

Thermodynamics and energetics I: Exercise 6, Solution

1. (a) For each of the four components, we can state the first law for an open system. In steady state and by neglecting kinetic and potential energies we obtain:

$$\underbrace{\frac{dE}{dt}}_{0 \text{ (steady state)}} = \dot{Q} - \dot{W}_{CV} + \dot{m} \left(h_{in} + \cancel{\frac{w_{in}^2}{2}} + \cancel{gz_{in}} \right) - \dot{m} \left(h_{out} + \cancel{\frac{w_{out}^2}{2}} + \cancel{gz_{out}} \right)$$

$$\dot{m} (h_{out} - h_{in}) = \dot{Q} - \dot{W}_{CV}$$

It is therefore necessary to determine for each state the enthalpy (CoolProp code separately uploaded):

- state 1

$$\begin{aligned} p_1 &= 3 \text{ MPa}, T_1 = 400 \text{ }^\circ\text{C} \\ T_1 &> T_{sat}(p_1) \rightarrow \text{Superheated vapor} \\ h_1 &= 3231.69 \text{ kJ/kg} \end{aligned}$$

- state 2

$$\begin{aligned} p_2 &= 50 \text{ kPa}, x_2 = 0.950 \rightarrow \text{Liquid - vapor mixture} \\ h_2 &= h_f + x_2(h_g - h_f) = 2529.98 \text{ kJ/kg} \end{aligned}$$

- state 3

$$\begin{aligned} p_3 &= 50 \text{ kPa}, \text{ Saturated liquid} \\ h_3 &= h_f(p_3) = 340.54 \text{ kJ/kg} \end{aligned}$$

- state 4

$$\begin{aligned} p_4 &= 3 \text{ MPa}, T_4 = 82 \text{ }^\circ\text{C} \\ p_4 &> p_{sat}(T_4) \rightarrow \text{Compressed liquid} \\ h_4 &= 345.75 \text{ kJ/kg} \end{aligned}$$

- 1-2

$$\begin{aligned} \dot{Q}_{12} &= 0 \\ \dot{W}_{12} &= \dot{m}(h_1 - h_2) = 63.15 \text{ MW} \end{aligned}$$

- 2-3

$$\dot{W}_{23} = 0$$

$$\dot{Q}_{23} = \dot{m}(h_3 - h_2) = -197.05 \text{ MW}$$

- 3-4

$$\dot{Q}_{34} = 0$$

$$\dot{W}_{34} = \dot{m}(h_3 - h_4) = -0.47 \text{ MW}$$

- 4-1

$$\dot{W}_{41} = 0$$

$$\dot{Q}_{41} = \dot{m}(h_1 - h_4) = 259.73 \text{ MW}$$

The efficiency of the cycle is given by:

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{\dot{W}_{12} + \dot{W}_{34}}{\dot{Q}_{41}} = 0.24$$

(b) For the condenser we can state:

$$\dot{Q}_{23} + \dot{Q}_{56} = 0$$

$$\dot{m}(h_3 - h_2) + \dot{m}_{\text{cooling}}(h_6 - h_5) = 0$$

To calculate the enthalpy difference of the cooling water ($h_6 - h_5$)

$$(h_6 - h_5) = c_p(T_6 - T_5) + v(p_6 - p_5) \approx c_p(T_6 - T_5)$$

$$\dot{m}_{\text{cooling}} = \frac{\dot{m}(h_2 - h_3)}{c_p(T_6 - T_5)} = 3139 \text{ kg/s}$$

2. (a) From the first law of thermodynamics for an open system we derive:

$$\dot{W}_{\text{comp}} = \dot{m}_{\text{CO}_2}(h_1 - h_2)$$

$$h_1 - h_2 = \frac{1}{M} \int_{T_2}^{T_1} \tilde{c}_p(T) dT = \frac{-8970.351}{12 + 2 \cdot 16} = -203.87 \text{ kJ/kg}$$

$$\text{with } \tilde{c}_p(T) = 19.962 + 72.623 \cdot 10^{-3}T - 54.931 \cdot 10^{-6}T^2 + 16.645 \cdot 10^{-9}T^3 \text{ kJ/(kmol K)}$$

$$\dot{m}_{\text{CO}_2} = \frac{\dot{W}_{\text{comp}}}{(h_1 - h_2)} = \frac{-100}{-203.87} = 0.4905 \text{ kg/s}$$

(b) For the heat exchanger we can state:

$$-\dot{Q}_{23} = \dot{Q}_{45}$$

$$-\dot{m}_{\text{CO}_2}(h_3 - h_2) = \dot{m}_{\text{H}_2\text{O}}(h_5 - h_4)$$

Using incompressible substance model neglecting the pressure difference:

$$(h_5 - h_4) = c_p(T_5 - T_4) = 4.185 \cdot (30 - 20) = 41.85 \text{ kJ/kg}$$

$$h_2 - h_3 = \frac{1}{M} \int_{T_3}^{T_2} \tilde{c}_p(T) dT = \frac{6317.91}{12 + 2 \cdot 16} = 143.59 \text{ kJ/kg}$$

$$\dot{m}_{\text{H}_2\text{O}} = \frac{\dot{m}_{\text{CO}_2}(h_2 - h_3)}{(h_5 - h_4)} = 1.684 \text{ kg/s}$$

3. The conservation of mass after opening the valves:

$$\frac{dm}{dt} = \dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \dot{m} - \dot{m} = 0$$

The first law of thermodynamics for open systems without work and external heat transfer, neglecting kinetic and potential energy variations:

$$\frac{dE}{dt} = \frac{dU}{dt} = \dot{m}(h_{\text{in}} - h_{\text{out}})$$

Assume air to be a perfect gas: $du = c_v dT$

$$\frac{dU}{dt} = m \frac{du}{dt} + u \frac{dm}{dt} = mc_v \frac{dT}{dt} + 0$$

The properties of the injected gas at the input are known, and it is assumed that the air is evacuated at the outlet of the chamber with the temperature T . Thus,

$$h_{\text{in}} - h_{\text{out}} = c_p(T_{\text{in}} - T)$$

We rewrite the first law for open systems:

$$mc_v \frac{dT}{dt} = \dot{m}c_p(T_{\text{in}} - T)$$

This is a first order non-homogeneous differential equation of the form:

$$a \frac{df(x)}{dx} + bf(x) + c = 0, \text{ where } a, b, c \text{ are constants.}$$

$$\text{Homogeneous solution: } f(x) = Ae^{-\frac{b}{a}x} \rightarrow T_{\text{H}}(t) = Ae^{-\frac{t}{\tau}} \text{ with } \tau = \frac{mc_v}{\dot{m}c_p}$$

$$\text{Non-homogeneous solution: } f(x) = -\frac{c}{b} \rightarrow T_{\text{P}} = \frac{\dot{m}c_p T_{\text{in}}}{\dot{m}c_p} = T_{\text{in}}$$

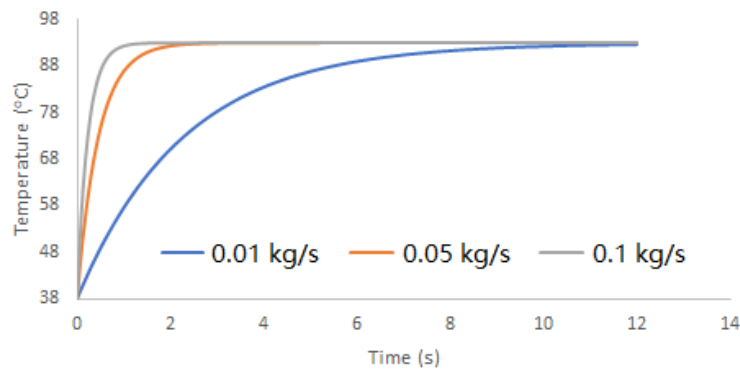
$$\text{General solution: } f(x) = Ae^{-\frac{b}{a}x} - \frac{c}{b} \rightarrow T(t) = T_{\text{H}}(t) + T_{\text{P}} = Ae^{-\frac{t}{\tau}} + T_{\text{in}}$$

$$\text{With the initial condition: } T(0) = T_0 = 38 \text{ }^\circ\text{C} \rightarrow A = T_0 - T_{\text{in}}$$

$$\text{And the final solution: } T(t) = T_{\text{in}} + (T_0 - T_{\text{in}})e^{-\frac{t}{\tau}}$$

Mass of air in the chamber: $m = \frac{p_0 V M}{\tilde{R} T_0} = 0.031 \text{ kg}$ and constant heat capacities: $\frac{c_v}{c_p} = \frac{1}{1.4}$

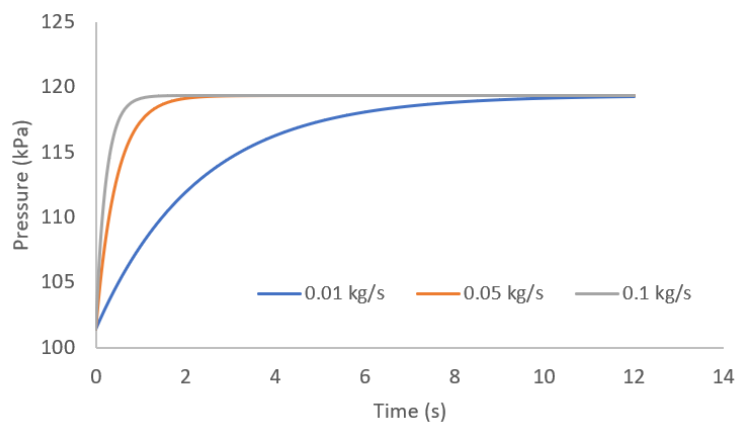
The characteristic time τ for mass flow rate of 0.01 kg/s, 0.05 kg/s and 0.1 kg/s is 2.27 s, 0.455 s and 0.227 s, respectively. After 5τ the temperature of the system can be considered equal to the temperature of the inlet flow: $T(t > 5\tau) \simeq T_{\text{in}}$



The pressure of the system in time can be obtained from the ideal gas law.

$$p(t) = \frac{m\tilde{R}}{MV} T(t) = \frac{m\tilde{R}}{MV} [T_{\text{in}} + (T_0 - T_{\text{in}})e^{-\frac{t}{\tau}}]$$

Note: The final pressure of the system is different from the pressure of the inlet flow. This is due to the fact that the pressure difference between the inlet flow and the system gives the thermodynamic driving force to guarantee a steady state flow of air.



4. We calculate for each cycle:

the efficiency $\eta_{th} = \frac{W_{cycle}}{Q_H}$,

the Carnot efficiency $\eta_{th,max} = 1 - \frac{T_C}{T_H}$,

the energy balance $Q_H - Q_C - W_{cycle}$ according to the first law of thermodynamics.

Cycle	η_{th}	$\eta_{th,max}$	$Q_H - Q_C - W_{cycle}$	
1	0.5	0.7	0	Irreversible ($\eta_{th} < \eta_{th,max}$)
2	0.7	0.7	0	Reversible ($\eta_{th} = \eta_{th,max}$)
3	0.4286	0.7	-100.0	Impossible (Violate 1st law)
4	0.75	0.7	0	Impossible ($\eta_{th} > \eta_{th,max}$)