SCHOOL OF ENGINEERING
MECHANICAL ENGINEERING

# Thermodynamics and energetics I: Exercise 9, Solution

1. Given: $p_1 = 1$ bar, $T_1 = 300$ K, $p_2 = 10$ bar, $p_3 = 10$ bar, $T_3 = 1400$ K, $p_4 = 1$ bar

   Additionally: $R_{\text{air}} = \dfrac{\widetilde{R}_{\text{air}}}{M_{\text{air}}} = 286.69\ \text{J}/\left(\text{kg}\cdot\text{K}\right)$

   In this exercise the gas is ideal but some considerations about how to sketch the T-s diagram can be derived for a perfect gas.

   An isentropic process appears as a vertical straight line on the T-s diagram.
   When an adiabatic process is not reversible, there is always an increase in entropy due to irreversibilities.

   Process 1-2 is an adiabatic compression with an isentropic efficiency of 80%. 1-2s is the same process if it were isentropic. As it is a compression, the temperature increases. Indeed, for a perfect gas:

   $$s - s_1 = c_p \ln\left(\frac{T}{T_1}\right) - R\ln\left(\frac{p}{p_1}\right) = 0 \text{ when } s = s_1$$

   $$T(p) = T_1\left(\frac{p}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \text{ where } \gamma = \frac{c_p}{c_v} \text{ and } R = c_p - c_v$$

   Thus, the processes 1-2 and 1-2s go upwards. Moreover, as 1-2 is not isentropic, there is an increase in entropy which is why we see the 1-2 line moving to the right (increasing entropy) as it goes upwards. States 2 and 2s are at the same pressure.

   Process 2-3 is an isobaric process in which heat is provided, which means that entropy of the system increases. Therefore, point 3 has higher temperature and entropy.
   An isobaric process for a perfect gas is an exponential curve on the T-s diagram as:

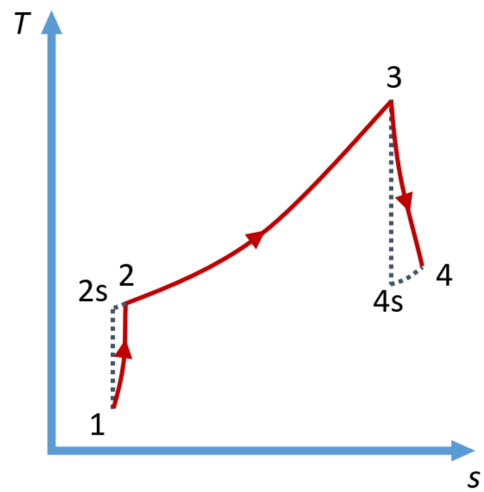
   $$s - s_1 = c_p \ln\left(\frac{T}{T_1}\right) - R\ln\left(\frac{p}{p_1}\right) = c_p \ln\left(\frac{T}{T_1}\right) \text{ when } p = p_1$$

   $$T(s) = T_1 \exp\left(\frac{s - s_1}{c_p}\right)$$

   The gas is this exercise is ideal but the curve will be somehow similar to an exponential function.

   Process 3-4 is an expansion with an isentropic efficiency of 90%. 3-4s is the same process if it were isentropic. As it is an expansion, the temperature decreases. Thus, 3-4 and 3-4s go downwards. Moreover, as 3-4 is not isentropic, there is an increase in entropy which is why we see the 3-4 line move to the right (increasing entropy) as it goes downwards. States 4 and 4s are at the same pressure.

   The $Ts$ diagram of these processes is therefore:




(a)

Ideal gas model: $s_{2s} - s_1 = s^0(T_{2s}) - s^0(T_1) - R \cdot \ln\left(\frac{p_2}{p_1}\right) = 0$

$T_{2s} = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$

For perfect gas, the relationship between the isentropic efficiency and temperature is:

$\eta_{c,is} = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{c_p(T_{2s} - T_1)}{c_p(T_2 - T_1)} = \frac{T_{2s} - T_1}{T_2 - T_1}$

Rearranging, we get:

$T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_{c,is}} = 651.375\,\text{K}$

Adiabatic compression ($\dot{Q}_{12} = 0$), first principle of thermodynamics for open systems:

$$\underbrace{\frac{\mathrm{d}E_{12}}{\mathrm{d}t}}_{0} = \underbrace{\dot{Q}_{12}}_{0} - \dot{W}_{12} + \dot{m} \cdot \left(h_1 + \cancel{\frac{w_1^2}{2} + gz_1}\right) - \dot{m} \cdot \left(h_2 + \cancel{\frac{w_2^2}{2} + gz_2}\right)$$

Given $c_p = 3.5R = 3.5 \times 286.69 = 1003.415\,\text{J}/(\text{kg} \cdot \text{K})$, we have:

$\dot{W}_{12} = \dot{m}(h_1 - h_2) \approx -2.04$ MW

(b)

Ideal gas model: $s_{4s} - s_3 = s^0(T_{4s}) - s^0(T_3) - R \cdot \ln\left(\frac{p_4}{p_3}\right) = 0$

For an isentropic process, we can calculate $T_{4s}$ directly using the following formula:

$$T_{4s} = T_3 \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}}$$

where $\gamma = 1.4$ for air as an ideal gas.

Using the isentropic efficiency $\eta_{t,is}$ of the turbine, we have:

$$T_4 = T_3 - \eta_{t,is} \cdot (T_3 - T_{4s}) = 790.16K$$

Since we assume air is an ideal gas, the enthalpy difference can be calculated as:

$$h_3 - h_4 = c_p(T_3 - T_4)$$

Adiabatic expansion ($\dot{Q}_{34} = 0$), first principle of thermodynamics for open systems:

$$\underbrace{\frac{\mathrm{d}E_{34}}{\mathrm{d}t}}_{0} = \underbrace{\dot{Q}_{34}}_{0} - \dot{W}_{34} + \dot{m} \cdot \left(h_3 + \cancel{\frac{w_3^2}{2} + gz_3}\right) - \dot{m} \cdot \left(h_4 + \cancel{\frac{w_4^2}{2} + gz_4}\right)$$

Finally, we can determine the power output of the turbine:

$$\dot{W}_{34} = \dot{m}(h_3 - h_4) = \dot{m} \cdot c_p \cdot (T_3 - T_4) \approx 3.56 \text{ MW}$$

(c)

First principle of thermodynamics for open systems:

$$\underbrace{\frac{\mathrm{d}E_{23}}{\mathrm{d}t}}_{0} = \dot{Q}_{23} - \underbrace{\dot{W}_{23}}_{0} + \dot{m} \cdot \left(h_2 + \cancel{\frac{w_2^2}{2} + gz_2}\right) - \dot{m} \cdot \left(h_3 + \cancel{\frac{w_3^2}{2} + gz_3}\right)$$

$$\dot{Q}_{23} = -\dot{m}(h_2 - h_3) = \dot{m} \cdot c_p \cdot (T_3 - T_2) \approx 4.35 \text{ MW}$$

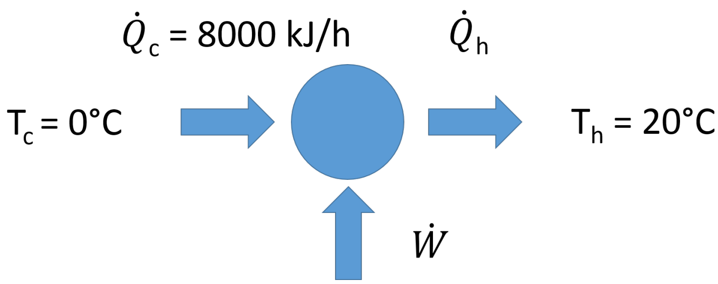
$$\eta_{\text{th}} = \frac{\dot{W}_{12} + \dot{W}_{34}}{\dot{Q}_{23}} = 0.35$$

2. Given: Flow rate $\dot{V}$, quality $x_1 = 1$, pressure $p_1$, $p_2$, isentropic efficiency of a compressor $\eta$.

   (a) Inflow (state 1) is saturated, thus thermodynamic properties can be obtained from a database (for example CoolProp). We get specific enthalpy $h_1$, specific volume $v_1$ , and specific entropy $s_1$ .
   For outflow (state 2), enthalpy can be calculated based on the definition of isentropic efficiency of a compressor,
   $$\eta = \frac{h_{2\mathrm{s}} - h_1}{h_2 - h_1} \Leftrightarrow h_2 = h_1 + \frac{h_{2\mathrm{s}} - h_1}{\eta},$$
   where $h_{2\mathrm{s}}$ is the enthalpy at the ideal exit state ($s_1 = s_{2\mathrm{s}}$), which can be obtained with the entropy and pressure $h_{2\mathrm{s}} = h(s_1, p_2)$ from a database.
   Mass conservation in steady states can be formulated as:
   $$\frac{dm}{dt} = \dot{m}_{\mathrm{in}} - \dot{m}_{\mathrm{out}} = 0 \Leftrightarrow \dot{m}_{\mathrm{in}} = \dot{m}_{\mathrm{out}} = \dot{m} = \frac{\dot{V}}{v_1} .$$
   Using the first law of thermodynamics (energy conservation) and $\dot{Q}_{12} = 0$ one can get
   $$\underbrace{\frac{dE}{dt}}_{0} = \underbrace{\dot{Q}}_{0} - \dot{W} + \dot{m} \cdot \left( h_1 + \cancel{\frac{w_1^2}{2} + gz_1} \right) - \dot{m} \cdot \left( h_2 + \cancel{\frac{w_2^2}{2} + gz_2} \right).$$
   Substituting $h_2$ and $\dot{m}$ obtained above, necessary work to drive this compressor is obtained.
   $$-\dot{W} = -\dot{m}(h_1 - h_2) = \frac{\dot{V}}{v_1} \cdot \frac{h_{2\mathrm{s}} - h_1}{\eta}.$$

   (b) Entropy balance for this compressor is:
   $$\frac{dS}{dt} = \frac{\dot{Q}}{T_0} + \dot{m}(s_1 - s_2) + \dot{\sigma}.$$
   When the system is running in a steady state and isentropically, this equation reduces to:
   $$0 = \frac{\dot{Q}}{T_\mathrm{b}} + \dot{\sigma}.$$
      (i) For an adiabatic compressor $\dot{Q} = 0$. Therefore, the required condition is $\dot{\sigma} = 0$. This means the compressor must be internally reversible to run isentropically.
      (ii) For a compressor without insulation, $\dot{Q} \neq 0$, and the additional requirement is:
      $$\dot{Q} = -T_b \dot{\sigma}.$$
      This means the internal irreversibility $\dot{\sigma}$ must be counterbalanced by heat transfer **out** of the system ($\dot{Q} < 0$) which is equal to $-T_\mathrm{b}\dot{\sigma}$.

3. The refrigeration cycle operates between a cold reservoir (subscript c) and a hot reservoir (subscript h).




(a)

From the definition of the coefficient of performance:

$$COP = \frac{\dot{Q}_c}{\dot{W}} = 2.5$$

$$\dot{W} = \frac{\dot{Q}_c}{COP} = 3.20 \text{ MJ/h} = 888.89 \text{ W}$$

(b) The coefficient of performance in the case of a reversible cooling cycle (Carnot inverse cycle) is given by:

$$COP_{\text{Carnot}} = \frac{T_c}{T_h - T_c} = 13.66$$

$$\dot{W}_{\text{Carnot}} = \frac{\dot{Q}_c}{COP_{\text{Carnot}}} = 585.76 \text{ kJ/h} = 172.61 \text{ W}$$