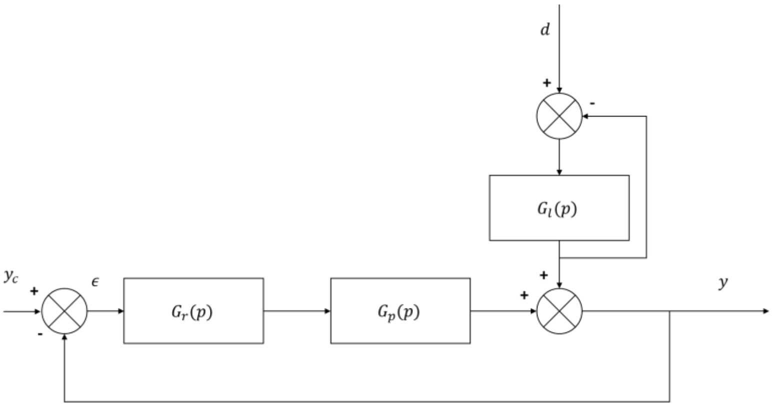
1. **Regulation and Control (35 points)** Consider the control system shown below. Given the transfer functions $G_l(s) = \frac{1}{s^2+2s-1}$ and $G_p(s) = \frac{1}{(s+2)}$



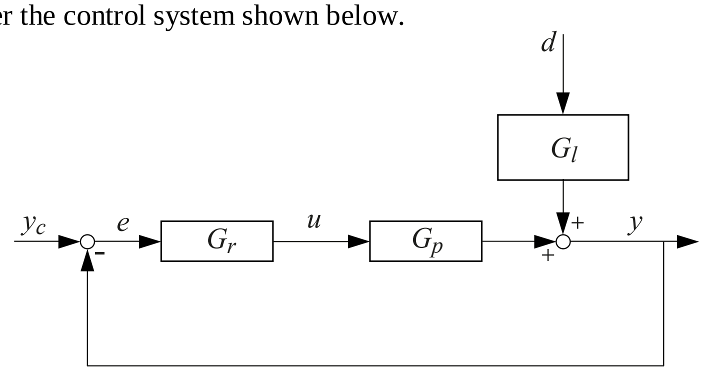
   a) Assuming that the system is controlled by a PID controller calculate the transfer function $H_c = \frac{Y}{Y_c}$
   b) Design a PID controller that eliminates the permanent error between the reference signal $y_c(t)$ and the output $y(t)$. Find an equation on the parameters of your controller so that the poles of your closed-loop system have the *fastest aperiodic* response.
   c) Calculate the transfer function $H_d = \frac{Y}{D}$.
   d) Does the controller designed in a) eliminates the influence of step disturbance $d(t) = D_0 e(t)$ at $t \to \infty$?
   e) We want the perturbation error at infinity to respect a margin: $\frac{y_\infty - D_0}{D_0} \leq 0.1$. Derive a condition on the PID parameters to verify this.
   f) Additionally, physical constraints impose $K = 10$. It is more expensive to reduce τ_I, as it requires more accurate instruments. Choose the cheapest τ_I respecting e), and from the constraint derived in b), determine τ_D for having a controller with the *fastest aperiodic* response.

2. **(20 points)** Consider the following dynamical system

$$y^{(iv)}(t) + 2y^{(iii)}(t) + 2y^{(ii)}(t) + 4y^{(i)}(t) + 4y(t) = 2u^{(i)}(t) + u(t),$$

   with $y(t)$ being the output, $u(t)$ the input of the system, and $.^{(k)}$ denotes the k-th derivative.
   a) Calculate the static gain and the transfer function of the system.
   b) Determine the stability of the system. If the system is unstable, determine how many of its poles have a positive real part.
   c) The closed-loop system is controlled by a P controller. Find the range of the controller's parameter that ensures the stability of the closed-loop system.

3. **20 points)** Consider the control system shown below.



The behavior of a closed-loop system is specified by the transfer function $G_{BF}(s) = \frac{Y(s)}{Y_c(s)} = K_d \frac{\omega_0^2}{s^2+2\xi\omega_0 s+\omega_0^2}$ with the damping factor $\xi = 0.7$, and the natural frequency $\omega_0 = 1\ rad/s$.

a) Choose K_d so that the steady-state error e between the reference signal $y_c(t)$ and the output $y(t)$ is equal to zero.
b) Given $G_p(s) = \frac{K}{(s+1)(s+1.4)}$, calculate a controller to satisfy $G_{BF}(s)$. Express the parameters of the obtained controller knowing that it is in a PID form (K_R or K_R, τ_D or K_R, τ_I or K_R, τ_I, τ_D).
c) For $G_l(s) = \frac{1}{s^2(s+1)}$, does the controller obtained in b) eliminate the influence of disturbance $\mathrm{d}(t)$ at $t \to \infty$?

4. **(20 points)** A dynamical system is given by its step response:

$$y(t) = 0.25 - 0.25e^{-t}\cos\sqrt{3}\,t + 0.25\sqrt{3}e^{-t}\sin\sqrt{3}\,t, \quad t \geq 0$$

a) Calculate the transfer function of the system.
b) Calculate the static gain, the natural frequency, the damping factor and the time constant(s) of the system.
c) Calculate and sketch the impulse response of this system.

5. **(20 points)** On considère le réacteur batch de la figure 1. On souhaite garder la température du réacteur T constante à l'aide d'une commande en cascade qui manipule la température à l'entrée du manteau $T_{m,in}$.
a) Dessiner le schéma fonctionnel d'une telle commande en cascade en identifiant toutes les variables du schéma.
b) On a estimé les fonctions de transfert $\frac{T_m(s)}{T_{m,in}(s)} = \frac{0{,}2}{5s+1}$ et $\frac{T(s)}{T_m(s)} = \frac{0{,}5\,e^{-s}}{10s+1}$. Calculer les régulateurs d'une commande PI-P avec les propriétés suivantes : (i) la boucle secondaire a une constante de temps de 1, et (ii) le régulateur primaire est calculé selon les règles de Ziegler-Nichols.
c) **Bonus (5 points):** Est-il possible de déstabiliser le système commandé en augmentant le gain du régulateur P (justification qualitative suffit) ?