Thermodynamics and energetics I: Exercise 8, Solution

1. A Carnot power cycle is operated with steam/water as the working fluid. During the isothermal expansion, the water is heated from an initial state 1, with pressure p_1, temperature T_1 and the quality 15 %, to saturated vapor state 2. The vapor then expands isentropically to a state 3 with temperature T_3 and pressure $p_3 < p_1$. Isothermal compression then bring state 3 to state 4, and then back to state 1 with an isentropic compression.

   Note the specific entropies at the saturate states satisfies $s_g(p_3) < s_g(p_1) < s_f(p_1) < s_f(p_3)$

   (a) Sketch the states and the processes in the pv diagram. (Don't be concerned about the specific shape of the isentropic curve)

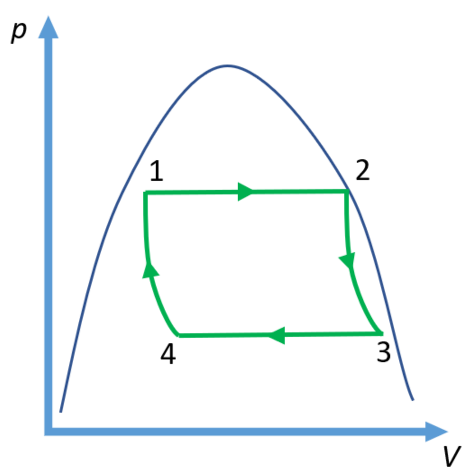
   (b) Describe how you would obtain the specific internal energy and specific entropy for each of the 4 states in the Carnot cycle. Provide an analytical expression, not a numerical solution.

   (c) Determine the expressions for heat and work transfer for each process, assuming you know the specific internal energy and specific entropy for each of the 4 states in the Carnot cycle. Provide an analytical expression, not a numerical solution.

   (d) Provide an expression for the thermal efficiency.

   (a) The Carnot cycle consists of the following processes:

   - 1-2: Isothermal expansion
   - 2-3: Adiabatic, reversible expansion (isentropic)
   - 3-4: Isothermal compression
   - 4-1: Adiabatic, reversible compression (isentropic)



(b)

- state 1

  $x_1 = 0.150$
  $T_1 = T_{\text{sat}}(p_1)$
  $u_1 = u_{\text{f}} + x_1(u_{\text{g}} - u_{\text{f}})$
  $s_1 = s_{\text{f}} + x_1(s_{\text{g}} - s_{\text{f}})$

- state 2

  $x_2 = 1$
  Process 1-2 is isothermal: $T_2 = T_1$
  In the two-phase region it holds that if $T =$ constant also $p =$ constant. $p_2 = p_1$
  $u_2 = u_{\text{g}}(p_2)$
  $s_2 = s_{\text{g}}(p_2)$

- state 3

  Process 2-3 is isentropic: $s_3 = s_2$
  $x_3 = \dfrac{s_3 - s_{\text{f}}}{s_{\text{g}} - s_{\text{f}}}$
  $T_3 = T_{\text{sat}}(p_3)$
  $u_3 = u_{\text{f}} + x_3(u_{\text{g}} - u_{\text{f}})$

- state 4

  Process 3-4 is isothermal: $T_4 = T_3, p_4 = p_3$
  Process 4-1 is isentropic: $s_4 = s_1$
  $x_4 = \dfrac{s_4 - s_{\text{f}}}{s_{\text{g}} - s_{\text{f}}}$
  $u_4 = u_{\text{f}} + x_4(u_{\text{g}} - u_{\text{f}})$

(c) First law of thermodynamics (closed system): $\Delta U = Q - W$

- 1-2 (isobaric and isothermal)

$$W_{12} = m\int_1^2 p\mathrm{d}v = mp_1(v_2 - v_1)$$
$$Q_{12} = m(u_2 - u_1) + W_{12}$$
or
$$Q_{12} = m\int_1^2 T\mathrm{d}s = T_1 m(s_2 - s_1)$$

Note: Although 1-2 is an isothermal process, $\Delta U \neq 0$ because it is not an ideal gas/liquid. In the last equation.

- 2-3 (adiabatic)

$$Q_{23} = 0$$
$$W_{23} = -U_{23} = -m(u_3 - u_2)$$

- 3-4 (isobaric and isothermal)

$$W_{34} = m\int_3^4 p\mathrm{d}v = mp_3(v_4 - v_3)$$
$$Q_{34} = U_{34} + W_{34} = m(u_4 - u_3) + W_{34}$$
or
$$Q_{34} = m\int_3^4 T\mathrm{d}s = T_3 m(s_4 - s_3)$$

- 4-1 (adiabatic)

$$Q_{41} = 0$$
$$W_{41} = -U_{41} = -m(u_1 - u_4)$$

(d)

$$\eta_{\mathrm{th}} = \frac{W_{12} + W_{23} + W_{34} + W_{41}}{Q_{12}}$$

Or

$$\eta_{\mathrm{Carnot}} = 1 - \frac{T_{\mathrm{C}}}{T_{\mathrm{H}}} = 1 - \frac{T_3}{T_1} = 1 - \frac{T_4}{T_2}$$

2.

$$M_{\mathrm{air}} = 29 \text{ g/mol} \rightarrow R_{\mathrm{air}} = \frac{\widetilde{R}}{M_{\mathrm{air}}} = 286.7 \text{ J/(kg K)}$$

(a)

$$T_1 = \frac{p_1 V_1}{m R_{\mathrm{air}}} = 586.0\ \mathrm{K} = 312.85\ {}^\circ\mathrm{C} = T_2 = T_{\mathrm{H}}$$
$$\eta_{\mathrm{Carnot}} = 1 - \frac{T_{\mathrm{C}}}{T_{\mathrm{H}}}$$
$$T_{\mathrm{C}} = (1 - \eta_{\mathrm{Carnot}}) T_{\mathrm{H}} = 234.4\ \mathrm{K} = -38.77\ {}^\circ\mathrm{C} = T_3 = T_4$$

(b) First law of thermodynamics (closed system): $\Delta U = Q - W$

- 1-2 (isothermal)

  Under the hypothesis of perfect gas, $\Delta U_{12} = 0$
  $W_{12} = Q_{12} = 40\ \mathrm{kJ}$
  $$pV = m R_{\mathrm{air}} T_1 = p_1 V_1 \rightarrow p = p_1 \frac{V_1}{V}$$
  $$W_{12} = \int_1^2 p \mathrm{d}V = \int_1^2 p_1 \frac{V_1}{V} \mathrm{d}V = p_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$$
  $$V_2 = V_1 \cdot \exp\left(\frac{W_{12}}{p_1 V_1}\right) = 0.3045\ \mathrm{m}^3$$

(c)

$$p_2 = \frac{m R_{\mathrm{air}} T_2}{V_2} = 5.517\ \mathrm{bar}$$
Process 2-3 is isentropic:
$$pV^\gamma = \mathrm{constant} \rightarrow \frac{m R_{\mathrm{air}} T}{V} V^\gamma = \mathrm{constant} \rightarrow T \cdot V^{(\gamma-1)} = \mathrm{constant}$$
$$V_3 = \left(\frac{T_2}{T_3} V_2^{(\gamma-1)}\right)^{\frac{1}{\gamma-1}} = V_2 \left(\frac{T_2}{T_3}\right)^{\frac{1}{\gamma-1}} = 3.009\ \mathrm{m}^3$$
Process 4-1 is isentropic:
$$V_4 = V_1 \left(\frac{T_1}{T_4}\right)^{\frac{1}{\gamma-1}} = 2.372\ \mathrm{m}^3$$
$$p_4 = \frac{m R_{\mathrm{air}} T_4}{V_4} = 0.2833\ \mathrm{bar}$$

state	1	2	3	4
p[bar]	7	5.517	0.2233	0.2833
V[m^3]	0.240	0.3045	3.009	2.372
T [°C]	312.8	312.8	-38.77	-38.77
T [K]	586.0	586.0	234.4	234.4

(d)

- 2-3 (isentropic)

  $Q_{23} = 0$

  $pV^\gamma = \text{constant} \ \rightarrow p = p_2 \frac{V_2^\gamma}{V^\gamma}$

  $$W_{23} = \int_2^3 p\mathrm{d}V = \int_2^3 p_2 \frac{V_2^\gamma}{V^\gamma}\mathrm{d}V = \frac{p_2 V_2^\gamma}{1-\gamma}(V_3^{1-\gamma} - V_2^{1-\gamma}) = \frac{p_2 V_2 - p_3 V_3}{\gamma - 1} = 252.0 \text{ kJ}$$

- 3-4 (isothermal)

  $$W_{34} = p_3 V_3 \ln\left(\frac{V_4}{V_3}\right) = -16 \text{ kJ}$$

  $\Delta U_{34} = 0 \rightarrow W_{34} = Q_{34} = -16$ kJ

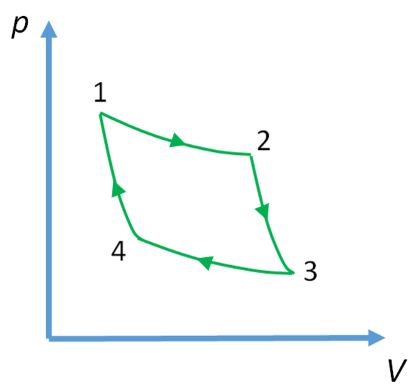
- 4-1 (isentropic)

  $Q_{41} = 0$

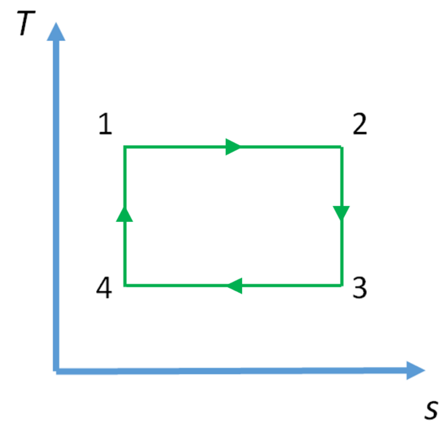
  $$W_{41} = \int_4^1 p\mathrm{d}V = \frac{p_1 V_1 - p_4 V_4}{1-\gamma} = -252.0 \text{ kJ}$$

	W[kJ]	Q[kJ]
1-2	40	40
2-3	252.0	0
3-4	-16	-16
4-1	-252.0	0

(e)



3. (a) *Note*: This is a Carnot cycle.



(b)

$$M_{\text{air}} = 29 \text{ g/mol} \rightarrow R_{\text{air}} = 286.7 \ \frac{\text{J}}{\text{kg K}}$$

For perfect gas: $c_{\text{v}} = \dfrac{1}{\gamma - 1} R_{\text{air}} = 716.8 \ \dfrac{\text{J}}{\text{kg K}}$

$$V_1 = \frac{m R_{\text{air}} T_1}{p_1} = 0.014 \text{ m}^3$$

$$V_2 = \frac{m R_{\text{air}} T_2}{p_2} = 0.072 \text{ m}^3$$

Process 2-3 is isentropic (see 3c): $V_3 = V_2 \left(\dfrac{T_2}{T_3} \right)^{\frac{1}{\gamma - 1}} = 0.125 \text{ m}^3$

$$p_3 = \frac{m R_{\text{air}} T_3}{V_3} = 4.579 \text{ bar}$$

Process 4-1 is isentropic: $V_4 = V_1 \left(\dfrac{T_1}{T_4} \right)^{\frac{1}{\gamma - 1}} = 0.025 \text{ m}^3$

$$p_4 = \frac{m R_{\text{air}} T_4}{V_4} = 22.90 \text{ bar}$$

	p [bar]	V [m^3]	T [K]
1	50	0.014	500
2	10	0.072	500
3	4.579	0.125	400
4	22.90	0.025	400

(c)

- 1-2 (isothermal)

$$W_{12} = p_1 V_1 \ln\left(\frac{V_2}{V_1}\right) = 115.4 \text{ kJ}$$
$\Delta U_{12} = 0$, therefore: $Q_{12} = W_{12} = 115.4$ kJ

- 2-3 (isentropic)

$Q_{23} = 0$
$W_{23} = -\Delta U_{23} = 35.84$ kJ

- 3-4 (isolthermal)

$$W_{34} = p_3 V_3 \ln\left(\frac{V_4}{V_3}\right) = -92.29 \text{ kJ}$$
$\Delta U_{34} = 0$, therefore: $Q_{34} = W_{34} = -92.29$ kJ

- 4-1 (isentropic)

$Q_{41} = 0$
$W_{41} = -\Delta U_{41} = -35.84$ kJ

	W[kJ]	Q[kJ]	ΔU [kJ]
1-2	115.4	115.4	0
2-3	35.84	0	-35.84
3-4	-92.29	-92.29	0
4-1	-35.84	0	35.84

(d)

$Q_{\text{net}} = Q_{12} + Q_{34} = 23.07$ kJ
$W_{\text{net}} = W_{12} + W_{23} + W_{34} + W_{41} = 23.07$ kJ

(e)

$$\eta_{\text{th}} = \frac{W_{12} + W_{23} + W_{34} + W_{41}}{Q_{12}} = 0.2$$
Note: $\eta_{\text{Carnot}} = 1 - \frac{T_4}{T_2} = 0.2$

(f) Each process is internally reversible, therefore it is possible to write:

$$\mathrm{d}S = \left(\frac{\delta Q}{T}\right)_{\substack{\text{int}\\\text{rev}}} \;\rightarrow\; \Delta S = \int \frac{\delta Q}{T}$$

$$\Delta S_{12} = \int_1^2 \frac{\delta Q}{T} = \frac{Q_{12}}{T_1} = \frac{115.4}{500} = 0.23\ \frac{\text{kJ}}{\text{K}}$$

$$\Delta S_{23} = \int_2^3 \frac{\delta Q}{T} = 0$$

$$\Delta S_{34} = \int_3^4 \frac{\delta Q}{T} = \frac{Q_{34}}{T_3} = \frac{-98.29}{400} = -0.23\ \frac{\text{kJ}}{\text{K}}$$

$$\Delta S_{41} = \int_4^1 \frac{\delta Q}{T} = 0$$

$$\Delta S_{\text{cycle}} = \oint \frac{\delta Q}{T} = \Delta S_{12} + \Delta S_{23} + \Delta S_{34} + \Delta S_{41} = 0$$

Note: The entropy variation in adiabatic and internally reversible processes is zero (isentropic processes). Being a state property, in every cycle (either reversible or irreversible) the total variation of entropy is zero.