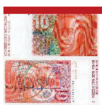
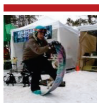
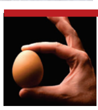
Plates

Part IV



Shells

Syllabus:

	Date	Lecture #	Topic
 Part I	11 Sep.	Lecture 1	(Review: Elasticity 3D & Beams) + Introduction.
	18 Sep.	Lecture 2	Dimensional Analysis. Scalings.
	25 Sep.	Lecture 3	Calculus of Variations. Euler's Elastica & Buckling. [by Dr. Luna Lin]
	02 Oct.	Lecture 4	Nonlinear curved beams.
 Part II	09 Oct.	Lecture 5	Kirchhoff's theory of rods I (Basis)
	16 Oct.	Lecture 6	Kirchhoff's theory of rods II (Applications)
	23 Oct.	EPFL Fall Break – No Classes!	
 Part III	30 Oct.	Lecture 7	Plates I: Pure bending
	06 Nov.	Lecture 8	Plates II: Nonlinear plate theory
 Part IV	13 Nov.	Lecture 9	Plates III: Foppl-von Kármán theory of plates. Buckling of plates.
	20 Nov.	Lecture 10	Shells I: Pressure vessels & differential geometry of surfaces
	27 Nov.	Lecture 11	Shells II: Dimensional reduction & Linear shell theory.
	04 Dec.	Lecture 12	Shells III: Shell indentation, Nonlinear shells, Shell Buckling (onset).
	11 Dec.	Lecture 13	Final Review of Course & Tour of fleXLab
	18 Dec.	No Lecture	Happy Holidays!

Studios (Exercise classes):

Studio 01: A Truss structure (Crane).
Internal forces, method of sections, method of joints, buckling.

Studio 02: Euler buckling from dimensional analysis, Periodic folding of a sheet of paper.

Studio 03: Elasto-gravity bending of a sheet of paper.

Studio 04: Matched approximations of a loaded beam.

Studio 05: Naturally curved hair in 2D.

Studio 06: Mechanics of a helical Kirchhoff rod.

Studio 07: Tearing of an adhesive thin film.

Studio 08: Deformation of a circular plate under distributed load.

Studio 09: Buckling of plates.

Studio 10: Cylinders: pressure and geometry.

Studio 11: Membrane theory of shells.

Studio 12: Linear shell theory.

ME-411: Main Learning objectives

1. Understand the power of dimensional analysis and scalings to gain physical insight into other difficult-to-tackle structures.
2. Recognize and analyze slender structural systems composed of slender elements (beams, rods, plates, shells) toward predicting their mechanical performance.
3. Model and analytically solve simple problems of statics and stress analysis.
4. Analyze and design assemblies of simple mechanical elements in the framework of statics and buckling.
5. Model with analytical (or numerical) tools the nonlinear response of structures and materials.
6. Recognize the power of classical analytical tools of structural mechanics to rationalize the mechanical response of complex systems within modern contexts of research and engineering practice.

ME-411: What I expect you to expect

- **A** Familiarity with structural analysis of elastic systems (bars, trusses, beams, trusses, frames, and mechanisms) and boundary value problems in elasticity.

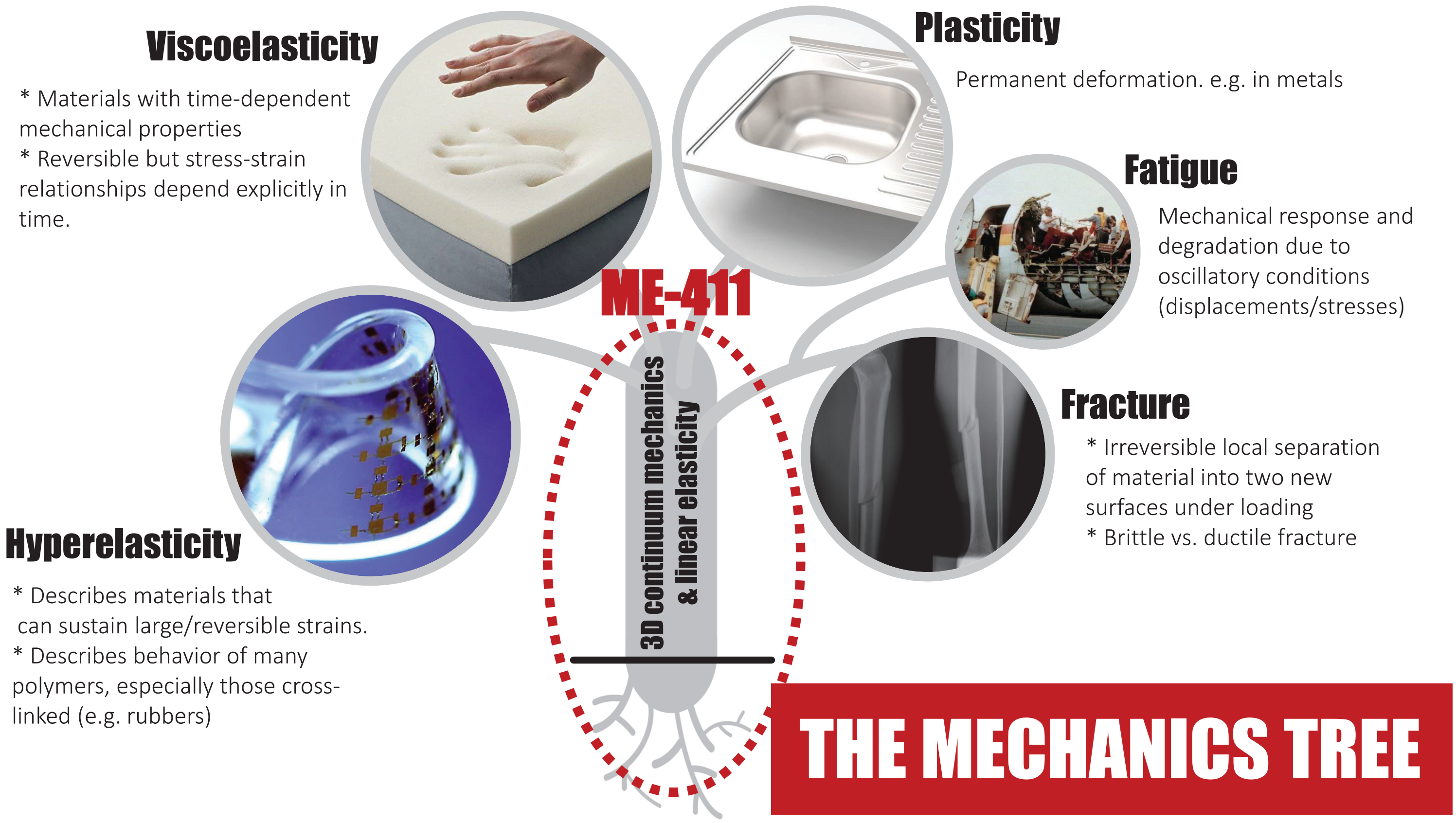
- **B** The emphasis of the course is primarily on analytical methods to rationalize and model the mechanics of slender structure, hence, an affinity to mathematical and theoretical approaches in problem solving is expected.

The goal of Engineering Mechanics is to:

Predict how

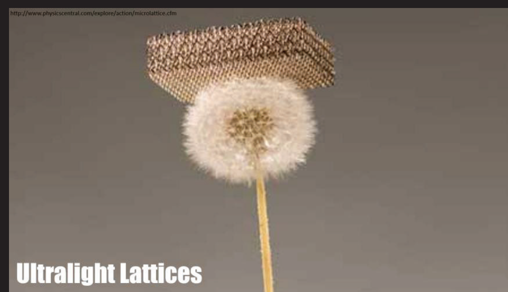
objects		respond	to	load
Materials:	**Geometries:**			
Metals	Continuous bodies	Change shape		Impact
Ceramics	Structures	Fracture		Static loading
Polymers	Simple elements	Yield		Gravity
Composites		Creep		Stretching
				Shearing
				Bending

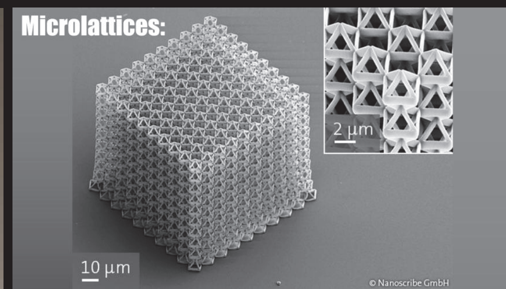
to be able to analyze and/or design mechanical systems.



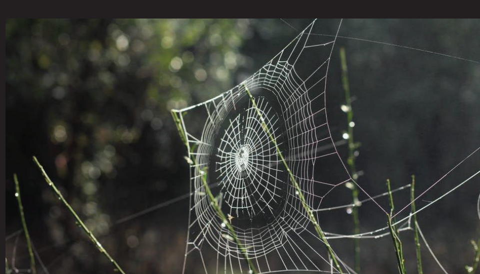
Trusses and frames (large-scale)

Trusses and frames (small-scale)



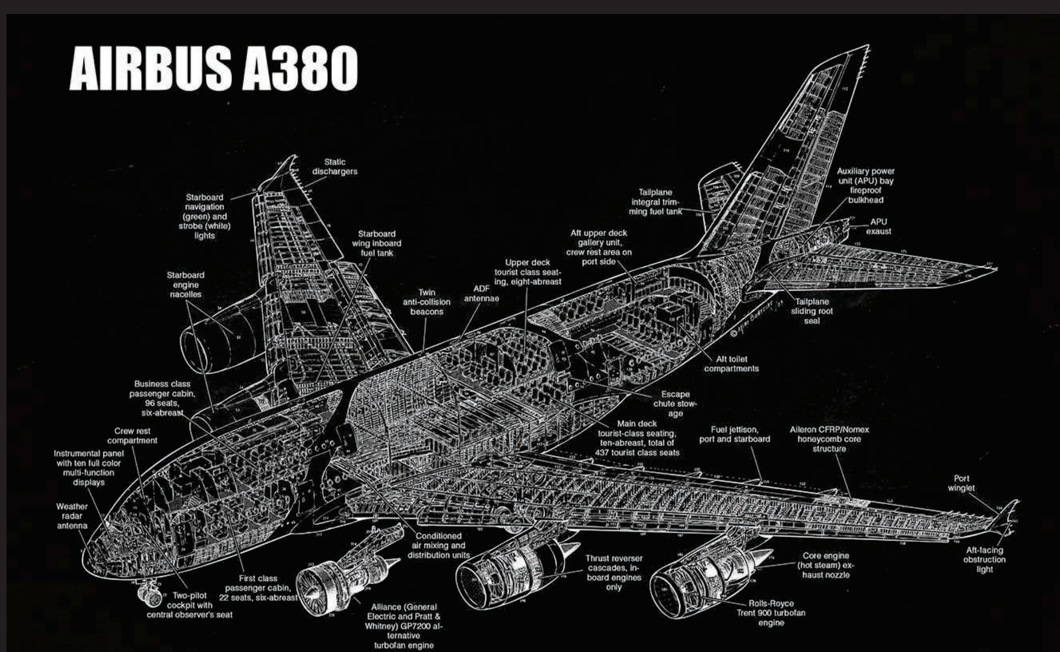


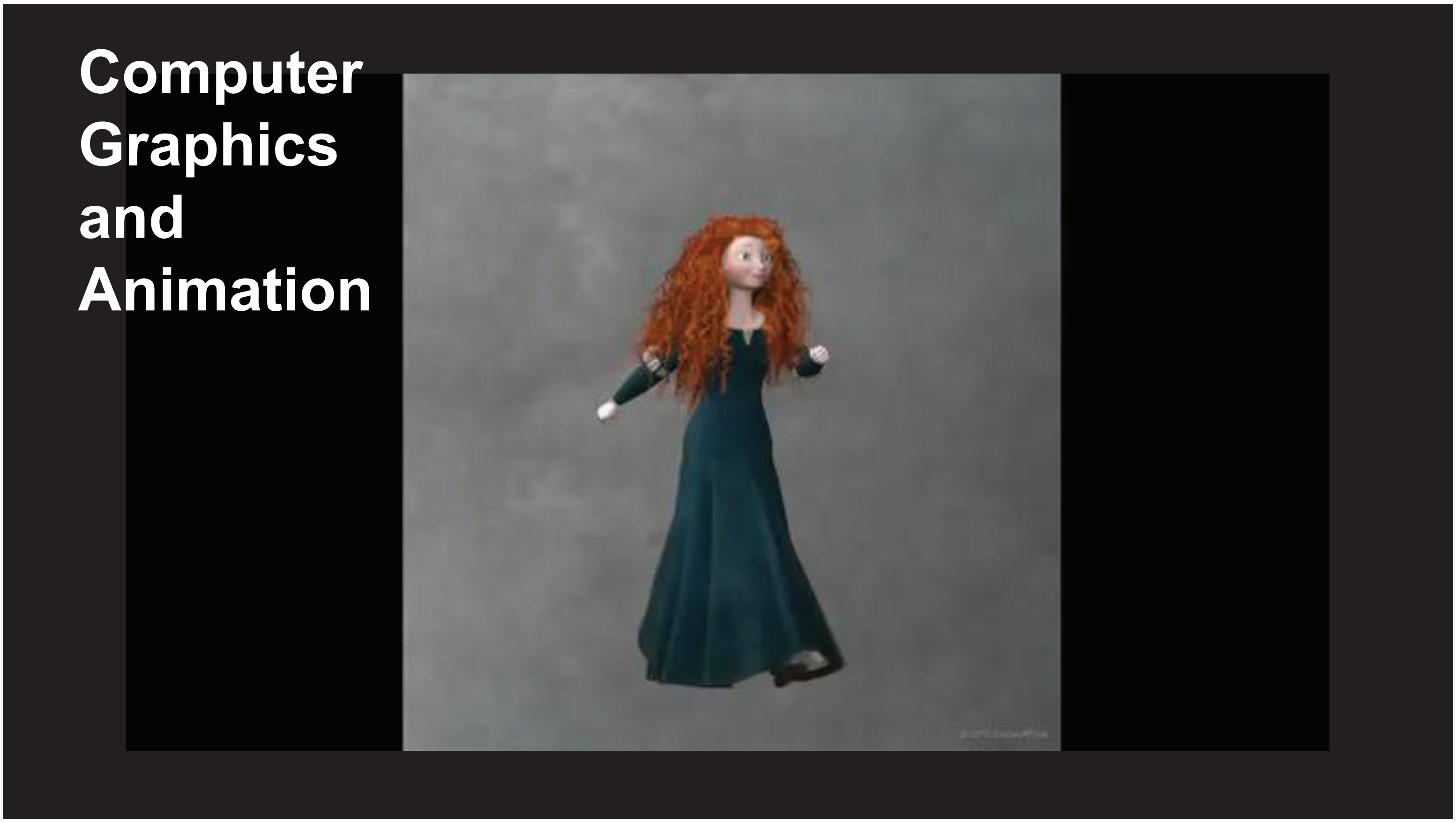
Strings, Beams and Arches



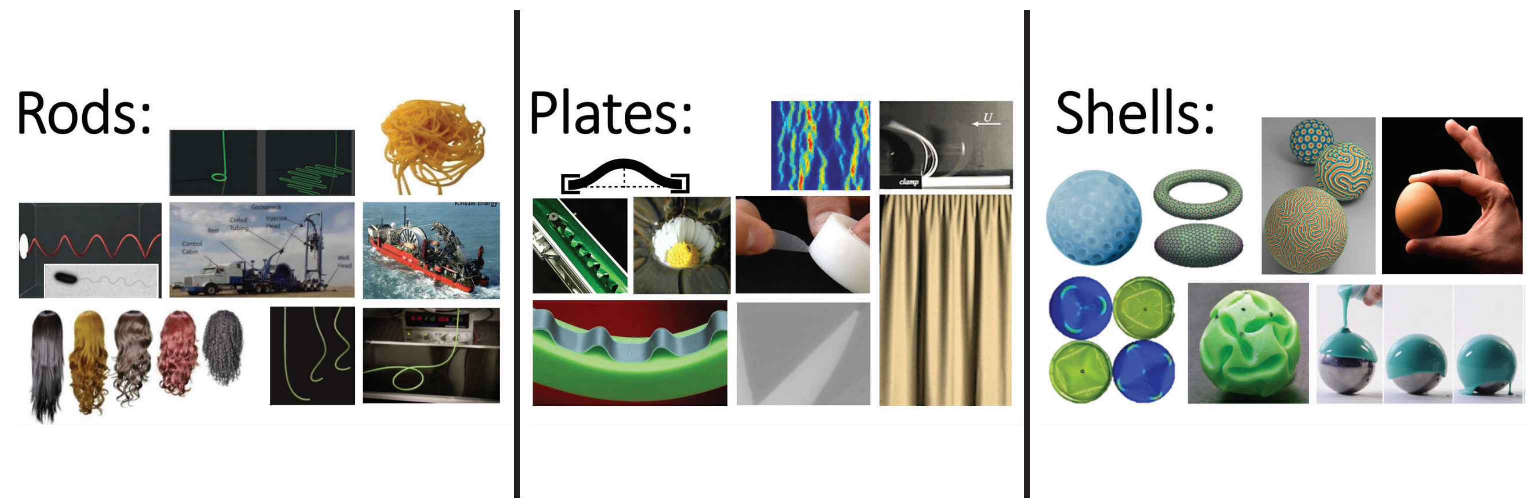
Tubes and shells

Compound structures





Slender structures:



(this is essentially the structure of the course)

Slender structures:

Thickness *Length*

$h \ll L$

(small strain, large displacement)

Width $w \sim h$

rods

inextensible

bending + twist

$$F = \frac{EI}{L^2}$$

$w \gg h$

$L \gg w \gg h$

ribbons

Plates
many isometries
(soft bending modes)

shells
less isometries.
structual stiffening

Geometry

Gauss (1777-1855)
theorema egregium

Dimensional Reduction in Slender Structures

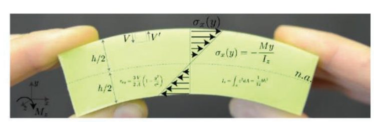
3D Elasticity

kinematics: $\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x^j} + \frac{\partial u_j}{\partial x^i} + \frac{\partial u_k}{\partial x^i}\frac{\partial u_k}{\partial x^j}\right)$

constitutive law: $\sigma_{ij} = \frac{E}{1+\nu}\left(\epsilon_{ij} + \frac{\nu}{1-2\nu}\epsilon_{kk}\delta_{ij}\right)$

$$\mathcal{E} = \frac{1}{2}\int_{\mathcal{B}} \mathrm{d}V\, \sigma_{ij}\epsilon_{ij}$$

balance equations: $\delta\mathcal{E} = 0 \quad \Rightarrow \quad \frac{\partial \sigma_{ij}}{\partial x^j} = f_{\text{ext},\, i}$



D. P. Holmes: http://www.bu.edu/moss

Simplifying assumptions:

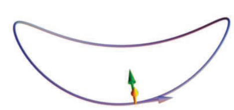
state of stress
kinematics

1D Elasticity

$\mathcal{B} = \mathcal{C} \times \underbrace{\mathcal{S}_{cs}}_{\text{av.}}$

$$\mathcal{E} \approx \frac{1}{2}\int_{\mathcal{C}} \mathrm{d}s\, f_{\mathrm{B}}(\omega_1, \omega_2, \omega_3)$$

Beams, bars, rods, strings, ribbons, etc



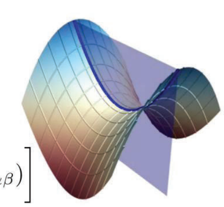
$\mathcal{S} = \mathcal{C} \times \underbrace{[-w/2, w/2]}_{\text{av.}} \quad \longrightarrow \quad D \sim \frac{wh^3E}{12}$

2D Elasticity

$\mathcal{B} = \mathcal{S} \times \underbrace{[-h/2, h/2]}_{\text{av.}}$

$$\mathcal{E} \approx \frac{1}{2}\int_{\mathcal{S}} \mathrm{d}A \left[hE\, f_{\mathrm{S}}(\boldsymbol{\epsilon}^{(0)}) + \frac{h^3E}{12} f_{\mathrm{B}}(\mathbf{K}_{\alpha\beta})\right]$$

Plates and shells

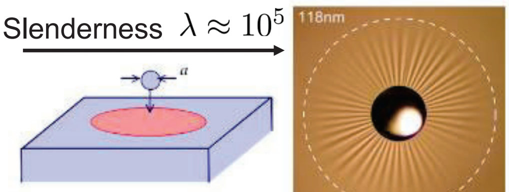


How slender are we talking about?

Slenderness: $\lambda = \dfrac{\ell}{h}$

Thickness $h = 50\,\mathrm{cm}$
Length $\ell \approx 30\,\mathrm{m}$
Slenderness $\lambda \approx 10^2$

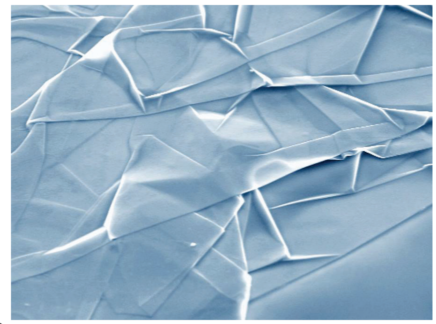
Slenderness $\lambda \approx 10^5$



Huang *et al. Science* (2007)

Thickness $h = 4\,\mathrm{m}$
Length $\ell = 342\,\mathrm{m}$
Slenderness $\lambda \approx 10^2$

Millau Viaduct, France



Thickness $h = 0.3\,\mathrm{nm}$
Length $\ell \approx \mathrm{cm}$
Slenderness $\lambda \approx 10^8$

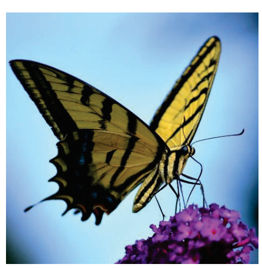
Andre Geim & Kostya Novoselov, Nobel Prize 2010

How slender are we talking about?

Slenderness: $\lambda = \dfrac{\ell}{h}$

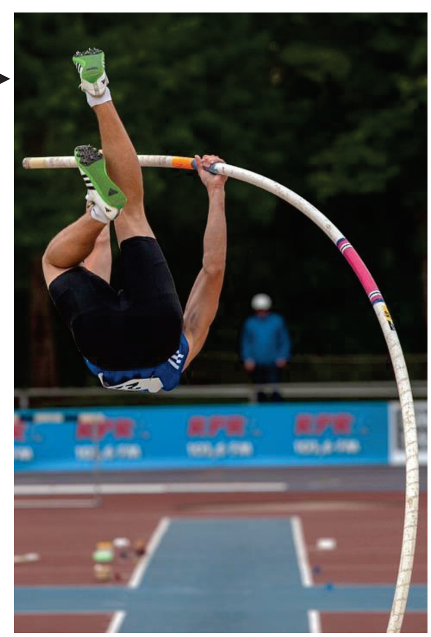
$\lambda \approx 10$

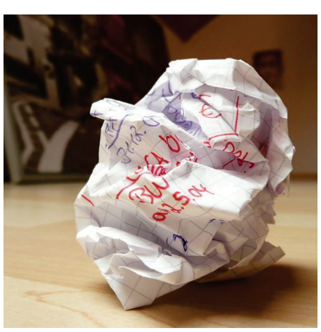
$\lambda \approx 10^2$



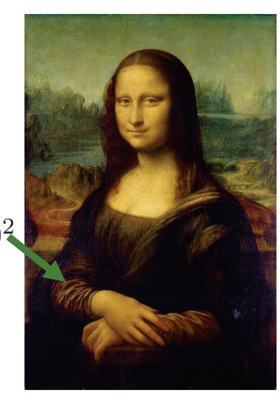
$\lambda \approx 10^2$

$\lambda \approx 10^2$



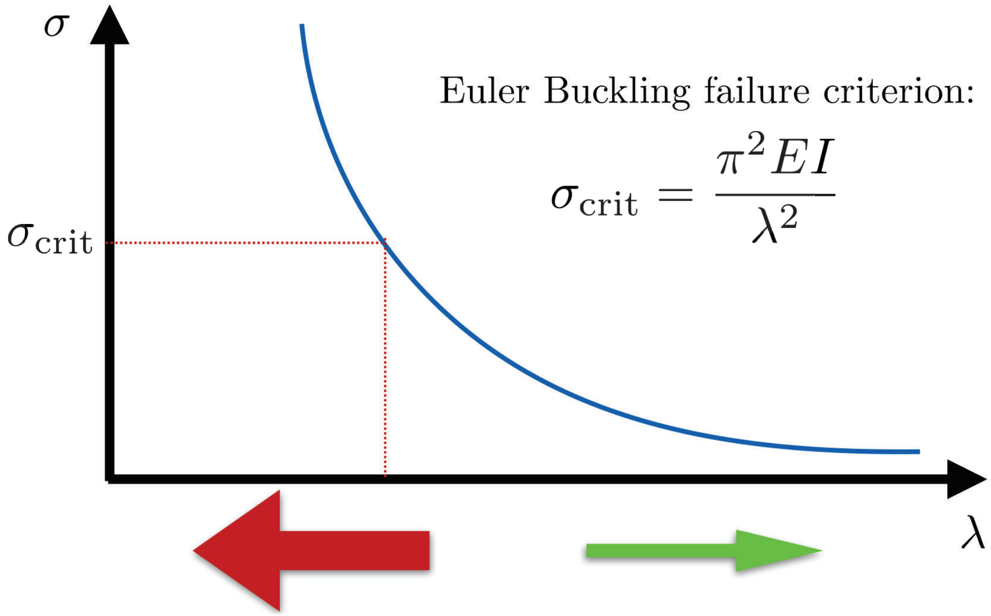


$\lambda \approx 10^3$

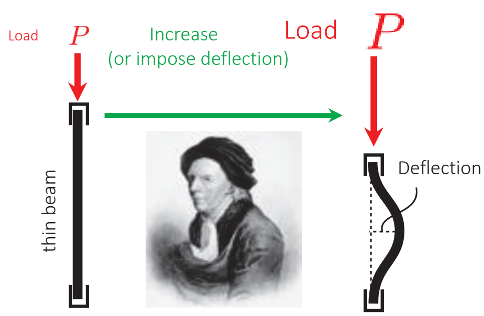


$\lambda \approx 10^2$

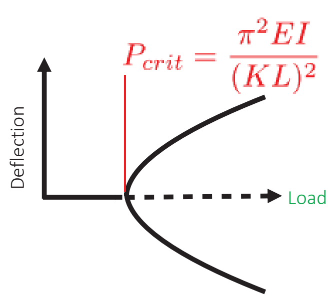
Why do we care? Buckling!

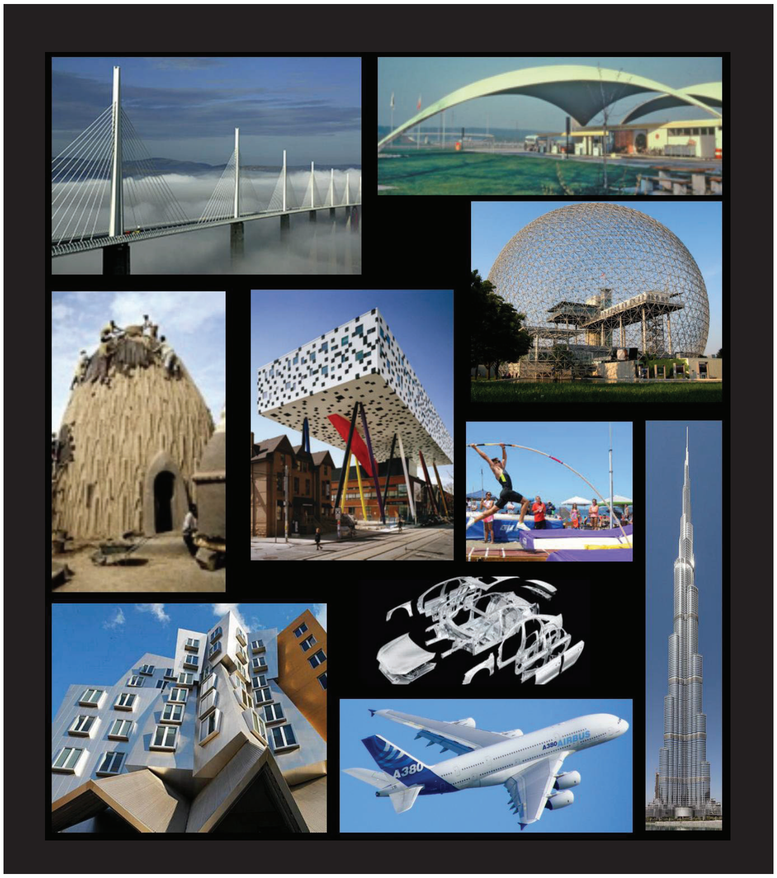


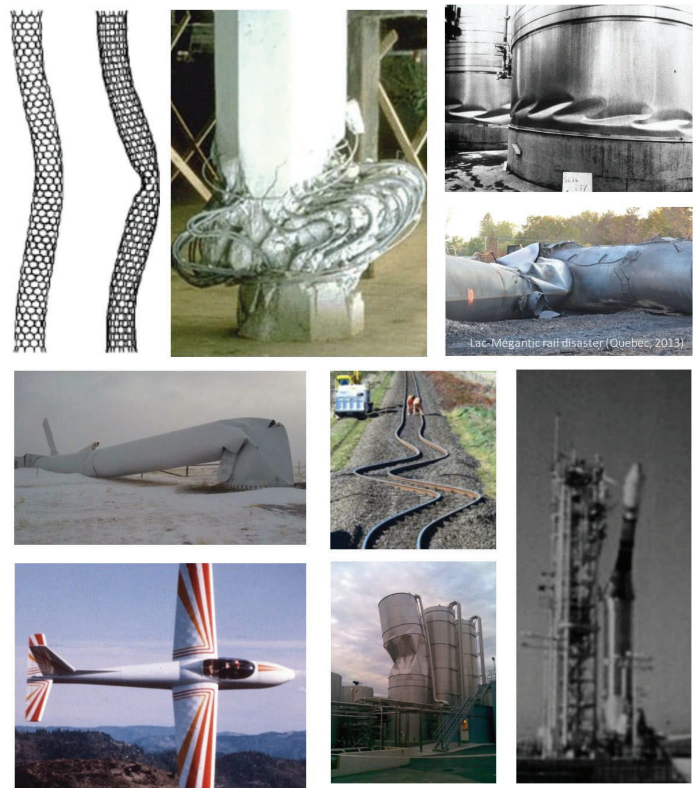
Buckling:



[Euler 1744 '*Elastica*']







Imploding of a railroad tank car

Demonstration of the implosion of a railroad tank car due to incorrect operating procedures.

[youtu.be/Zz95_VvTxZM]

Everybody loves a buckling problem!

Trends in Solid Mechanics 1979

Proceedings of the Symposium dedicated to the 65th Birthday of W.T. Koiter

Delft University of Technology
June 13 - 15, 1979

J.F. Besseling/A.M.A. van der Heijden/*editors*

Buckling: Progress and Challenge

*B. Budiansky and J.W. Hutchinson**

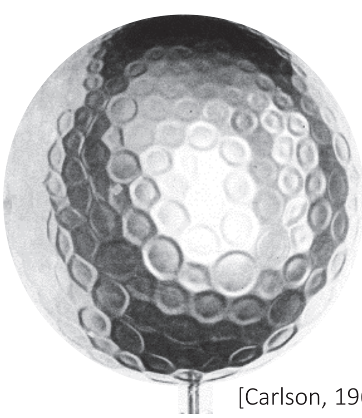
Summary

The general theories of elastic and plastic buckling and post-buckling behavior of structures are summarized briefly, and several special topics of current interest are discussed.

1. Introduction

It is tempting to begin this survey of the subject of buckling with a long backward look at its history and to muse on the special fascination it has held for so many engineers and scientists. Everybody loves a buckling problem! Without sentimentality, however, we will limit ourselves to the simple but confident introductory assertion that great progress in the understanding of buckling phenomena has been achieved in the last four decades. It is symptomatic of the vigor of the subject that surveys, assessments, and recapitulations of this progress have been appearing with unusual frequency [e.g. 1, 2, 3]. Nevertheless, we welcome with pleasure this opportunity to present our own overview at this symposium in honor of the central contemporary figure in the field of buckling and stability of structures, Warner Koiter.

Buckliphobia



[Carlson, 1967]

Imperfection sensitivity in the buckling of thin shells.

Buckliphilia

[Terwagne, 2014]

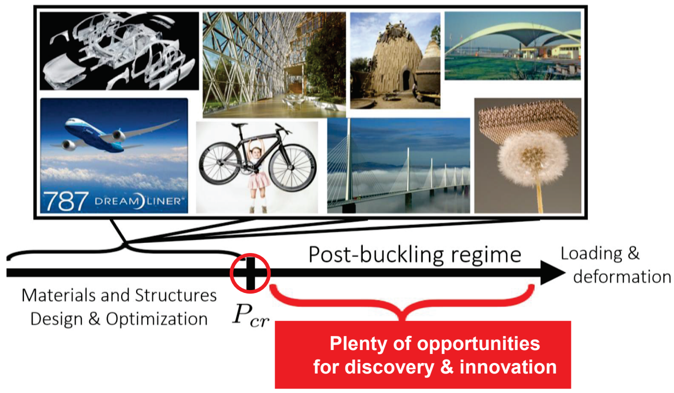
Wrinkling on curved surfaces for switchable & tunable aerodynamic drag reduction.

FLEXLAB

FLEXIBLE STRUCTURES LABORATORY

http://flexlab.epfl.ch
: @flexlab_epfl

Mechanics of slender structures: *'Buckliphobia'* & *'Buckliphilia'*:

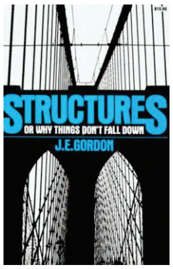


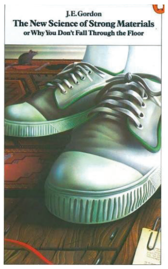
With a focus on P.D.E.s (Precision Desktop Experiments)

Some (non-exhaustive list of) recommended books

Maths | Elementary | General & comprehensive | Rods | Thin films | Plates / Shells

Popular Science





History