Problem 1 solution
Solution

a)

$$H_c = \frac{Y}{Y_c} = \frac{G_r G_p}{1 + G_r G_p}$$

$$H_c = \frac{K\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)}{s + 2 + K\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)}$$

$$H_c = \frac{K(\tau_I \tau_D s^2 + \tau_I s + 1)}{(\tau_I + \tau_I \tau_D K)s^2 + (2\tau_I + K\tau_I)s + K}$$

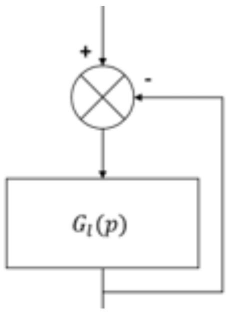
b) Fastest aperiodic poles are determined by

$$\Delta = (2\tau_I + K\tau_I)^2 - 4K(\tau_I + \tau_I \tau_D K) = 0$$

Since $\tau_I \neq 0$,

$$\tau_I (2 + K)^2 - 4K(1 + \tau_D K) = 0$$

c) First, let's simplify the looped G_l:



The equivalent transfer function $l(s)$ of this is:

$$H_l(s) = \frac{G_l(s)}{1 + G_l(s)}$$

$$H_l(s) = \frac{\frac{1}{s^2 + 2s - 1}}{1 + \frac{1}{s^2 + 2s - 1}}$$

$$H_l(s) = \frac{1}{s(s+2)}$$

From there we calculate $\frac{Y}{D}$:

$$Y = H_l D + G_p G_r * (-Y)$$

$$\frac{Y}{D} = \frac{H_l}{1 + G_p G_r}$$

$$\frac{Y}{D} = \frac{1}{s(s+2) + s(s+2) * K \frac{\left(1 + \frac{1}{\tau_I s} + \tau_D s\right) * 1}{s+2}}$$

$$\frac{Y}{D} = \frac{1}{s(s+2) + K\left(\tau_D s^2 + s + \frac{1}{\tau_I}\right)}$$

$$\frac{Y}{D} = \frac{1}{(K\tau_D + 1)s^2 + (2+K)s + \frac{K}{\tau_I}}$$

d) The step disturbance $d(t) = D_0 e(t)$ at $t \to \infty$ can be calculated:

$$\lim_{t\to\infty} Y(t) = \lim_{s\to 0} s \cdot \frac{Y}{D} \cdot \frac{D_0}{s} = \frac{\tau_I}{K} D_0$$

e)

$$\frac{y_\infty - D_0}{D_0} \leq 0.1 \Leftrightarrow \frac{\tau_I}{K} \leq 1.1$$

f) The cheapest τ_I is the largest, and since $K = 10$ and $\tau_I \leq 1.1K$, we have $\tau_I = 11[s]$
From b),

$$\tau_I(2+K)^2 - 4K(1+\tau_D K) = 0$$

$$\tau_D = \tau_I \left(\frac{2+K}{2K}\right)^2 - \frac{1}{K} = 3.86[s]$$

Problem 2 solution

We assume the derivatives to be zero.

In the Laplace domain, the equation becomes:

$$s^4Y(s) + 2s^3Y(s) + 2s^2Y(s) + 4sY(s) + 4Y(s) = 2sU(s) + U(s)$$

From there,

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2s+1}{s^4 + 2s^3 + 2s^2 + 4s + 4}$$

2pts

The static gain K is given by:

$$K = \lim_{s\to 0} \frac{Y(s)}{U(s)}$$

Hence $K = \frac{1}{4}$ **2pts**

We can then use the Routh-Hurwitz criterion to study the stability of this system

s^4	1	2	4
s^3	2	4	
s^2	0→ ϵ	4	
s^1	$\frac{4\epsilon - 8}{\epsilon}$		
s^0	4		

4pts

If $\epsilon > 0$:	If $\epsilon < 0$:
$4\epsilon - 8 < 0$ $\frac{(4\epsilon - 8)}{\epsilon} < 0$	$4\epsilon - 8 < 0$ $\frac{(4\epsilon - 8)}{\epsilon} > 0$

1pt

In any case there are two sign changes, hence two poles with a positive real part. **2pts**

If we loop the system with a P regulator $G_r(s) = K_r$, we get the new transfer function:

$$H(s) = \frac{G_r(s)G(s)}{1 + G_r(s)G(s)}$$

$$H(s) = \frac{(2s+1)K_r}{s^4 + 2s^3 + 2s^2 + (4+2K_r)s + 4 + K_r}$$

2pts

We can use again the Routh-Hurwitz criterion to study the stability of the system:

s^4	1	2	$4 + K_r$
s^3	2	$4 + 2K_r$	
s^2	$-K_r$	$4 + K_r$	
s^1	$\frac{2}{K_r}(K_r^2 + 3K_r + 4)$		
s^0	$4 + K_r$		

4pts

From s^2 we get: $K_r < 0$
From s^1 we get: $K_r > 0$
From s^0 we get: $K_r > -4$
Hence, the system cannot be stabilized by any value of K_r

3pts

Problem 3 solution

a) We need to study $\frac{E(s)}{Y_c(s)} = \frac{(Y_c(s) - Y(s))}{Yc(s)}$:

$$\frac{E(s)}{Y_c(s)} = \frac{Y_c(s) - Y(s)}{Y_c(s)} = 1 - G_{BF}(s)$$

$$\frac{E(s)}{Y_c(s)} = \frac{s^2 + 2\xi\omega_0 s + \omega_0^2 - K_d\omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

3pts

We want to determine the static error:

$$\lim_{t\to+\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s * \frac{E(s)}{Y_c(s)} * Y_c(s), \qquad with\, Y_c(s) = \frac{1}{s}\ (step\ response)$$

2pts

$$\lim_{t\to+\infty} e(t) = \lim_{s\to 0} \frac{E(s)}{Y_c(s)} = \frac{\omega_0^2 - K_d\omega_0^2}{\omega_0^2}$$

The condition for the static error to be nil is :

$$\lim_{t\to+\infty} e(t) = 0 \Leftrightarrow \frac{\omega_0^2 - K_d\omega_0^2}{\omega_0^2} = 0$$

3pts

$$\lim_{t\to+\infty} e(t) = 0 \Leftrightarrow K_d = 1$$

b)

$$G_{BF}(s) = \frac{G_r(s)G_p(s)}{1 + G_r(s)G_p(s)}$$

$$G_r(s) = \frac{G_{BF}(s)}{G_p(s) - G_p(s)G_{BF}(s)}$$

3pts

$$G_r(s) = \frac{s+1}{Ks}$$

Gr(s) has an integrator and a zero, i.e., it has the structure of a PI controller:

$$G_{r\_\mathrm{PI}}(s) = \mathrm{Kr}(1 + 1/\, s\ \mathrm{Tau\_i})$$

A direct comparison of Gr_PI(s) and Gr(s) yields K_r =1/K and Tau_i=1

3pts

c) We need to study $\frac{E(s)}{D(s)}$:

$$\frac{E(s)}{D(s)} = \frac{-G_l(s)}{1 + G_p(s)G_r(s)}$$

$$\frac{E(s)}{D(s)} = \frac{-1}{s^2(s+1)\left(1 + \frac{K}{(s+1)(s+1.4)} \cdot \frac{s+1}{Ks}\right)}$$

3pts

$$\frac{E(s)}{D(s)} = \frac{-(s+1.4)}{s(s+1)(s^2+1.4s+1)}$$

We want to determine the static error in perturbation:

$$\lim_{t\to+\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s * \frac{E(s)}{D(s)} * D(s), \qquad with\ D(s) = \frac{1}{s}\ (step\ perturbation)$$

$$\lim_{t\to+\infty} e(t) = \lim_{s\to 0^+} \frac{E(s)}{D(s)} = -\infty$$

The corrector in b) cannot eliminate the influence of the perturbation. Since the perturbation has a double integrator, it would take at least a double integrator to rectify the perturbation to a non-divergent error, and a triple integrator to get it to 0. Other means of control might be considered at that point, at the level of the perturbation.

3pts

Problem 4 solution

ME-323 Commande de procédés February 2nd, 2018

Examen

1) **(20 points)** Un système dynamique est donné par sa réponse indicielle :

$$y(t) = 0.25 - 0.25e^{-t}\cos\sqrt{3}\,t + 0.25\sqrt{3}e^{-t}\sin\sqrt{3}\,t,\;\; t \geq 0$$

a) Calculer la fonction de transfert de ce système.
b) Évaluer le gain statique, la fréquence naturelle, l'amortissement, et les constantes de temps de ce système.
c) Calculer et dessiner la réponse impulsionnelle du système.

Solution (si nécessaire, continuez au verso) **:**

10 points

a) Fonction de transfert : $G(s) = \frac{Y(s)}{U(s)}$

$$Y(s) = 0.25\cdot\frac{1}{s} - 0.25\frac{s+1}{(s+1)^2+3} + 0.25\sqrt{3}\,\frac{\sqrt{3}}{(s+1)^2+3} = 0.25\left(\frac{1}{s} - \frac{s+1}{(s+1)^2+3} + \frac{3}{(s+1)^2+3}\right) =$$

$$= 0.25\left(\frac{s^2+2s+4-s^2-s+3s}{s\left((s+1)^2+3\right)}\right) = \frac{s+1}{s\left[(s+1)^2+3\right]}$$

$U(s) = \frac{1}{s}$ (indicielle) $\Rightarrow$ $G(s) = \frac{Y(s)}{U(s)} = \frac{s(s+1)}{s\left((s+1)^2+3\right)} = \frac{s+1}{(s+1)^2+3}$

5 points

b) Gain statique : $K = \lim_{s\to 0} G(s) = \frac{1}{4}$

* Forme canonique $G(s)$: $\frac{k(\tau s+1)}{s^2+2\xi\omega_0 s+\omega_0^2}$ $\Rightarrow$ Fréquence : $\omega_0 = 2$

Amortissement : $\xi = 0.5$

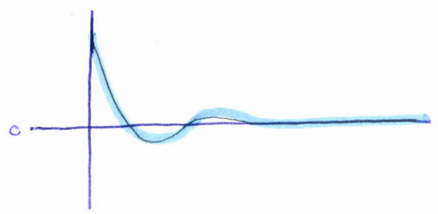
Const. temps : $\tau = \frac{1}{\omega_0} = \frac{1}{2}$

5 points

c) $U(s) = 1$ (impulsionnelle).

$G(s) = \frac{Y(s)}{U(s)}$; $Y(s) = G(s)\cdot U(s) = G(s)$

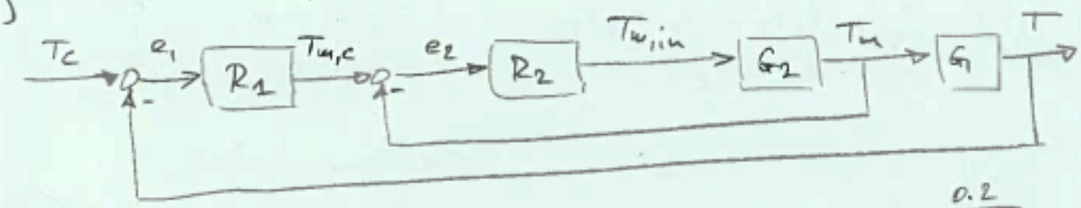
$$y(t) = \mathcal{L}^{-1}[G(s)] = \mathcal{L}^{-1}\left[\frac{s+1}{(s+1)^2+3}\right] = e^{-t}\cdot\cos(\sqrt{3}\cdot t)$$



Etudiant: Sciper no:

Problem 5 solution

a)



(0.25)

b) Boucle secondaire

$$\frac{T_m(s)}{T_{m,c}(s)} = \frac{R_2(s)\,G_2(s)}{1+R_2(s)\,G_2(s)} = \frac{K_{R,2}\frac{0.2}{5s+1}}{1+K_{R,2}\frac{0.2}{5s+1}}$$

(0.25)

$$= \frac{0.2\,K_{R,2}}{5s+1+0.2\,K_{R,2}} = \frac{\frac{0.2\,K_{R,2}}{1+0.2\,K_{R,2}}}{\frac{5}{1+0.2K_{R,2}}\,s+1} = \frac{K_2}{\tau_{BF,2}\,s+1}$$

$$\tau_{BF,2} = \frac{5}{1+0.2\,K_{R,2}} \overset{!}{=} 1 \longrightarrow K_{R,2} = 20$$

(0.25)

$$K_2 = \frac{0.2*20}{1+0.2*20} = \frac{4}{5} = 0.8$$

Boucle primaire

$$\frac{T(s)}{T_{m,c}(s)} = \frac{T(s)}{T_m(s)}\,\frac{T_m(s)}{T_{m,c}(s)} = \frac{0.5e^{-s}}{10s+1}\cdot\frac{0.8}{s+1} \simeq \frac{0.4}{10s+1}e^{-2s}$$

(0.25)

Régulateur PI selon ZN :

$$K_{R,1} = 0.9\,\frac{\tau}{\theta K} = 0.9\,\frac{10}{2*0.4} = 11.25$$

(Si nécessaire, continuez au verso)

$$\tau_{I,1} = 3.33\,\theta = 6.66$$

c) Oui. Si $K_{R,2}$ ↗, alors K_2 ↗ et K ↗

(0.25)

Pour $K_{R,1}$ donné, un grand K déstabilisera le système commandé.