

Thermodynamics and energetics 1

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Summary last lecture

- Describe ideal gas mixture composition in terms of mass fractions or mole fractions.
- Use the *Dalton model* to relate pressure, volume, and temperature and to calculate changes in U , H , and S for ideal gas mixtures.
- Apply mass, energy, and entropy balances to systems involving ideal gas mixtures, including mixing processes.

Learning outcomes of today's lecture

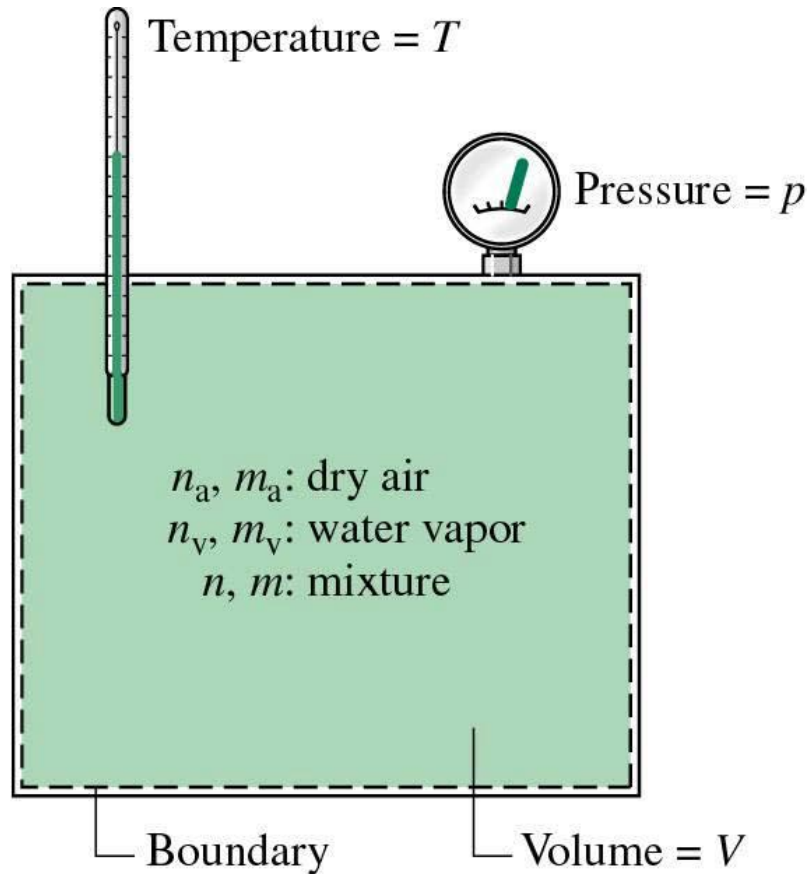
- Demonstrate understanding of psychrometric terminology, including humidity ratio, relative humidity, mixture enthalpy, and dew point temperature.
- Use the psychrometric chart to represent common air-conditioning processes and to retrieve data.
- Apply mass, energy, and entropy balances to analyze air-conditioning processes and cooling towers.

Psychrometrics

- **Psychrometrics**: the study of systems involving dry air and water (both vapour and condensed water may be present)
- **Moist air**: a mixture of dry air and water vapour in which the dry air is treated as if it were a pure component
- For the psychrometric applications under present considerations, we will assume that the overall mixture and all its components behave as ideal gases

Moist air

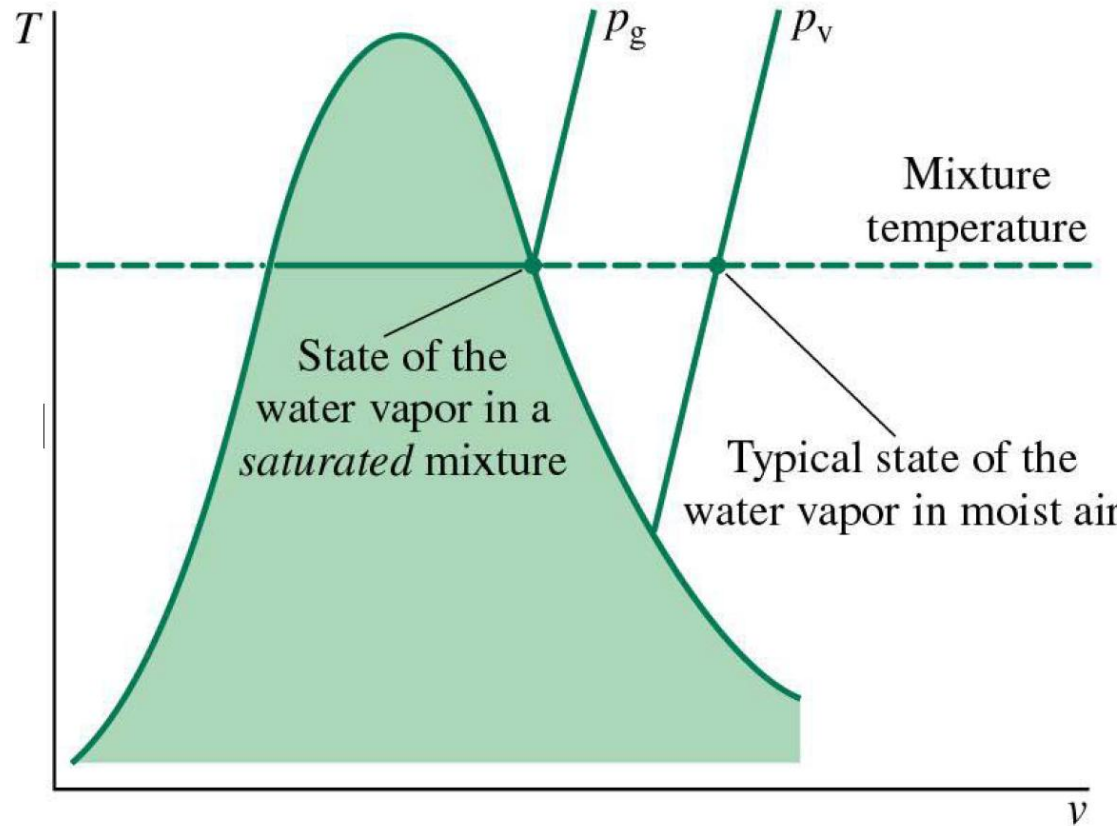
- Moist air = dry air + water vapour



$$\rho = \frac{n\bar{R}T}{V} = \frac{m(\bar{R}/M)T}{V}$$
$$p_a = \frac{n_a\bar{R}T}{V} = \frac{m_a(\bar{R}/M_a)T}{V} = y_a p$$
$$p_v = \frac{n_v\bar{R}T}{V} = \frac{m_v(\bar{R}/M_v)T}{V} = y_v p$$

Moist air

- T - v diagram for water vapour in an air-water mixture



- p_g : saturation pressure of water at the mixture temperature
- **Saturated air**: a mixture of dry air and saturated water vapor

Humidity ratio and relative humidity

- **Humidity ratio**

$$\omega = \frac{m_v}{m_a}$$

$$\omega = \frac{m_v}{m_a} = \frac{\frac{M_v p_v V}{\bar{R}T}}{\frac{M_a p_a V}{\bar{R}T}} = \frac{M_v p_v}{M_a p_a} = 0.622 \frac{p_v}{p_a} = 0.622 \frac{p_v}{p - p_v}$$

- **Relative humidity**

$$\phi = \left. \frac{y_v}{y_{v,\text{sat}}} \right)_{T,p}$$

$$\left. \begin{array}{l} p_v = y_v p \\ p_g = y_{v,\text{sat}} p \end{array} \right\} \rightarrow \phi = \left. \frac{p_v}{p_g} \right)_{T,p}$$

Enthalpy, internal energy and entropy of moist air

- H , U , and S are found by adding contributions of each component at the condition at which the component exists in the mixture

- Enthalpy of moist air

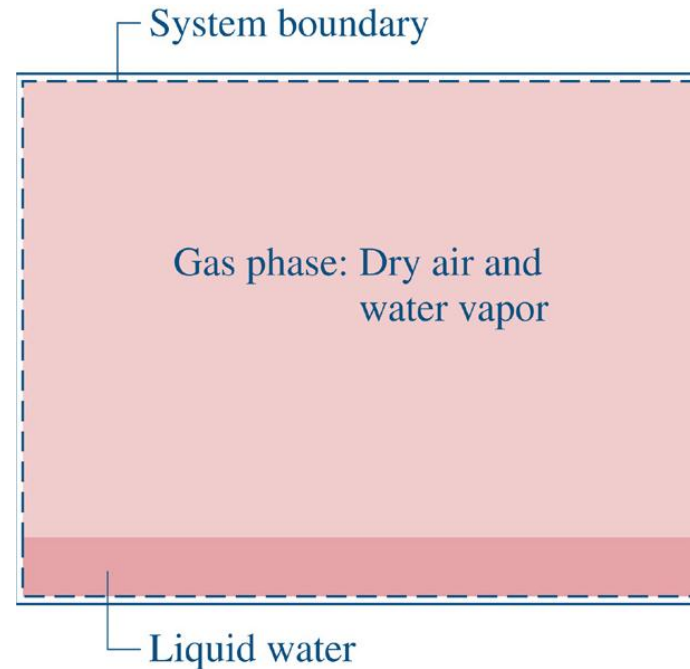
$$H = H_a + H_v = m_a h_a + m_v h_v$$
$$\frac{H}{m_a} = h_a + \frac{m_v}{m_a} h_v = h_a + \omega h_v$$
$$h_v \approx h_g(T)$$

- Enthalpy of dry air can be obtained from the ideal gas database
- Do not use ideal gas database to obtain the enthalpy of water vapour

- Entropy of water vapour in moist air

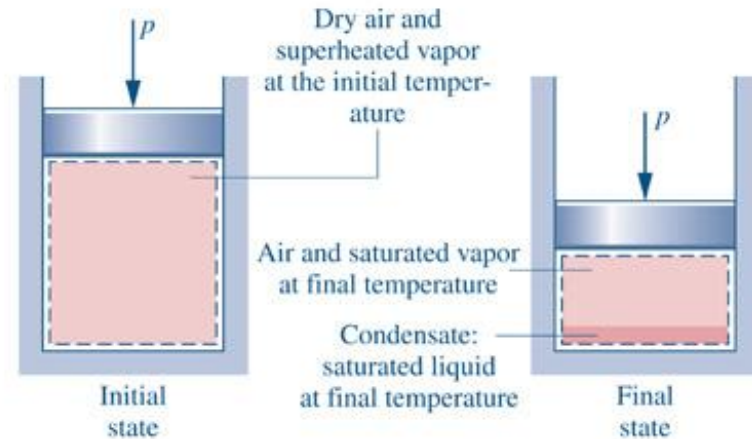
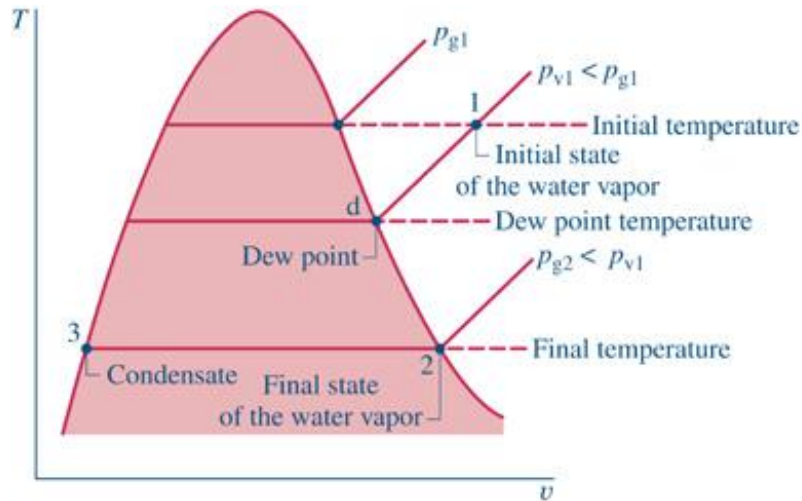
$$s_2(T_2, p_2) - s_1(T_1, p_1) = \int_{T_1}^{T_2} c_p(T) \frac{dT}{T} - R \ln \left(\frac{p_2}{p_1} \right) \xrightarrow[\rho_1 = \rho_g, \rho_2 = \rho_v]{T_1 = T_2 = T} s_v(T, p_v) = s_g(T) - R \ln \phi$$

Moist air in equilibrium with liquid water



- Idealisations:
 - Dry air and $\text{H}_2\text{O}(\text{g})$ behave as ideal gases
 - Equilibrium between $\text{H}_2\text{O}(\text{g})$ and $\text{H}_2\text{O}(\text{l})$ is not disturbed by the presence of air
 - $\text{H}_2\text{O}(\text{l})$ will evaporate until gas phase attains saturation at equilibrium
 - Thus, at equilibrium, $p_v = p_g(T)$

Dew point temperature



- Moist air undergoing cooling at constant pressure
 - **Initial state 1:** dry air + superheated water vapour @ T_1
 composition of moist air = constant
 $p = \text{constant}$
- } $\rightarrow p_v = y_v p = \text{constant}$

- **State d:** dew point

Definition: T @ state "d" = dew point temperature = T_d = saturation temperature @ p_v

Further cooling: $T < T_d \rightarrow \text{H}_2\text{O}(g)$ condenses $\rightarrow p_v < p_{g1}$

- **Final state 2:** dry air + $\text{H}_2\text{O}(g)$ in equil. with $\text{H}_2\text{O}(l)$ @ $T_2 \rightarrow p_v = p_{g2}$

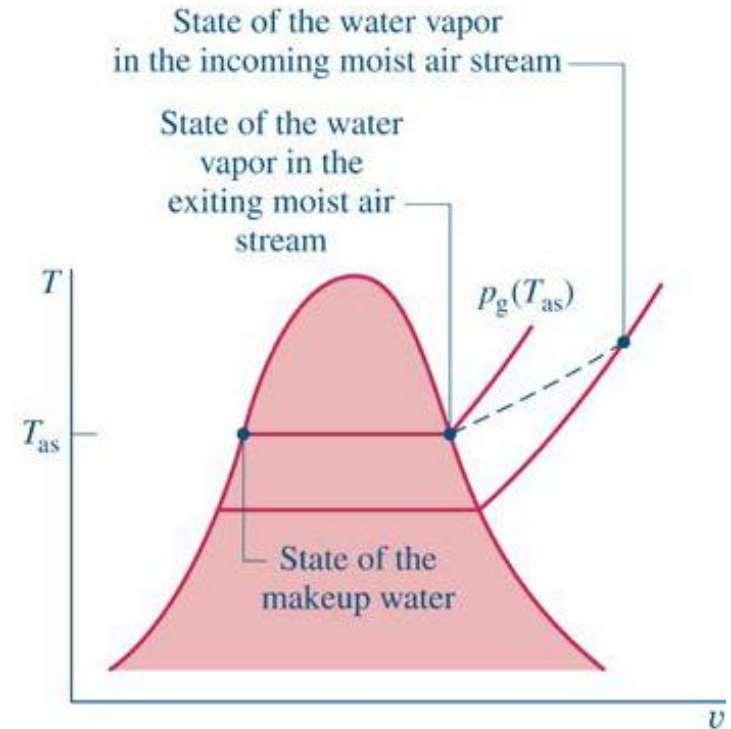
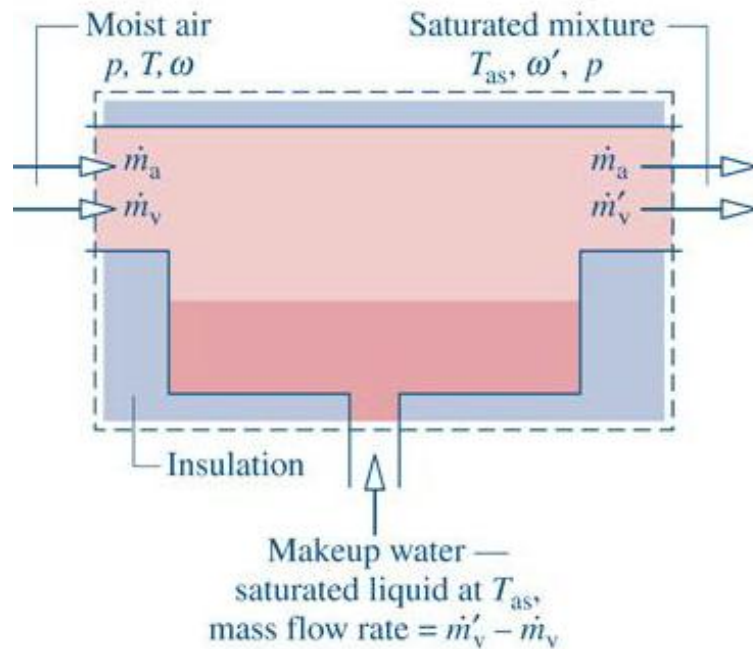
Example

- Example 9.3: A 1 kg sample of moist air initially at 21°C, 1 bar, and 70% relative humidity is cooled to 5°C while keeping the pressure constant.

Determine:

- (a) the initial humidity ratio,
- (b) the dew point temperature, in °C,
- (c) the amount of water vapor that condenses, in kg.

Adiabatic-saturation temperature



- Steady-state energy conservation:

$$(\dot{m}_a h_a + \dot{m}_v h_v)_{\text{moist air entering @ } T} + [(\dot{m}'_v - \dot{m}_v) h_w]_{\text{makeup water @ } T_{as}} = (\dot{m}_a h_a + \dot{m}'_v h_v)_{\text{moist air exiting @ } T_{as}}$$

Adiabatic-saturation temperature

- Steady-state energy balance:

$$(\dot{m}_a h_a + \dot{m}_v h_v)_{\substack{\text{moist air} \\ \text{entering} \\ @ T}} + [(\dot{m}'_v - \dot{m}_v) h_w]_{\substack{\text{makeup} \\ \text{water} \\ @ T_{as}}} = (\dot{m}_a h_a + \dot{m}'_v h_v)_{\substack{\text{moist air} \\ \text{exiting} \\ @ T_{as}}}$$

$$(h_a + \omega h_v)_{\substack{\text{moist air} \\ \text{entering} \\ @ T}} + [(\omega' - \omega) h_w]_{\substack{\text{makeup} \\ \text{water} \\ @ T_{as}}} = (h_a + \omega' h_v)_{\substack{\text{moist air} \\ \text{exiting} \\ @ T_{as}}}$$

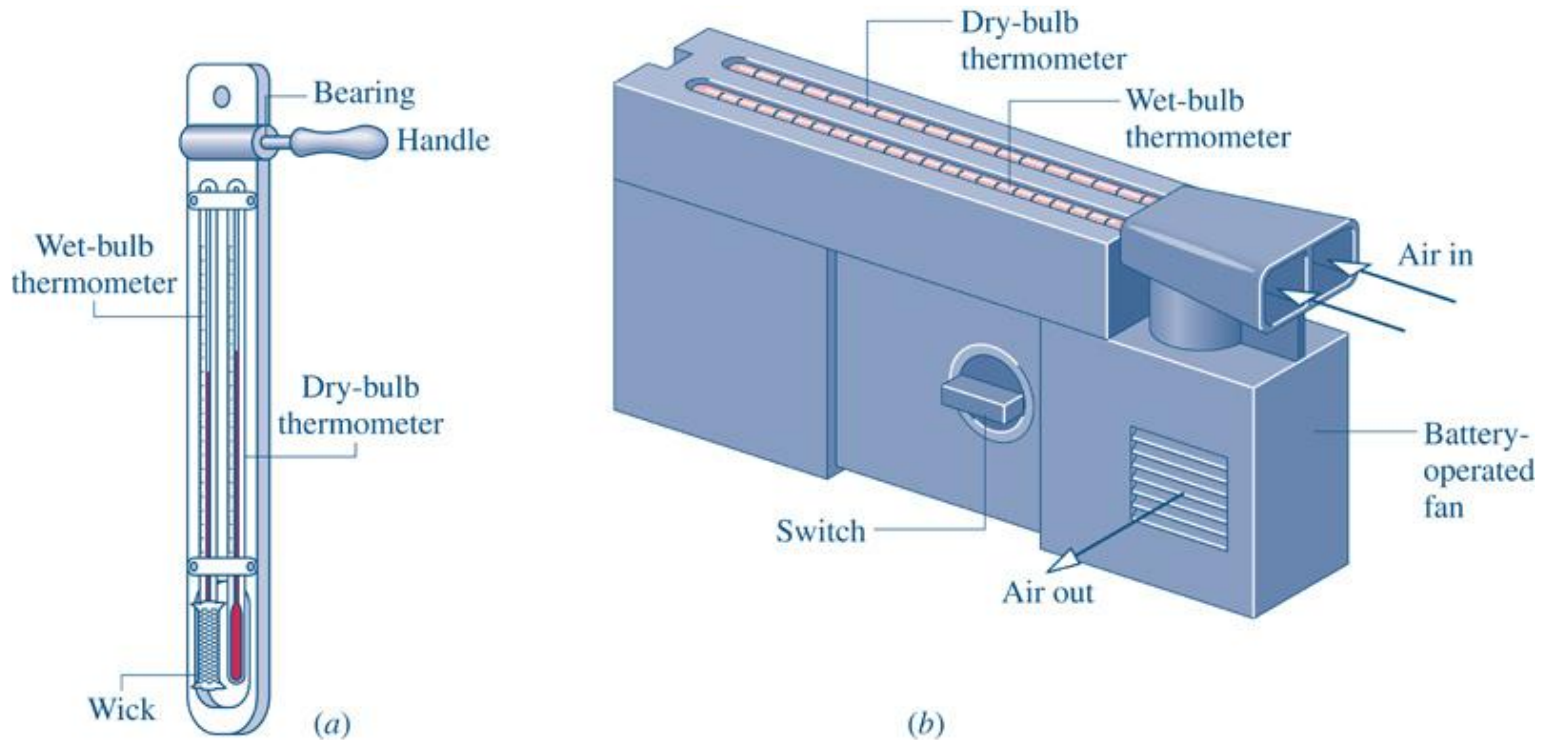
$$| [h_v(T) - h_w(T_{as})] \omega + [h_w(T_{as}) - h_v(T_{as})] \omega' = h_a(T_{as}) - h_a(T)$$

$$h_v \approx h_g(T) \rightarrow [h_g(T) - h_f(T_{as})] \omega + [h_f(T_{as}) - h_g(T_{as})] \omega' = h_a(T_{as}) - h_a(T)$$

$$\rightarrow \omega = \frac{h_a(T_{as}) - h_a(T) + \omega' [h_g(T_{as}) - h_f(T_{as})]}{h_g(T) - h_f(T_{as})}$$

$$\text{where } \omega' = 0.622 \frac{p_g(T_{as})}{p - p_g(T_{as})}$$

Wet-bulb and dry-bulb temperatures



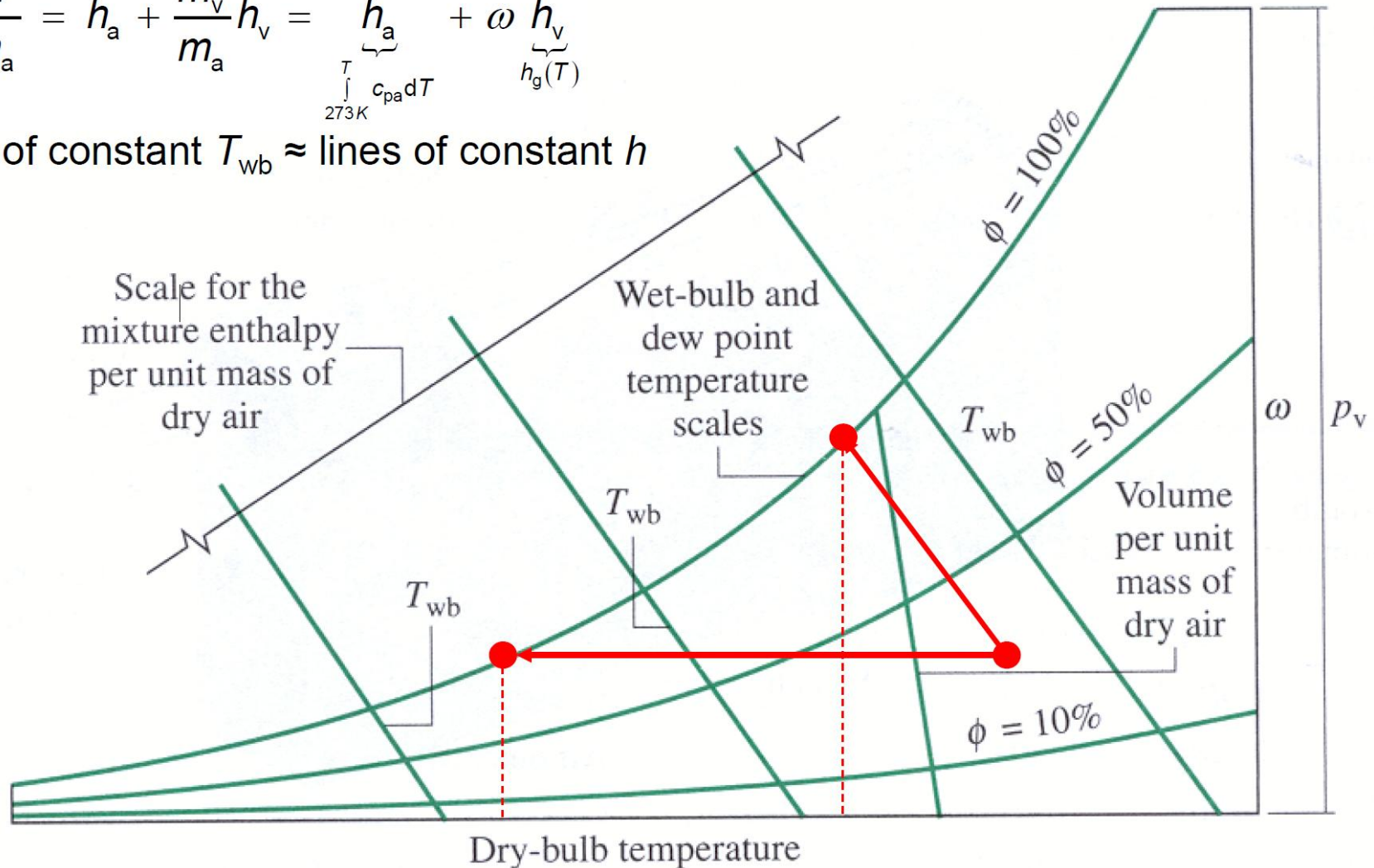
- Dry-bulb temperature $T_{db} = T$ of moist air mixture
- Wet-bulb temperature $T_{wb} \approx T_{as}$
- At saturation, $T_{db} = T_{wb}$

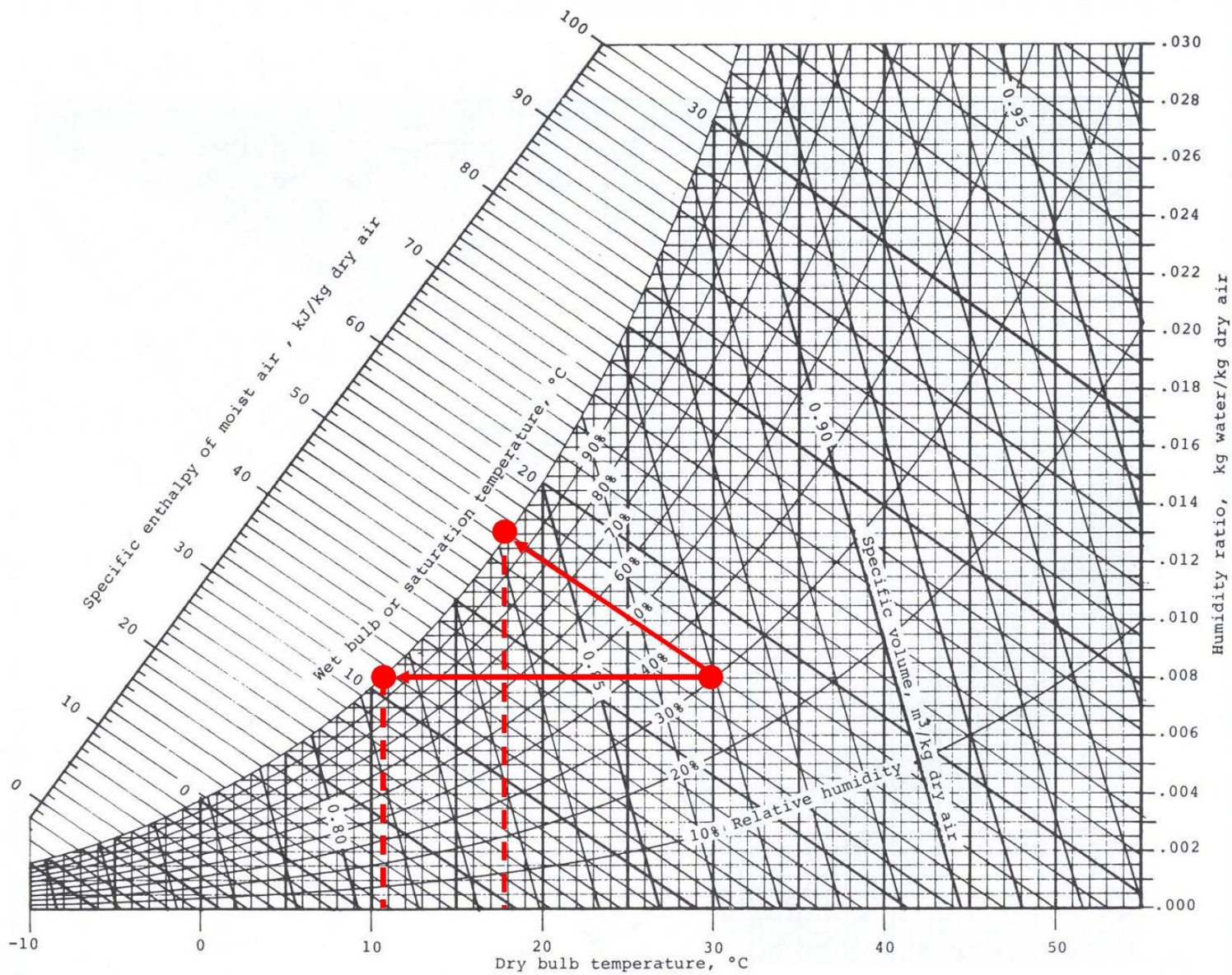
Psychrometric chart @ 1 bar

$$\omega = 0.622 \frac{p_v}{p - p_v}$$

$$h = \frac{H}{m_a} = h_a + \frac{m_v}{m_a} h_v = \underbrace{h_a}_{\int_{273K}^T c_{pa} dT} + \omega \underbrace{h_v}_{h_g(T)}$$

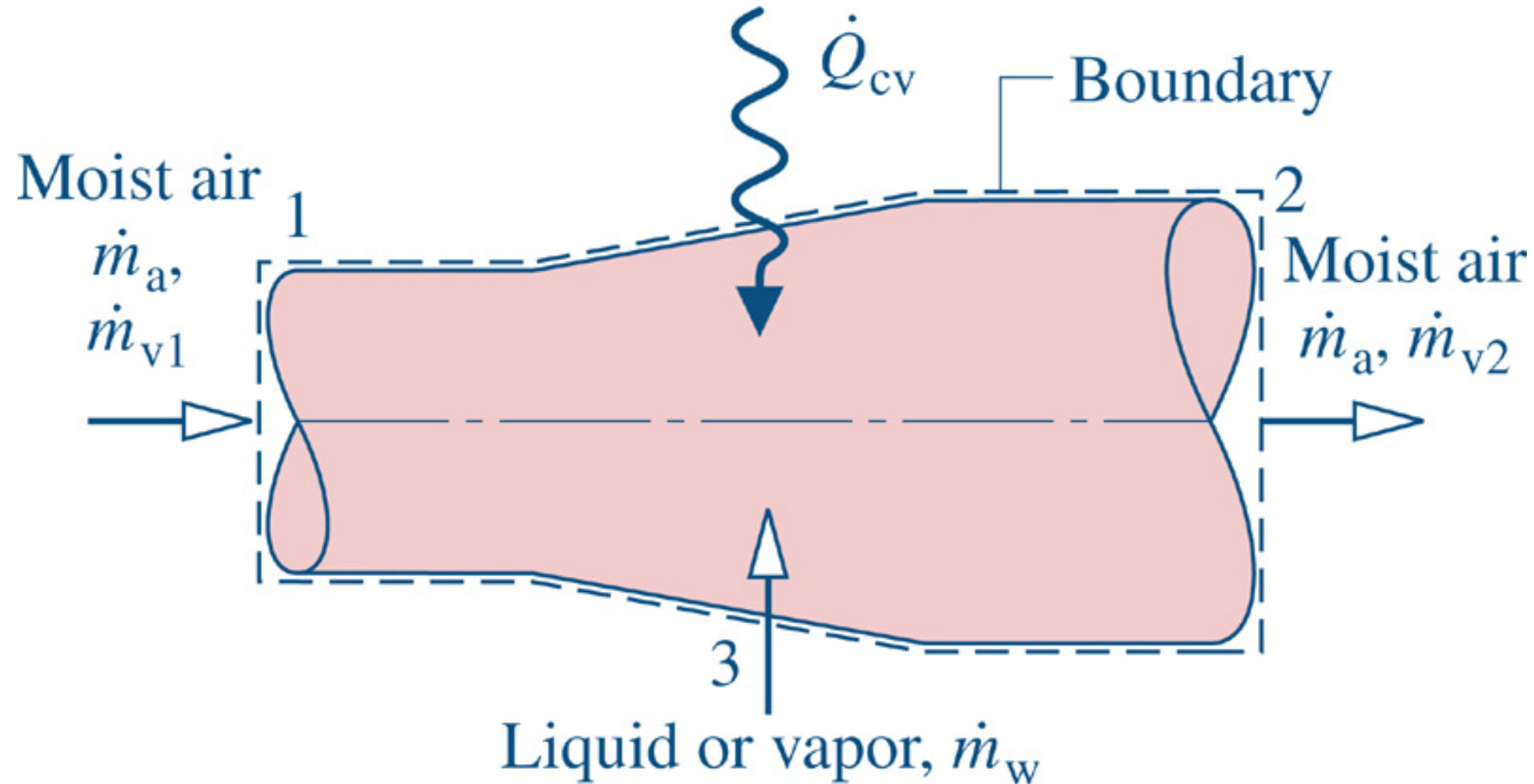
Lines of constant $T_{wb} \approx$ lines of constant h





Source: ASHRAE Transactions, 1988.

Air-conditioning process



Air-conditioning process

- Mass balance:

$$\text{dry air} \rightarrow \dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$$

$$\text{water} \rightarrow \dot{m}_{v_1} + \dot{m}_w = \dot{m}_{v_2} \rightarrow \dot{m}_w = \dot{m}_a (\omega_2 - \omega_1)$$

- Energy balance:

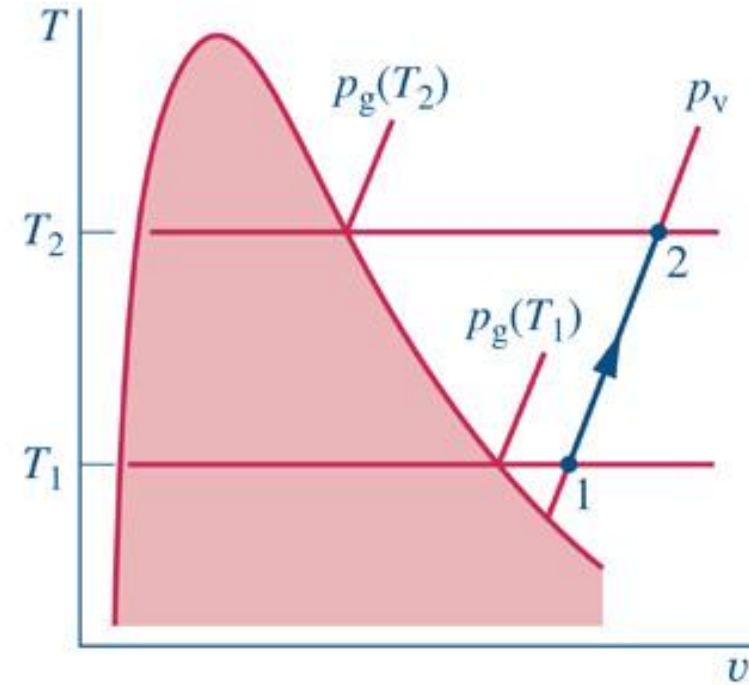
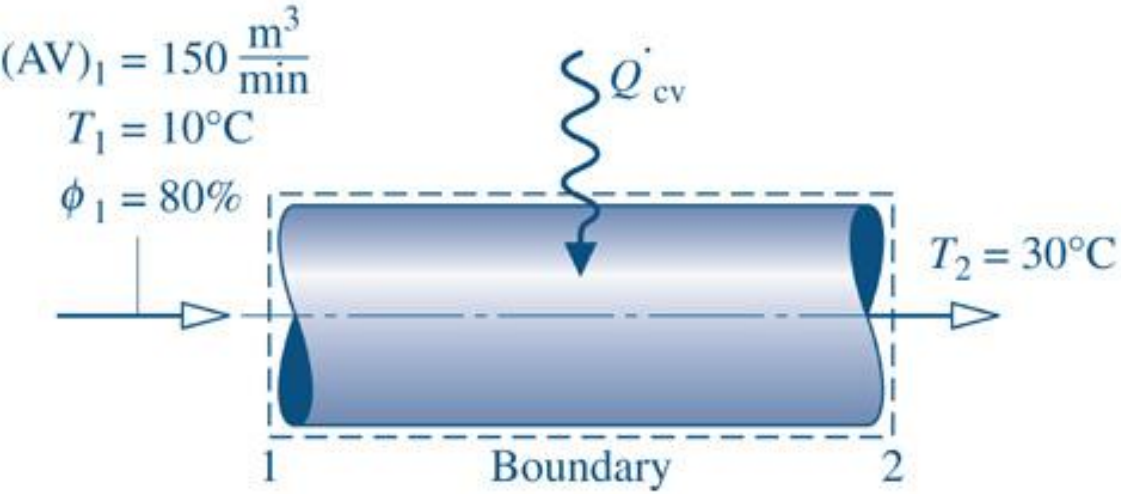
$$0 = \underbrace{\dot{W}_{CV}}_{=0} + \dot{Q}_{CV} + \left(\dot{m}_a h_{a_1} + \dot{m}_{v_1} h_{v_1} \right) + \dot{m}_w h_w - \left(\dot{m}_a h_{a_2} + \dot{m}_{v_2} h_{v_2} \right)$$

$$0 = \underbrace{\dot{W}_{CV}}_{=0} + \dot{Q}_{CV} + \left(\dot{m}_a h_{a_1} + \dot{m}_{v_1} h_{g_1} \right) + \dot{m}_w h_w - \left(\dot{m}_a h_{a_2} + \dot{m}_{v_2} h_{g_2} \right)$$

$$0 = \dot{Q}_{CV} + \dot{m}_a \left(h_{a_1} + \omega_1 h_{g_1} \right) + \underbrace{\dot{m}_w}_{\dot{m}_a (\omega_2 - \omega_1)} h_w - \dot{m}_a \left(h_{a_2} + \omega_2 h_{g_2} \right)$$

$$0 = \dot{Q}_{CV} + \dot{m}_a \left[\left(h_{a_1} - h_{a_2} \right) + \left(\omega_2 - \omega_1 \right) h_w + \omega_1 h_{g_1} - \omega_2 h_{g_2} \right]$$

Heating moist air



Heating moist air

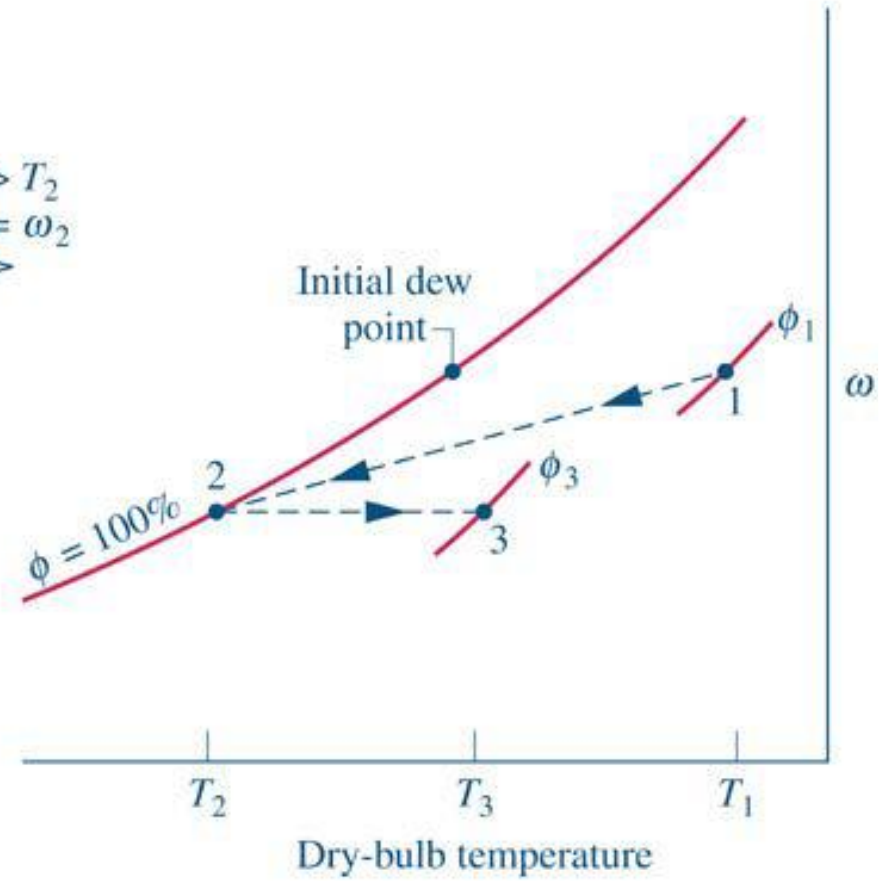
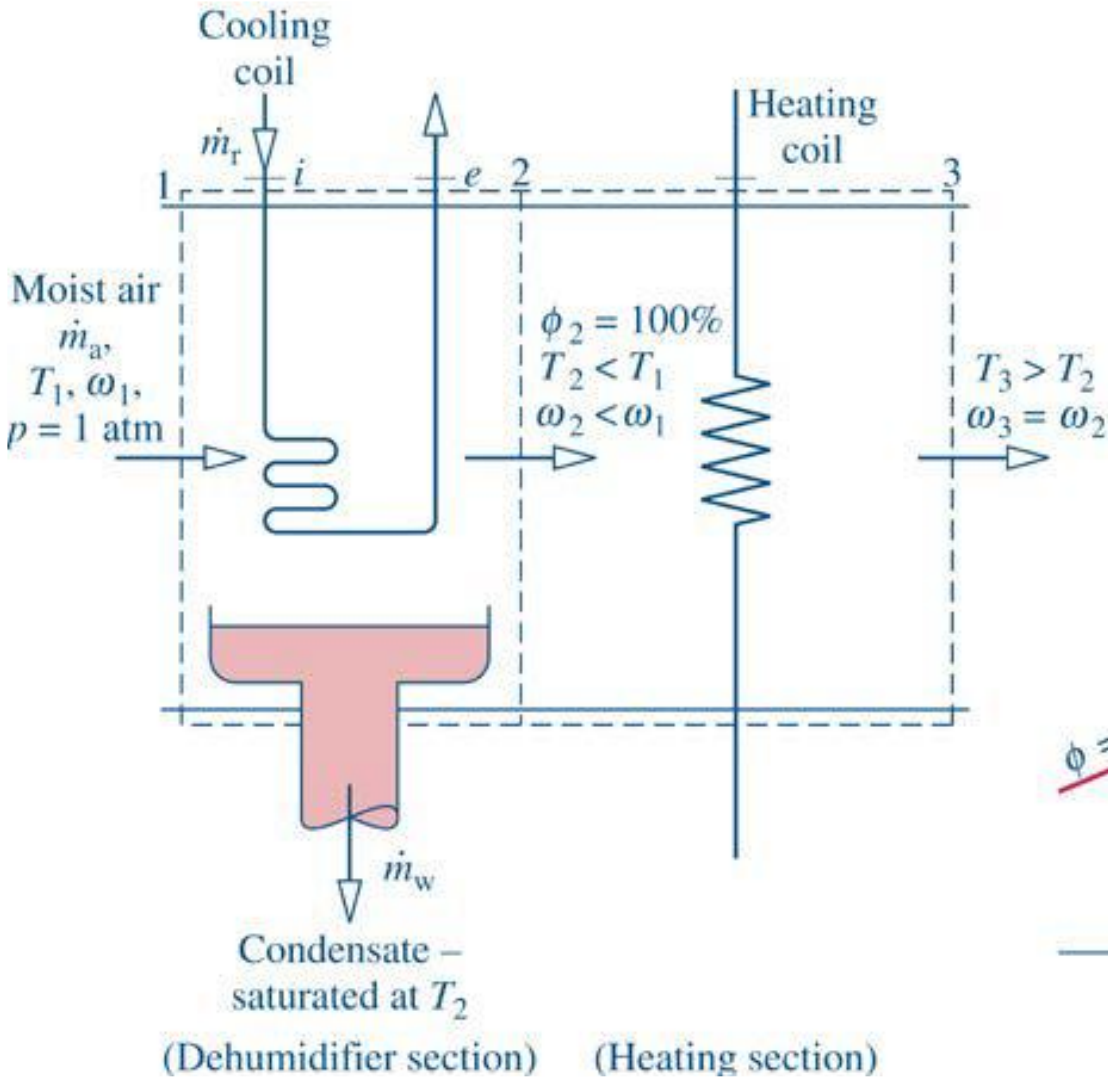
- For constant composition

$$0 = \dot{Q}_{CV} + \dot{m}_a \left[\left(h_{a_1} - h_{a_2} \right) + \underbrace{\left(\omega_2 - \omega_1 \right) h_w}_{=0} + \omega_1 h_{g_1} - \omega_2 h_{g_2} \right]$$

$$\rightarrow \dot{Q}_{CV} = \dot{m}_a \left[\left(h_{a_2} - h_{a_1} \right) + \omega \left(h_{g_2} - h_{g_1} \right) \right]$$

$$\rightarrow \dot{Q}_{CV} = \dot{m}_a \left[\left(h_{a_2} + \omega h_{g_2} \right) - \left(h_{a_1} + \omega h_{g_1} \right) \right]$$

Dehumidification



Dehumidification

- Mass balance:

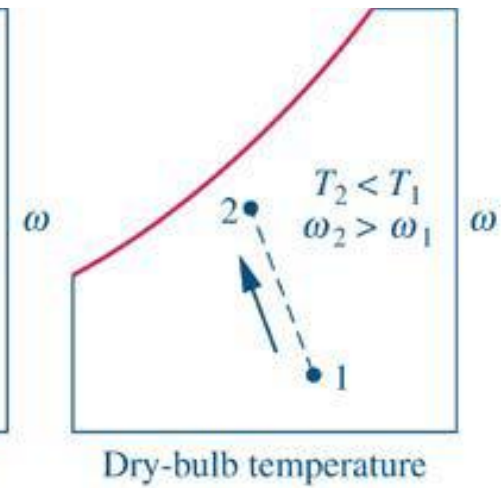
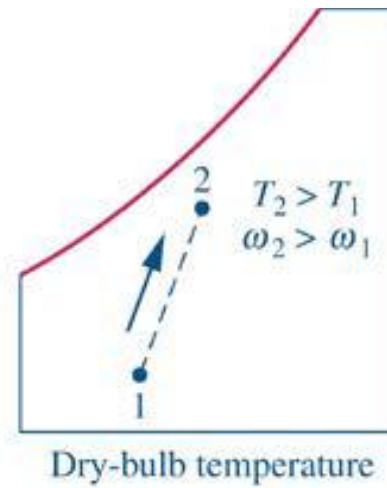
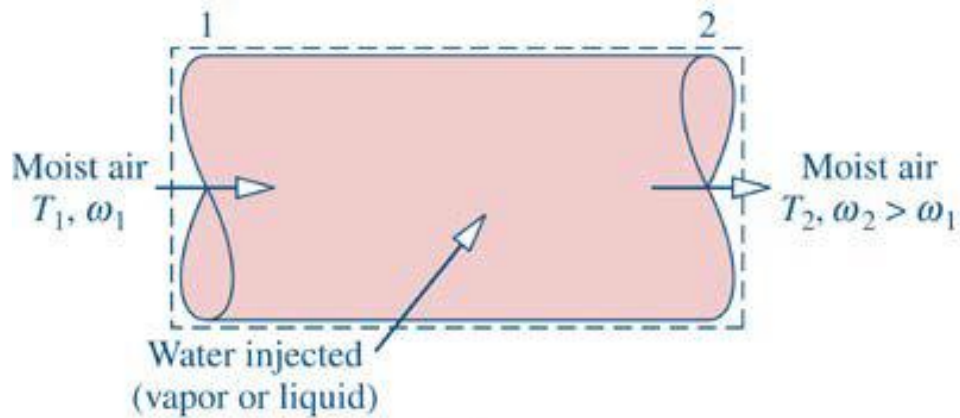
$$\left. \begin{array}{l} \text{dry air} \rightarrow \dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a \\ \text{water} \rightarrow \dot{m}_{v_1} = \dot{m}_w + \dot{m}_{v_2} \end{array} \right\} \rightarrow \dot{m}_w = \dot{m}_a (\omega_1 - \omega_2) = \dot{m}_a (\omega_1 - \omega_3)$$

- Energy balance:

$$0 = \underbrace{\dot{W}_{CV}}_{=0} - \Delta H_{\text{cooling}} + (\dot{m}_a h_{a_1} + \dot{m}_{v_1} h_{v_1}) - (\dot{m}_a h_{a_2} + \dot{m}_{v_2} h_{v_2}) - \dot{m}_w h_w$$

$$\begin{aligned} \Delta H_{\text{cooling}} &= (\dot{m}_a h_{a_1} - \dot{m}_a h_{a_2}) + (\dot{m}_{v_1} h_{v_1} - \dot{m}_{v_2} h_{v_2}) - \dot{m}_w h_w \\ &= \dot{m}_a \left[(h_{a_1} - h_{a_2}) + (\omega_1 h_{v_1} - \omega_2 h_{v_2}) - (\omega_1 - \omega_2) h_w \right] \\ &= \dot{m}_a \left[(h_{a_1} - h_{a_2}) + (\omega_1 h_{g_1} - \omega_2 h_{g_2}) - (\omega_1 - \omega_2) h_{f2} \right] \\ &= \dot{m}_a \left[(h_{a_1} + \omega_1 h_{g_1}) - (h_{a_2} + \omega_2 h_{g_2}) - (\omega_1 - \omega_2) h_{f2} \right] \end{aligned}$$

Humidification



Humidification

- Mass balance:

$$\left. \begin{array}{l} \text{dry air} \rightarrow \dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a \\ \text{water} \rightarrow \dot{m}_{v_1} + \dot{m}_w = \dot{m}_{v_2} \end{array} \right\} \rightarrow \dot{m}_w = \dot{m}_a (\omega_2 - \omega_1)$$

- Energy balance:

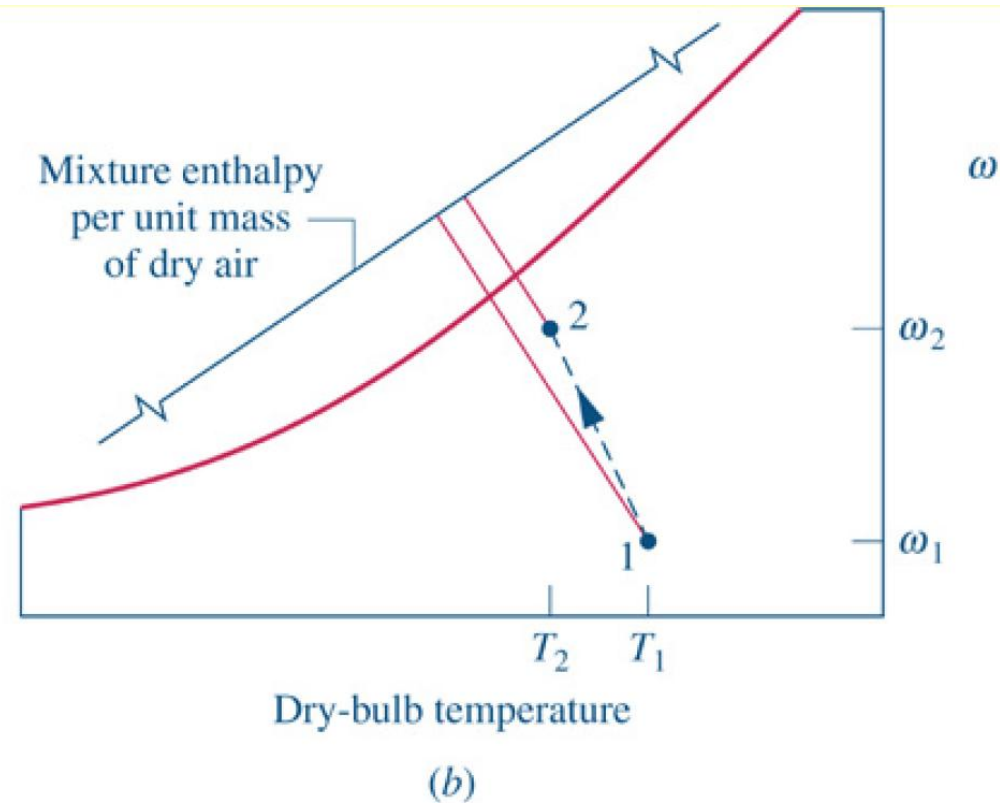
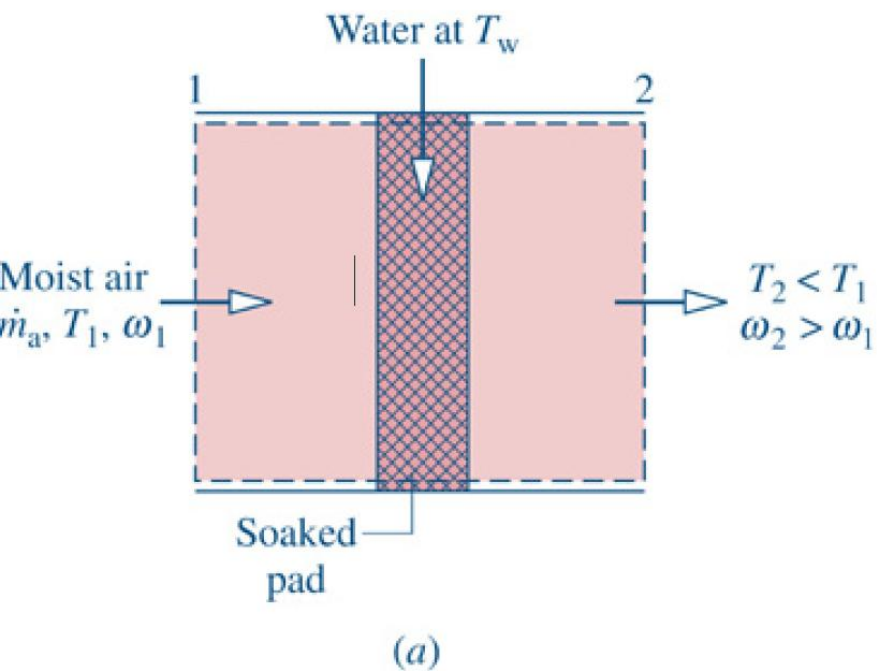
$$0 = \underbrace{\dot{W}_{CV}}_{=0} + (\dot{m}_a h_{a_1} + \dot{m}_{v_1} h_{v_1}) - (\dot{m}_a h_{a_2} + \dot{m}_{v_2} h_{v_2}) + \dot{m}_w h_w$$

$$0 = \dot{m}_a \left[(h_{a_1} - h_{a_2}) + (\omega_1 h_{v_1} - \omega_2 h_{v_2}) + (\omega_2 - \omega_1) h_w \right]$$

$$0 = (h_{a_1} - h_{a_2}) + (\omega_1 h_{g_1} - \omega_2 h_{g_2}) + (\omega_2 - \omega_1) h_{g_3}$$

$$0 = (h_{a_1} + \omega_1 h_{g_1}) - (h_{a_2} + \omega_2 h_{g_2}) + (\omega_2 - \omega_1) h_{g_3}$$

Evaporative cooling



Evaporative cooling

- Mass balance:

$$\left. \begin{array}{l} \text{dry air} \rightarrow \dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a \\ \text{water} \rightarrow \dot{m}_{v_1} + \dot{m}_w = \dot{m}_{v_2} \end{array} \right\} \rightarrow \dot{m}_w = \dot{m}_a (\omega_2 - \omega_1)$$

- Energy balance:

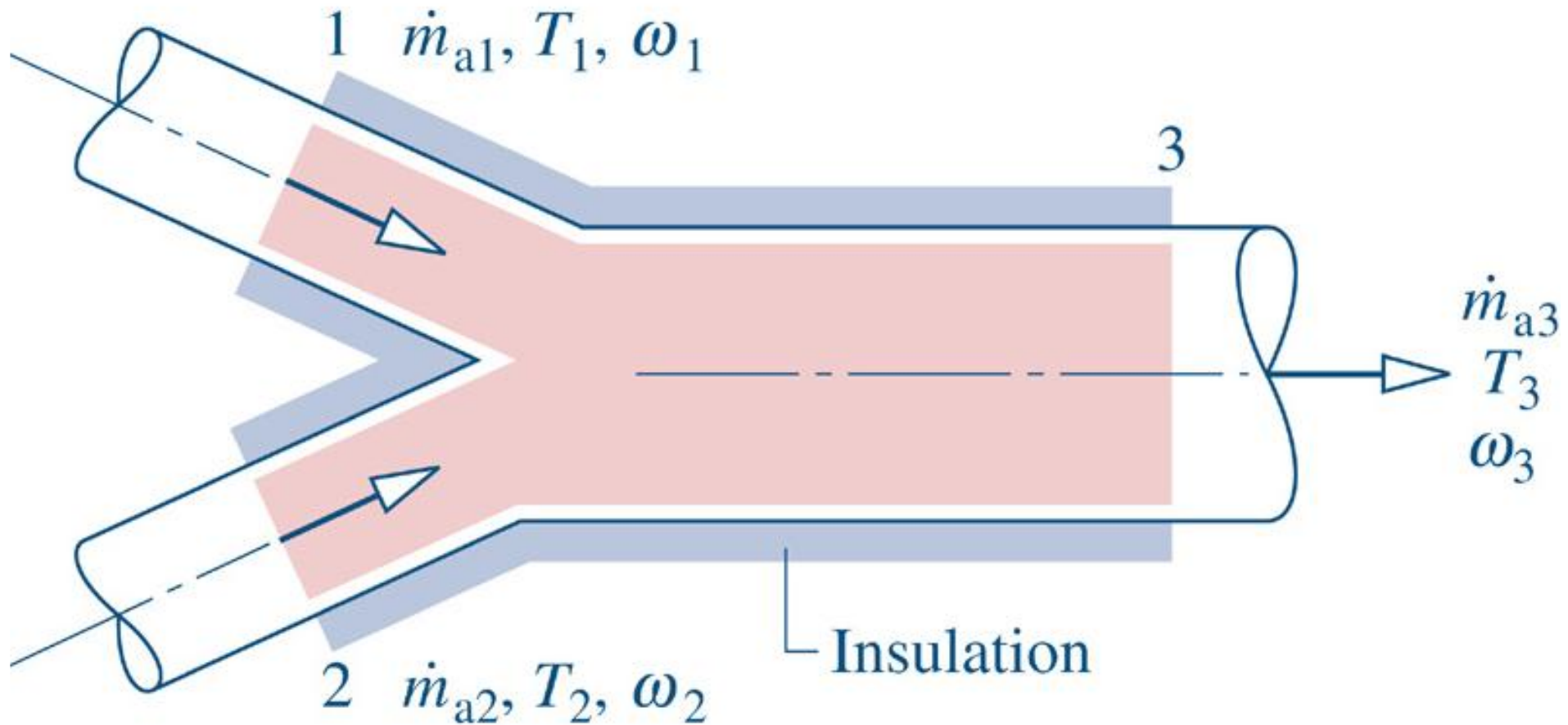
$$0 = \underbrace{\dot{W}_{CV}}_{=0} + (\dot{m}_a h_{a_1} + \dot{m}_{v_1} h_{v_1}) - (\dot{m}_a h_{a_2} + \dot{m}_{v_2} h_{v_2}) + \dot{m}_w h_w$$

$$0 = \dot{m}_a \left[(h_{a_1} - h_{a_2}) + (\omega_1 h_{v_1} - \omega_2 h_{v_2}) + (\omega_2 - \omega_1) h_w \right]$$

$$0 = (h_{a_1} - h_{a_2}) + (\omega_1 h_{g_1} - \omega_2 h_{g_2}) + (\omega_2 - \omega_1) h_{f3}$$

$$0 = (h_{a_1} + \omega_1 h_{g_1}) - (h_{a_2} + \omega_2 h_{g_2}) + (\omega_2 - \omega_1) h_{f3}$$

Adiabatic mixing



Adiabatic mixing

- Mass balance:

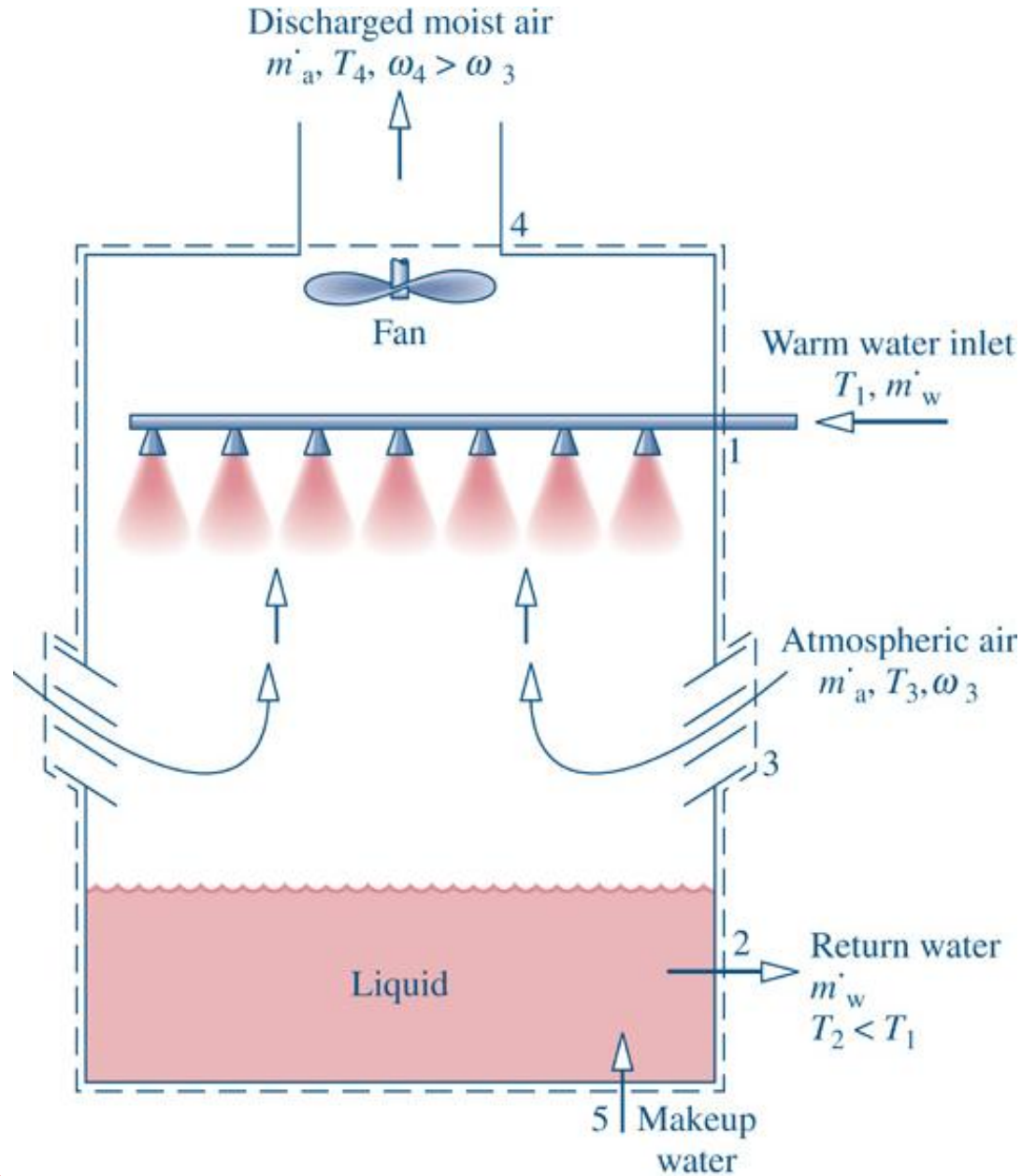
$$\left. \begin{array}{l} \text{dry air} \rightarrow \dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3} \\ \text{water vapor} \rightarrow \dot{m}_{v_1} + \dot{m}_{v_2} = \dot{m}_{v_3} \end{array} \right\} \rightarrow \omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2} = \omega_3 \dot{m}_{a_3}$$

- Energy balance:

$$\dot{m}_{a_1} (h_{a_1} + \omega_1 h_{v_1}) + \dot{m}_{a_2} (h_{a_2} + \omega_2 h_{v_2}) - \dot{m}_{a_3} (h_{a_3} + \omega_3 h_{v_3}) = 0$$

$$\dot{m}_{a_1} (h_{a_1} + \omega_1 h_{g_1}) + \dot{m}_{a_2} (h_{a_2} + \omega_2 h_{g_2}) - \dot{m}_{a_3} (h_{a_3} + \omega_3 h_{g_3}) = 0$$

Cooling towers



- Mass balance:

$$\left. \begin{array}{l} \text{dry air} \rightarrow \dot{m}_{a_3} = \dot{m}_{a_4} = \dot{m}_a \\ \text{water} \rightarrow \dot{m}_{w_1} + \dot{m}_{v_3} + \dot{m}_{w_5} = \underbrace{\dot{m}_{w_2}}_{=\dot{m}_{w_1}} + \dot{m}_{v_4} \end{array} \right\} \rightarrow \dot{m}_{w_5} = \dot{m}_{v_4} - \dot{m}_{v_3} = \dot{m}_a (\omega_4 - \omega_3)$$

- Energy balance:

$$0 = \Delta H = (H_1 + H_3 + H_5) - (H_2 + H_4)$$

$$0 = \dot{m}_{w_1} h_{w_1} + (\dot{m}_a h_{a_3} + \dot{m}_{v_3} h_{v_3}) + \dot{m}_{w_5} h_{w_5} - \dot{m}_{w_2} h_{w_2} - (\dot{m}_a h_{a_4} + \dot{m}_{v_4} h_{v_4})$$

$$0 = \dot{m}_{w_1} h_{f_1} + \left(\dot{m}_a h_{a_3} + \underbrace{\dot{m}_{v_3}}_{\dot{m}_a \omega_3} h_{g_3} \right) + \underbrace{\dot{m}_{w_5}}_{\dot{m}_a (\omega_4 - \omega_3)} h_{f_5} - \underbrace{\dot{m}_{w_2}}_{\dot{m}_{w_1}} h_{f_2} - \left(\dot{m}_a h_{a_4} + \underbrace{\dot{m}_{v_4}}_{\dot{m}_a \omega_4} h_{g_4} \right)$$

$$0 = \dot{m}_{w_1} (h_{f_1} - h_{f_2}) + \dot{m}_a \left[(h_{a_3} + \omega_3 h_{g_3}) + (\omega_4 - \omega_3) h_{f_5} - (h_{a_4} + \omega_4 h_{g_4}) \right]$$

Learning outcomes of today's lecture

- Demonstrate understanding of psychrometric terminology, including humidity ratio, relative humidity, mixture enthalpy, and dew point temperature.
- Use the psychrometric chart to represent common air-conditioning processes and to retrieve data.
- Apply mass, energy, and entropy balances to analyze air-conditioning processes and cooling towers.