SCHOOL OF ENGINEERING
MECHANICAL ENGINEERING
LRESE - Laboratory of Renewable Energy Sciences and Engineering

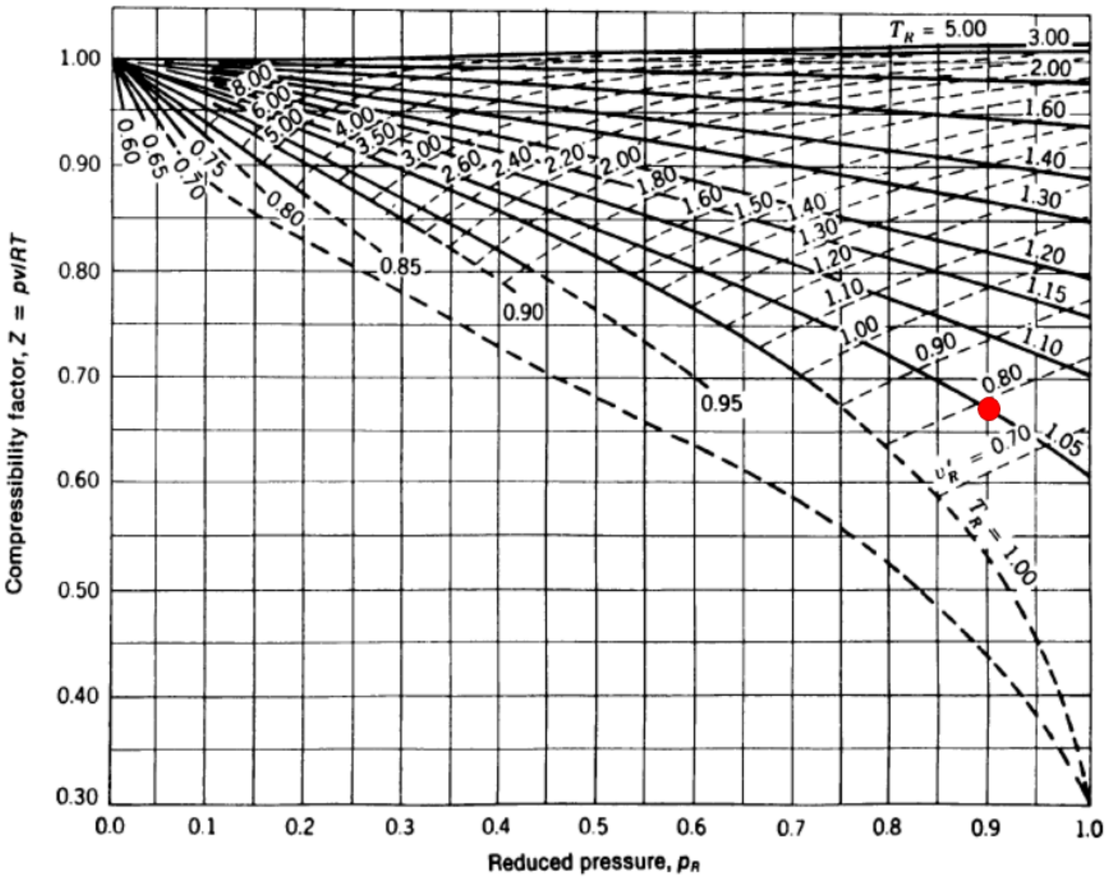
First Name: ______________ Last Name: ______________ Sciper: ______________

Thermodynamics and energetics I: Exam, Part I

Date: January 23, 2024
Duration: part I, 30 minutes
No documents allowed, calculator only. Answers on these sheets only.
7 exercises - Total: [4.3125 points]

Part I, Exercise 1 [1 points]

Carbon dioxide (CO_2) is stored in a tank at 46.2 °C and 66.4 bar. The critical temperature and pressure of CO_2 are $T_C = 31$ °C and $p_C = 73.8$ bar.
Estimate from the chart below the compressibility factor of CO_2 in the tank. Can the gas in this state be accurately described by the ideal gas model? If not, which model would be more accurate?



The compressibility factor has to be calculated from the reduced temperature and pressure of CO_2 in the tank.

$$T_R = \frac{T}{T_C}\mathbf{(0.125)} = \frac{46.2 + 273.15\ \mathrm{K}}{31 + 273.15\ \mathrm{K}}\mathbf{(0.0625)} = 1.05\mathbf{(0.0625)}$$

$$p_R = \frac{p}{p_C}\mathbf{(0.125)} = \frac{66.4\ \mathrm{bar}}{73.8\ \mathrm{bar}}\mathbf{(0.0625)} = 0.9\mathbf{(0.0625)}$$

From the chart, the compressibility factor can be estimated to be $z = 0.67$ **(0.125)**.
Ideal gas model does not accurately describe CO_2 in the tank **(0.125)** due to the compressibility factor less than one **(0.125)**.
Van der Waals, Redlich Kwong or even more complicated equations of state considering the interactions between molecules and their finite volume would more accurately describe CO_2 in the tank **(0.125)**.

Part I, Exercise 2 [0.75 points]

Assume steady state in an adiabatic turbine. Explain if the specific entropy of the fluid increases, decreases or stays constant during:

1. a reversible expansion;
2. an irreversible expansion.

What is the sign of the work transfer rate in the two processes? In which case is the work transfer rate maximum?

1. In a reversible expansion, the specific entropy of the fluid stays constant **(0.125)**. Applying the entropy balance for open systems:

$$\cancelto{0}{\frac{\mathrm{d}S_{\mathrm{CV}}}{\mathrm{d}t}} = \cancelto{0}{\sum_j \frac{\dot{Q}_j}{T_{\mathrm{b}}}} + \dot{m}\cdot s_{\mathrm{i}} - \dot{m}\cdot s_{\mathrm{e}} + \cancelto{0}{\dot{\sigma}_{\mathrm{CV}}}\ \mathbf{(0.125)}$$

$$s_{\mathrm{e}} = s_{\mathrm{i}}$$

2. In an irreversible expansion, the specific entropy of the fluid increases **(0.125)**.

$$\cancelto{0}{\frac{\mathrm{d}S_{\mathrm{CV}}}{\mathrm{d}t}} = \cancelto{0}{\sum_j \frac{\dot{Q}_j}{T_{\mathrm{b}}}} + \dot{m} \cdot s_{\mathrm{i}} - \dot{m} \cdot s_{\mathrm{e}} + \dot{\sigma}_{\mathrm{CV}} \mathbf{(0.0625)}$$

$$s_{\mathrm{e}} = s_{\mathrm{i}} + \underbrace{\frac{\dot{\sigma}_{\mathrm{CV}}}{\dot{m}}}_{>0} \mathbf{(0.0625)}$$

In the two processes, the work transfer rate is positive because the work is extracted from the system **(0.125)**.
The work transfer rate is maximum in the reversible process because it is an ideal process without losses, which are associated to irreversibilities **(0.125)**.

Part I, Exercise 3 **[0.5 point]**
Write the formula of the exergy transfer by heat. In an evaporator of a refrigeration cycle, is the exergy provided to or extracted from the working fluid? Why?

Exergy transfer by heat:

$$\dot{E}x_Q = \int \left(1 - \frac{T_0}{T}\right) \delta\dot{Q} = \left(1 - \frac{T_0}{\overline{T}}\right) \dot{Q} \mathbf{(0.125)}$$

In an evaporator of a refrigeration cycle, the heat transfer is provided to the working fluid and therefore it is positive ($\dot{Q}_{\mathrm{evap}} > 0$) **(0.125)**. The temperature at which heat transfer takes place is less then the ambient temperature ($T_{\mathrm{evap}} < T_0$) **(0.125)**. The exergy is therefore extracted from the working fluid because the exergy transfer is negative.

$$\dot{E}x_{Q,\mathrm{evap}} = \left(1 - \frac{T_0}{\overline{T}_{\mathrm{evap}}}\right) \dot{Q}_{\mathrm{evap}} < 0 \mathbf{(0.125)}$$

Part I, Exercise 4 **[0.75 points]**

Assume a reversible isobaric liquid-vapour phase change of a fluid in a closed system. Derive the relationship between the liquid-vapour specific enthalpy change ($h_\mathrm{g} - h_\mathrm{f}$) and specific entropy change ($s_\mathrm{g} - s_\mathrm{f}$) from the definition of enthalpy ($\mathrm{d}h = \mathrm{d}u + p\mathrm{d}v + v\mathrm{d}p$).

The infinitesimal variation of specific enthalpy for an isobaric process can be derived from the definition of enthalpy:

$$\mathrm{d}h = du + p\mathrm{d}v + \cancelto{0}{v\mathrm{d}p}\,\mathbf{(0.0625)}$$

Applying the First Law of Thermodynamics for closed systems:

$$\mathrm{d}u + p\mathrm{d}v = \mathrm{d}u + \delta w\mathbf{(0.0625)} = \delta q\mathbf{(0.125)}$$

From the definition of entropy for a reversible process:

$$\delta q = T\mathrm{d}s\mathbf{(0.125)}$$

Therefore, the relationship between the infinitesimal variations of enthalpy and entropy can be obtained:

$$\mathrm{d}h = T\mathrm{d}s\mathbf{(0.125)}$$

The isobaric phase change is also isothermal **(0.125)**. Integrating between saturated liquid and saturated vapour:

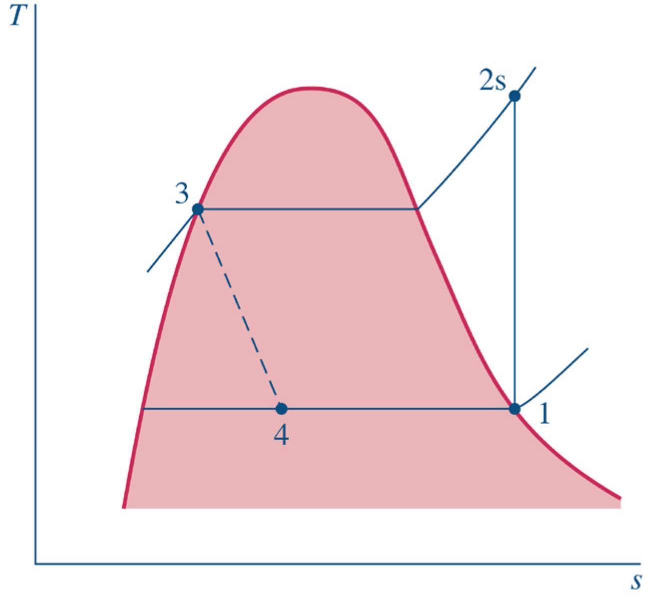
$$\int_\mathrm{f}^\mathrm{g} \mathrm{d}h = h_\mathrm{g} - h_\mathrm{f} = \int_\mathrm{f}^\mathrm{g} T\mathrm{d}s = T(s_\mathrm{g} - s_\mathrm{f})\mathbf{(0.125)}$$

Part I, Exercise 5 **[0.375 points]**

COP=Q_{in}/W **(0.0625)** $= 5/1.5 = 3.33$ **(0.03125)**
COP$_{Carnot}$ **(0.0625)** $= \frac{T_c}{T_h - T_c}$ **(0.0625)** $= \frac{18+273}{37-18} = 15.316$ **(0.03125)**
If the refrigerant is changed (while everything else stayed the same), do you expect the best possible COP would change? No **(0.03125)**, only dependent on temperature level of reservoir **(0.03125)**.
Changing temperatures **(0.03125)**, i.e. cold and hot temperatures closer together **(0.03125)**.

First Name: ________ Last Name: ________ Sciper: ________

Part I, Exercise 6 [0.625 points]



(0.03125) x 4 for each point, and **(0.03125)** x 4 for line (If reverse Carnot cycle was instead correctly drawn, (0.09375) was given)
non-idealities: non-isentropic compression, superheating in evaporator, supercooling in condenser, pressure losses in condenser, pressure losses in evaporator,heat transfer with finite differences of temperature from the heat reservoirs **(0.0625)** x 3 for each non-ideality (not more than 3)
refrigerant(-134) is a typical working fluid because of the saturation temperatures of the fluid are aligned with the heat pump needs at practical operating pressures**(0.0625)**
summer COP is higher than winter COP **(0.03125)** since temperature of source is higher **(0.03125)**
temperature of ground water is higher than of air in winter **(0.03125)**, therefore COP of water-source heat pump is higher in winter than air-source heat pump **(0.03125)**

Part I, Exercise 7 [0.3125 points]

What is the Joule-Thomson coefficient for an ideal gas: 0 **(0.0625)**
JT coefficient of H_2: $\mu_{JT} = (\frac{\partial T}{\partial p})_h$**(0.0625)** $\approx \frac{\Delta T}{\Delta p}$ **(0.0625)**
Temperature decreases during compression **(0.0625)**, implying that fueling becomes more difficult because heat is provided to the gas and therefore counteracts the compression. **(0.0625)**

SCHOOL OF ENGINEERING
MECHANICAL ENGINEERING
LRESE - Laboratory of Renewable Energy Sciences and Engineering

EPFL

First Name: ____________ Last Name: ____________ Sciper: ____________

Thermodynamics and energetics I: Exam, Part II

Date: January 23, 2024
Duration: part II, 2 hours 30 minutes
Allowed material: Calculator, personal summary (8 pages A4), and dictionary.
Three exercises - Total: 29.375 points.

Exercise 1: Regenerative Rankine cycle analysis [13.5 points]

(a) Calculate the specific enthalpy of every state in the cycle. **[6.625 points]**

Feel free to use the following table as a support.

State	p [bar]	T [°C]	x [-]	h [kJ/kg]	s [kJ/(kg K)]
1	0.08		0	173.88	
2s	100			183.87	
2	100			184.98	
3	100			555.78	
4	100			578.39	
5	100	560		3526.00	6.7864
6s	7.00			2798.39	6.7864
6	7.00			2798.39	6.7864
7	7.00	500		3481.70	7.9299
8s	0.08		0.961	2482.97	7.9299
8	0.08			2632.78	
9	7.00		0	697.22	
10s	100			706.52	
10	100			706.52	

State 1
Water leaves the condenser as saturated liquid ($x_1 = 0$).

$$h_1 = h_\mathrm{f}(p_1)\mathbf{(0.125)} = 173.88\mathrm{kJ/kg}\mathbf{(0.125)}$$

State 2s
Compressed liquid water behaves as an incompressible substance with constant specific volume

v, isentropic compression up to 100 bar is considered.

$$ds = c_p \frac{dT}{T} = 0 \Rightarrow dT = 0$$

$$dh = c_p dT + vdp = vdp$$

$$h_{2s} = h_1 + v(p_2 - p_1)\mathbf{(0.25)} = 173.88 + 0.001 \cdot (100 - 0.08) \cdot 10^2 = 183.87 \text{kJ/kg}\mathbf{(0.125)}$$

State 2
Isentropic efficiency of pump 1 $\eta_{s,P1} = 90\%$.

$$h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_{s,P1}}\mathbf{(0.25)} = 173.88 + \frac{183.87 - 173.88}{0.9} = 184.98 \text{kJ/kg}\mathbf{(0.125)}$$

State 5
Superheated water vapour at 100 bar and 560°C.

$$h_5 = 3526.00 \text{kJ/kg}\mathbf{(0.125)} \qquad s_5 = 6.7864 \text{kJ/(kg K)}\mathbf{(0.125)}$$

State 6s and 6
Isentropic efficiency of turbine 1 $\eta_{s,T1} = 100\%$. Isentropic expansion, state 6s corresponds to state 6.

$$s_6 = s_5\mathbf{(0.125)} = 6.7864 \text{kJ/(kg K)}\mathbf{(0.125)} > s_g(p_6) \Rightarrow \text{superheated vapour } \mathbf{(0.125)}$$

Superheated vapour at 7 bar.

$$h_6 = h_{sat} + \frac{s_6 - s_{sat}}{s_{180^\circ\text{C}} - s_{sat}}(h_{180^\circ\text{C}} - h_{sat})\mathbf{(0.25)} =$$

$$= 2763.5 + \frac{6.7864 - 6.7080\mathbf{(0.125)}}{6.7880\mathbf{(0.125)} - 6.7080}(2799.1\mathbf{(0.125)} - 2763.5\mathbf{(0.125)}) =$$

$$= 2798.39 \text{kJ/kg}\mathbf{(0.125)}$$

State 7
Superheated water vapour at 7 bar and 500°C.

$$h_7 = 3481.70 \text{kJ/kg}\mathbf{(0.125)} \qquad s_7 = 7.9299 \text{kJ/(kg K)}\mathbf{(0.125)}$$

State 8s
Isentropic expansion up to 0.08 bar is considered.

$$s_{8s} = s_7\mathbf{(0.125)} = 7.9299 \text{kJ/(kg K)}$$

$$s_{8s} < s_g(p_8) \Rightarrow \text{Liquid-vapour equilibrium}\mathbf{(0.125)}$$

$$x_{8s} = \frac{s_{8s} - s_f(p_8)}{s_g(p_8) - s_f(p_8)}\mathbf{(0.25)} = \frac{7.9299 - 0.5926\mathbf{(0.125)}}{8.2287\mathbf{(0.125)} - 0.5926} = 0.961\mathbf{(0.125)}$$

$$h_{8s} = h_f(p_8) + x_{8s}(h_g(p_8) - h_f(p_8))\mathbf{(0.25)} =$$

$$= 173.88 + 0.961(2577\mathbf{(0.125)} - 173.88\mathbf{(0.125)}) = 2482.97 \text{kJ/kg}\mathbf{(0.125)}$$

State 8
Isentropic efficiency of turbine 2 $\eta_{s,T2} = 85\%$.

$$h_8 = h_7 - \eta_{s,T2}(h_7 - h_{8s})\mathbf{(0.25)} = 3481.7 - 0.85(3481.7 - 2482.97) = 2632.78\text{kJ/kg}\mathbf{(0.125)}$$

State 9
Water leaves the FWH as saturated liquid ($x_9 = 0$).

$$h_9 = h_\mathrm{f}(p_9)\mathbf{(0.125)} = 697.22\text{kJ/kg}\mathbf{(0.125)}$$

State 10s and 10
Isentropic efficiency of pump 2 $\eta_{s,P2} = 100\%$. Isentropic compression **(0.125)**, state 10s corresponds to state 10. Compressed liquid water behaves as an incompressible substance with constant specific volume v.

$$h_{10} = h_9 + v(p_{10} - p_9)\mathbf{(0.25)} = 697.22 + 0.001 \cdot (100 - 7) \cdot 10^2 = 706.52\text{kJ/kg}\mathbf{(0.125)}$$

State 3
First principle of Thermodynamics for open systems applied to the closed feedwater heater. Steady state operation, negligible kinetic and potential energy variations, no external heat losses or work transfer in the heat exchanger. **(0.25)**

$$\cancel{\frac{\mathrm{d}E}{\mathrm{d}t}} = \cancel{\dot{Q}} - \cancel{\dot{W}} + \sum_\text{in} \dot{m}_\text{in}\left(h_\text{in} + \cancel{\frac{w_\text{in}^2}{2} + gz_\text{in}}\right) - \sum_\text{out} \dot{m}_\text{out}\left(h_\text{out} + \cancel{\frac{w_\text{out}^2}{2} + gz_\text{out}}\right)$$

$$y\dot{m}_{tot}h_6 + (1-y)\dot{m}_{tot}h_2 - y\dot{m}_{tot}h_9 - (1-y)\dot{m}_{tot}h_3 = 0$$

$$h_3 = h_2 + \frac{y(h_6 - h_9)}{(1-y)}\mathbf{(0.25)} =$$

$$= 184.98 + \frac{0.15(2798.39 - 697.22)}{(1 - 0.15)} = 555.78\text{kJ/kg}\mathbf{(0.125)}$$

State 4
First principle of Thermodynamics for open systems applied to the mixing chamber. Steady state operation, negligible kinetic and potential energy variations, no heat losses or work transfer in the mixing chamber. **(0.25)**

$$\cancel{\frac{\mathrm{d}E}{\mathrm{d}t}} = \cancel{\dot{Q}} - \cancel{\dot{W}} + \sum_\text{in} \dot{m}_\text{in}\left(h_\text{in} + \cancel{\frac{w_\text{in}^2}{2} + gz_\text{in}}\right) - \sum_\text{out} \dot{m}_\text{out}\left(h_\text{out} + \cancel{\frac{w_\text{out}^2}{2} + gz_\text{out}}\right)$$

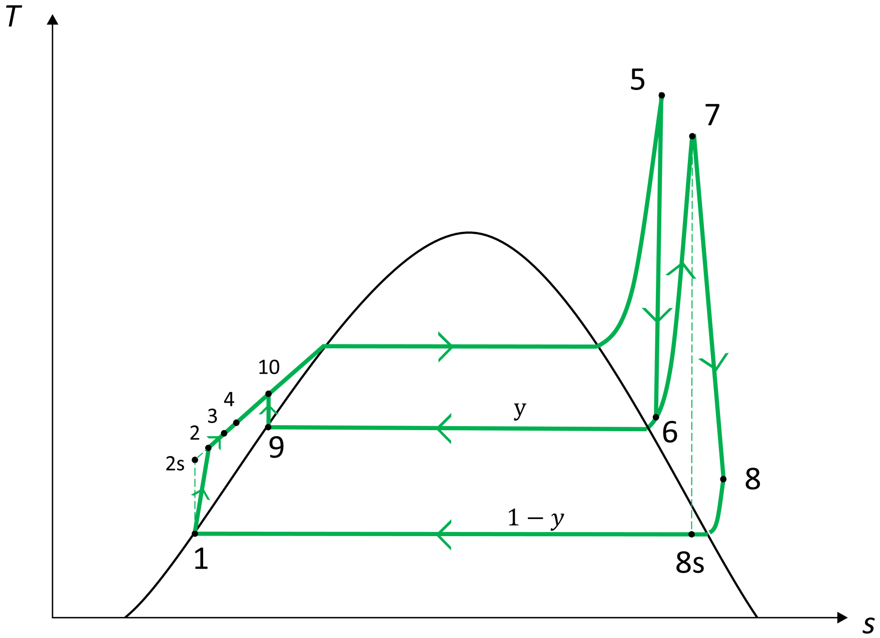
$$y\dot{m}_{tot}h_{10} + (1-y)\dot{m}_{tot}h_3 - \dot{m}_{tot}h_4 = 0$$

$$h_4 = yh_{10} + (1-y)h_3\mathbf{(0.25)} =$$

$$= 0.15 \cdot 706.52 + (1 - 0.15) \cdot 555.78 = 578.39\text{kJ/kg}\mathbf{(0.125)}$$

First Name: ____________ Last Name: ____________ Sciper: ____________

(b) Sketch the qualitative $T - s$ diagram of the cycle. **[3 points]**



States 1, 9 on saturated liquid line **2x(0.125)**
States 2, 3, 4, 10 in compressed liquid region and right order **4x(0.125)** $T_3 < T_9$ **(0.125)**
(if not in the right order but they are all specified in the drawing, they get minus 0.125),
States 5, 7, **2x(0.125)** 6, 8 **2x(0.25)** in superheated vapour region
States 2s, 8s are optional so they do not provide points

Process 1-2, 7-8 with increasing entropy **2x(0.125)**
Process 9-10, 5-6 vertical lines **2x(0.125)**
Processes 2-3, 3-4, 4-10, 10-5, 6-9, 6-7, 8-1 correctly reported **7x(0.125)**

(c) Calculate the total mass flow rate in the system. **[0.625 points]**

First principle of Thermodynamics for open systems applied to the boiler. Steady state operation, negligible kinetic and potential energy variations, no heat losses or work transfer in the heat exchanger. **(0.25)** (1st Principle of Thermodynamics has to be stated to get the points).

$$\cancel{\frac{\mathrm{d}E}{\mathrm{d}t}} = \dot{Q} - \cancel{\dot{W}} + \sum_{\text{in}} \dot{m}_{\text{in}} \left(h_{\text{in}} + \cancel{\frac{w_{\text{in}}^2}{2} + g z_{\text{in}}} \right) - \sum_{\text{out}} \dot{m}_{\text{out}} \left(h_{\text{out}} + \cancel{\frac{w_{\text{out}}^2}{2} + g z_{\text{out}}} \right)$$

$$\dot{Q}_{boiler} + \dot{m}_{tot} h_4 + (1-y)\dot{m}_{tot} h_6 - \dot{m}_{tot} h_5 - (1-y)\dot{m}_{tot} h_7 = 0$$

$$\dot{m}_{tot} = \frac{\dot{Q}_{boiler}}{h_5 - h_4 + (1-y)(h_7 - h_6)} \mathbf{(0.25)} =$$

$$= \frac{80 \cdot 10^3}{3526 - 578.39 + (1 - 0.15)(3481.7 - 2798.39)} = 22.67 \mathrm{kg/s} \mathbf{(0.125)}$$

(d) Calculate the work transfer rates of the turbines and of the pumps. **[1.75 points]**

First principle of Thermodynamics for open systems applied to the two turbines and the two pumps. Steady state operation, negligible kinetic and potential energy variations, adiabatic expansions and compressions. **(0.25)**

$$\cancel{\frac{\mathrm{d}E}{\mathrm{d}t}} = \cancel{\dot{Q}} - \dot{W} + \sum_{\text{in}} \dot{m}_{\text{in}} \left(h_{\text{in}} + \cancel{\frac{w_{\text{in}}^2}{2} + g z_{\text{in}}} \right) - \sum_{\text{out}} \dot{m}_{\text{out}} \left(h_{\text{out}} + \cancel{\frac{w_{\text{out}}^2}{2} + g z_{\text{out}}} \right)$$

$$\dot{W}_{\text{T1}} = \dot{m}_{\text{tot}}(h_5 - h_6) \mathbf{(0.25)} = 22.67 \cdot (3526 - 2798.39) =$$
$$= 16497.1 \mathrm{kW} \mathbf{(0.125)}$$

$$\dot{W}_{\text{T2}} = (1-y)\dot{m}_{\text{tot}}(h_7 - h_8) \mathbf{(0.25)} = (1 - 0.15) \cdot 22.67 \cdot (3481.7 - 2632.78) =$$
$$= 16360.5 \mathrm{kW} \mathbf{(0.125)}$$

$$\dot{W}_{\text{P1}} = (1-y)\dot{m}_{\text{tot}}(h_1 - h_2) \mathbf{(0.25)} = (1 - 0.15) \cdot 22.67 \cdot (173.88 - 184.98) =$$
$$= -214.0 \mathrm{kW} \mathbf{(0.125)}$$

$$\dot{W}_{\text{P2}} = y\dot{m}_{\text{tot}}(h_9 - h_{10}) \mathbf{(0.25)} = 0.15 \cdot 22.67 \cdot (697.22 - 706.52) =$$
$$= -31.6 \mathrm{kW} \mathbf{(0.125)}$$

First Name: ________________ Last Name: ________________ Sciper: ________________

(e) Calculate the thermal efficiency of the cycle. **[0.5 points]**

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{\dot{W}_{\text{T1}} + \dot{W}_{\text{T2}} + \dot{W}_{\text{P1}} + \dot{W}_{\text{P2}}}{\dot{Q}_{\text{boiler}}} \mathbf{(0.25)} = 0.408 \mathbf{(0.25)}$$

Possible expression $\eta_{\text{th}} = \dfrac{\dot{W}_{\text{T1}} + \dot{W}_{\text{T2}}}{\dot{Q}_{\text{boiler}} - \dot{W}_{\text{P1}} - \dot{W}_{\text{P2}}}$ also accepted as correct.

(f) Justify if a higher pressure of the condenser increases, decreases or does not modify the thermal efficiency of the cycle. **[0.5 points]**

A higher pressure of the condenser increases the average temperature at which heat is released $\overline{T}_{\text{C}}$ **(0.125)** and it can therefore decrease the maximum theoretical efficiency of the cycle**(0.125)**.

$$\eta_{\text{th, ideal}} = \frac{\dot{W}_{\text{net, int. rev.}}}{\dot{Q}_{\text{in, int. rev.}}} = 1 - \frac{\dot{Q}_{\text{out, int. rev.}}}{\dot{Q}_{\text{in, int. rev.}}} = 1 - \frac{\overline{T}_{\text{C}}}{\overline{T}_{\text{H}}} \mathbf{(0.125)}$$

If everything else stays the same, the lower maximum theoretical efficiency decreases the thermal efficiency of the cycle**(0.125)**.

(g) In absolute sign, is the exergy transfer by heat larger in the boiler or in the condenser? Why? **[0.5 points]**

The exergy transferred by heat to the boiler is much larger than the one released to the environment in the condenser **(0.125)**. Indeed, in the boiler the heat is transferred at a high temperature ($\overline{T}_{\text{H}} \gg T_0$) while in the condenser the heat is released at a temperature close to the one of the environment ($\overline{T}_{\text{C}} \approx T_0$) **(0.25)**.

$$|\dot{E}x_{Q,boiler}| = |\dot{Q}_{boiler}| \left(1 - \frac{T_0}{\overline{T}_{\text{H}}}\right) \gg |\dot{E}x_{Q,condenser}| = |\dot{Q}_{condenser}| \left(1 - \frac{T_0}{\overline{T}_{\text{C}}}\right) \mathbf{(0.125)}$$

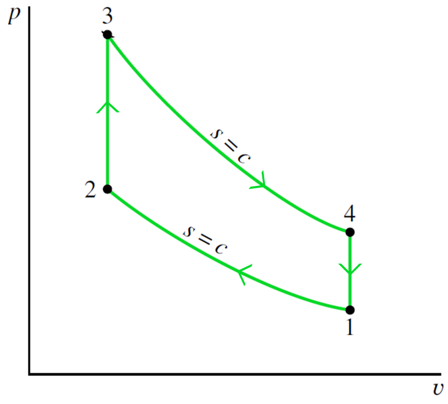
SCHOOL OF ENGINEERING
MECHANICAL ENGINEERING
LRESE - Laboratory of Renewable Energy Sciences and Engineering

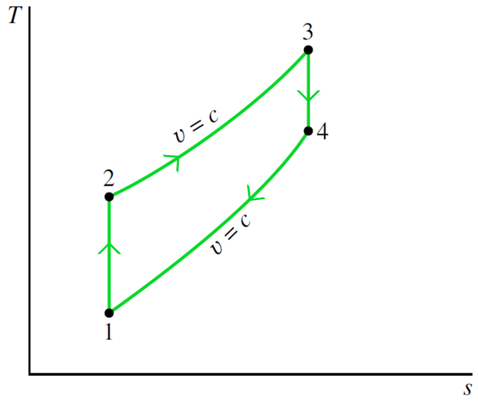
First Name: ______________ Last Name: ______________ Sciper: ______________

Exercise 2: Cold air-standard Otto cycle [9.5 points]

At the beginning of the compression process of a cold air-standard Otto cycle, $p_1 = 1$ bar $T_1 = 295$ K, $V_1 = 500$ cm^3. The maximum temperature in the cycle is 2100 K and the compression ratio is 9. Assume the specific heat of air to be $c_p = 1.1$ kJ/(kg K) and its molar mass $M = 29$ g/mol. State 1 can be assumed to be at ambient pressure and temperature.

(a) Plot the qualitative $p - v$ and the $T - s$ diagram of the cycle. **[1 point]**





$p - v$ diagram

- Curve line 1-2 **(0.125)**
- Vertical line 2-3 **(0.125)**
- Curve line 3-4 **(0.125)**
- Vertical line 4-1 **(0.125)**

$T - s$ diagram

- Vertical line 1-2 **(0.125)**
- Curve line 2-3 **(0.125)**
- Vertical line 3-4 **(0.125)**
- Curve line 4-1 **(0.125)**

First Name: ________________ Last Name: ________________ Sciper: ________________

(b) Calculate the heat addition and the net work of the cycle. **[4.625 points]**

The cold air-standard Otto cycle is characterized by the following assumptions:

- The air in the piston–cylinder assembly is the closed system.
- The compression and expansion processes are adiabatic.
- All the processes are internally reversible.
- The air is modeled as a perfect gas.
- Kinetic and potential energy effects are negligible.

Correct application of the assumptions. **(0.25)**

$$R = \frac{\tilde{R}}{M}\textbf{(0.125)} = \frac{8.314}{29 \cdot 10^{-3}} = 286.7\ \text{J/(kg K)}\textbf{(0.125)}$$

$$c_v = c_p - R\textbf{(0.125)} = 1100 - 286.7 = 813.31\ \text{J/(kg K)}\textbf{(0.125)}$$

$$k = \frac{c_p}{c_v}\textbf{(0.125)} = \frac{1100}{1100 - 286.7} = 1.35\textbf{(0.125)}$$

The mass of air in the cylinder can be calculated using the ideal gas law:

$$m = \frac{p_1 V_1}{R T_1}\textbf{(0.25)} = \frac{29 \cdot 10^{-3} \cdot 1 \cdot 10^5 \cdot 5 \cdot 10^{-4}}{8.314 \cdot 295} = 5.9 \cdot 10^{-4}\ \text{kg} = 0.59\ \text{g}\textbf{(0.25)}$$

State 2
Isentropic compression 1-2.

$$V_2 = V_1/r\textbf{(0.125)} = 5 \cdot 10^{-4}/9 = 5.56 \cdot 10^{-5}\ \text{m}^3\textbf{(0.125)}$$

$$pV^k = \text{const.} \Rightarrow TV^{-1}V^k = \text{const.} \Rightarrow TV^{k-1} = \text{const.}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{k-1} = T_1 \left(r\right)^{k-1}\ \textbf{(0.25)} = 295\left(9\right)^{1.35-1} = 640\ \text{K}\textbf{(0.25)}$$

Same points given if p_2 is calculated.
State 3
Isochoric heat addition 2-3.

$$V_3 = V_2\textbf{(0.125)} = 5.56 \cdot 10^{-5}\ \text{m}^3\textbf{(0.125)}$$

State 4

Isentropic compression 3-4, isochoric heat release 4-1.

$$V_4 = V_1 \mathbf{(0.125)} = 5 \cdot 10^{-4}\ \mathrm{m}^3 \mathbf{(0.125)}$$

$$T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{k-1} = T_3\,(1/r)^{k-1}\ \mathbf{(0.25)} = 2100\,(1/9)^{1.35-1} = 967.9\ \mathrm{K} \mathbf{(0.25)}$$

Same points given if p_4 is calculated.

For all the heat and work calculations first Law of Thermodynamics for closed systems is applied.

Process 2-3

Isochoric process, no work exchanged.

$$U_3 - U_2 = Q_{23} - \cancel{W_{23}} \mathbf{(0.125)}$$

$$Q_{23} = mc_v(T_3 - T_2) \mathbf{(0.25)} = 5.9 \cdot 10^{-4} \cdot 813.31(2100 - 640) = 700\ \mathrm{J} \mathbf{(0.125)}$$

Processes 1-2 and 3-4

Adiabatic compression, no heat transferred.

$$\Delta U = \cancel{Q} - W \Rightarrow W = -\Delta U \mathbf{(0.125)}$$

$$W_{12} = mc_v(T_1 - T_2) \mathbf{(0.25)} = 5.9 \cdot 10^{-4} \cdot 813.31(295 - 640) = -165.9\ \mathrm{J} \mathbf{(0.125)}$$

$$W_{34} = mc_v(T_3 - T_4) \mathbf{(0.25)} = 5.9 \cdot 10^{-4} \cdot 813.31(2100 - 967.9) = 544.3\ \mathrm{J} \mathbf{(0.125)}$$

$$W_{net} = W_{12} + W_{34} \mathbf{(0.125)} = -165.9 + 544.3 = 378.4\ \mathrm{J}$$

Alternative formula $W_{if} = \dfrac{p_i V_i - p_f V_f}{k-1}$ gives full points (0.25), half points (0.125) for $W_{if} = \int_i^f p dV$ and wrong integral.

(c) Calculate the mean effective pressure. **[0.5 points]**

$$mep = \frac{W_{net}}{V_1 - V_2} \mathbf{(0.25)} = \frac{378.4}{(5 - 0.556) \cdot 10^{-4}} = 8.51 \cdot 10^5\ \mathrm{Pa} = 8.51\ \mathrm{bar} \mathbf{(0.25)}$$

(d) Calculate the thermal efficiency. **[0.5 points]**

$$\eta = \frac{W_{net}}{Q_{23}} \mathbf{(0.25)} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{r^{k-1}} = \frac{378.4}{702} = 0.536 \mathbf{(0.25)}$$

(e) Calculate the exergy efficiency of the cycle. **[2.00 points]**

$$\varepsilon_{ex} = \frac{Ex_{W,34} + Ex_{W,12}}{Ex_{Q,23}} \mathbf{(0.25)}$$

Processes 1-2 and 3-4

$$Ex_{W,21} = W_{12} - p_0(V_2 - V_1)\mathbf{(0.125)} = -165.9 - 1 \cdot 10^5(0.556 - 5) \cdot 10^{-4} = -121.4 \text{ J}\mathbf{(0.25)}$$
$$Ex_{W,34} = W_{34} - p_0(V_4 - V_3)\mathbf{(0.125)} = 554.3 - 1 \cdot 10^5(5 - 0.556) \cdot 10^{-4} = 499.9 \text{ J}\mathbf{(0.25)}$$

Alternative solution

$$Ex_{W,net} = Ex_{W,21} + Ex_{W,34} = W_{12} + W_{34} = W_{net}\mathbf{(0.25)} = 378.5 \text{ J}\mathbf{(0.5)}$$

Process 2-3

Exergy balance for closed systems is applied considering the cold-air standard Otto cycle assumptions. Isochoric process ($\Delta V = 0$).

$$\Delta Ex = Ex_Q - \cancel{(W - p_0\Delta V)} - \cancel{Ex_d}$$
$$\Delta Ex = \Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} - T_0\Delta S + \cancel{p_0\Delta V}$$

Perfect gas $\rightarrow ds = c_v dT + \cancel{RdV}$

$$Ex_{Q,23} = U_3 - U_2 - T_0(S_3 - S_2)\mathbf{(0.25)} = Q_{23} - T_0\left[mc_v \ln\left(\frac{T_3}{T_2}\right)\right] \mathbf{(0.25)} =$$
$$= 702 - 295\left[5.9 \cdot 10^{-4} \cdot 813.31 \cdot \ln\left(\frac{2100}{640}\right)\right] = 533.5 \text{ J}\mathbf{(0.25)}$$

Alternative solution

Perfect gas $\rightarrow ds = c_v dT + \cancel{RdV}$

$$Ex_{Q,23} = \int_2^3 \left(1 - \frac{T_0}{T}\right)\delta Q = Q_{23} - T_0\int_2^3 \frac{\delta Q}{T} = U_3 - U_2 - T_0(S_3 - S_2)$$

When $\left(1 - \frac{T_0}{T}\right) Q$ is used, points only if they understood the T is not constant (**(0.25)**).

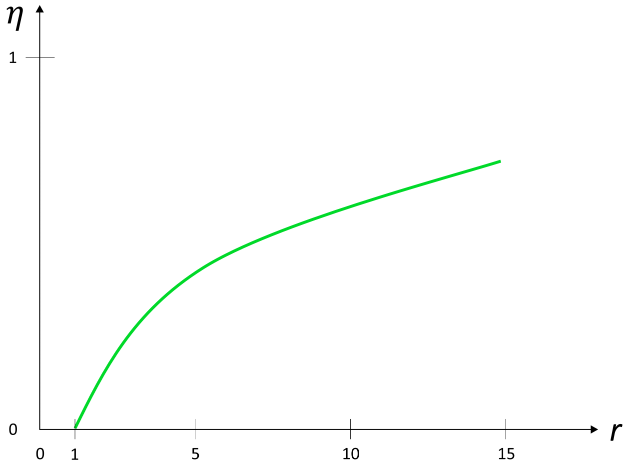
$$\varepsilon_{ex} = \frac{499.9 - 121.4}{533.5} = 0.709\mathbf{(0.25)}$$

First Name: ______________ Last Name: ______________ Sciper: ______________

(f) Considering a general cold air-standard Otto cycle, qualitatively plot the thermal efficiency as a function of the compression ratio. **[0.625 points]**

$$\eta = 1 - \frac{1}{r^{k-1}} \textbf{(0.125)}$$

If the formula is written in (d) in this expression with r, they also get the points.



- Intersection with r axis at 1 **(0.25)**
- Correct curvature **(0.125)**
- Monotonically increasing function **(0.125)**

Exercise 3: Entropy generation in two-compartment cylinder [6.375 points]

(a) Determine the final temperature of the helium. **[1.75 point]**
Realisation that helium undergoes an isentropic compression **(0.5)**. Points were also given if the process was recognized as adiabatic.
Polytropic process for isentropic process with ideal gas:
$p_1 v_1^k = p_2 v_2^k$ **(0.25)** → $p_1(\frac{T_1\tilde{R}}{p_1 M})^k = p_2(\frac{T_2\tilde{R}}{p_2 M})^k$ **(0.25)** → $T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(k-1)/k}$ **(0.25)**
Use of isentropic relation (also 0.75 p if no derivation): $T_{2,He} = T_{1,He}\left(\frac{p_2}{p_1}\right)^{(k-1)/k} = 321.7\text{K}$ **(0.125)**; with $k = c_p/c_v$ **(0.25)**=1.67 **(0.125)**

(b) Determine the final volume of the nitrogen. **[1.375 point]**
Final volume of nitrogen: $V_{2,N_2} = V_{1,N_2} + V_{1,He} - V_{2,He}$ **(0.5)** (half of these points were given if only $V_{tot} = V_{N_2} + V_{He}$ was specified)
Initial volume of helium: $V_{1,He} = \frac{\tilde{R}T_{1,He}m_{He}}{p_1 M_{He}}$ **(0.25)**=0.6406 m^3 **(0.125)**
Final volume of helium: $V_{2,He} = \frac{\tilde{R}T_{2,He}m_{He}}{p_2 M_{He}}$ **(0.25)**=0.5568 m^3 **(0.125)**
Final volume nitrogen: 0.2838 m^3 **(0.125)**

(c) Determine the heat transfer to the nitrogen. **[2.125 point]**
Energy balance, no work **(0.125)**: $U_2 - U_1 = Q$ **(0.125)** where $U_2 - U_1 = U_{2,N_2} - U_{1,N_2} + U_{2,He} - U_{1,He}$ **(0.25)**
Same pressure in the two compartments due to mechanical equilibrium **(0.25)**
Need mass of nitrogen and final temperature of nitrogen:
Mass of nitrogen: $m_{N_2} = \frac{p_1 V_{1,N_2} M_{N_2}}{\tilde{R}T_{2,N_2}}$ **(0.25)** $= 0.2185$ kg **(0.125)**
Final temperature of nitrogen: $T_{2,N_2} = \frac{p_2 V_{2,N_2} M_{N_2}}{m_{N_2}\tilde{R}}$ **(0.25)** $= 525.1$ K **(0.125)**
Internal energy differences: $U_{2,N_2} - U_{1,N_2} = m_{N_2} c_{v,N_2}(T_{2,N_2} - T_{1,N_2})$ **(0.25)** and $U_{2,He} - U_{1,He} = m_{He} c_{v,He}(T_{2,He} - T_{1,He})$ **(0.25)** → Q=46.6 kJ **(0.125)** (positive sign means incoming)

(d) Determine the entropy generation during this process. **[1.125 point]**
Entropy balance: $\sigma = S_2 - S_1 - \frac{Q}{T_{res}}$**(0.5)** $= S_{2,N_2} - S_{1,N_2} - \frac{Q}{T_{res}}$**(0.25)** $= m_{N_2}(c_{p,N_2} ln\frac{T_{2,N_2}}{T_{1,N_2}} - \tilde{R}/M_{N_2} ln\frac{p_2}{p_1}) - \frac{Q}{T_{res}}$**(0.25)** $= 0.057\text{kJ/K}$ **(0.125)**
(0.25 points were given if the entropy balance for an open system was instead written)