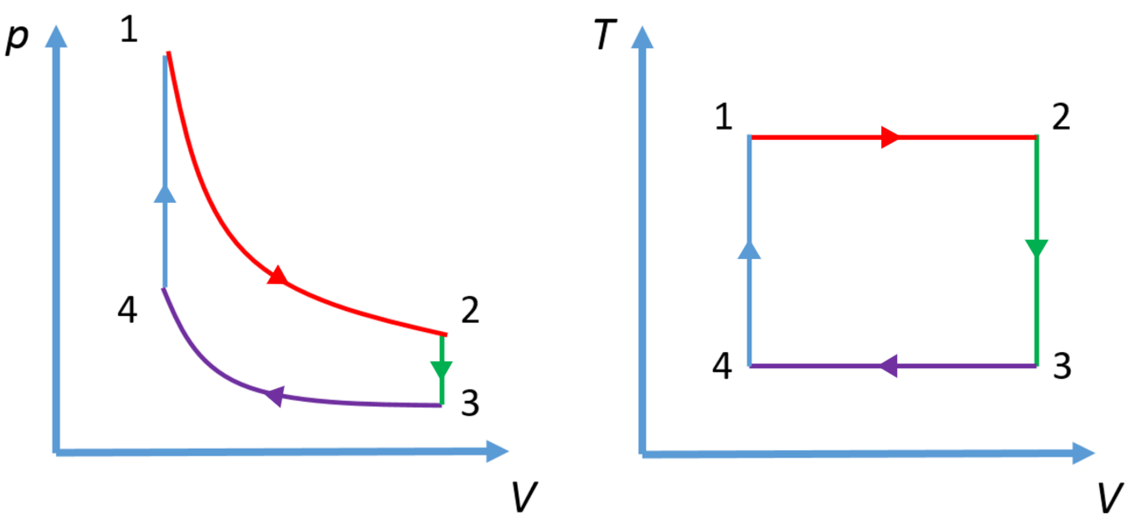
SCHOOL OF ENGINEERING
MECHANICAL ENGINEERING

EPFL

Thermodynamics and energetics I: Exercise 5, Solution

1. (a)



(b) We assume N_2 an ideal gas.

State	1	2	3	4
T [K]	2000	2000	300	300
p [bar]	20	6.67	1	3.00
V [m^3]	0.445	1.336	1.336	0.445

- State 1

  $T_1 = 2000$ K

  $p_1 = 20$ bar

  $$V_1 = \frac{m\widetilde{R}T_1}{M_{\mathrm{N_2}}p_1} = 0.445 \text{ m}^3 \text{ with } M_{\mathrm{N_2}} = 28 \text{ g/mol}$$

- State 3

  $T_3 = 300$ K

  $p_3 = 1$ bar

  $$V_3 = \frac{m\widetilde{R}T_3}{M_{\mathrm{N_2}}p_3} = 1.336 \text{ m}^3$$

- State 2

$$T_2 = 2000 \text{ K}$$
$$V_2 = V_3 = 1.336 \text{ m}^3$$
$$p_2 = \frac{m\widetilde{R}T_2}{M_{\mathrm{N_2}}V_2} = 6.67 \text{ bar}$$

- State 4

$$T_4 = T_3 = 300 \text{ K}$$
$$V_4 = V_1 = 0.445 \text{ m}^3$$
$$p_4 = \frac{m\widetilde{R}T_4}{M_{\mathrm{N_2}}V_4} = 3.00 \text{ bar}$$

(c)

$$c_v = \frac{1}{k-1} \cdot \frac{\tilde{R}}{M_{N_2}} = 891.68 \text{ J/kg/K}$$

Heat transfer:

$$Q_{23} = -Q_{41} = m(u_3 - u_2) = mc_v(T_3 - T_2) = -2273.78 \text{ kJ}$$

(d)

$$\Delta U_{12} = 0,\ W_{12} = p_1 V_1 \ln\left(\frac{V_2}{V_1}\right) = 978.628 \text{ kJ } = Q_{12}$$
$$W_{23} = 0,\ \Delta U_{23} = Q_{23}$$
$$\Delta U_{34} = 0,\ W_{34} = p_3 V_3 \ln\left(\frac{V_4}{V_3}\right) = -146.794 \text{ kJ } = Q_{34}$$
$$W_{41} = 0,\ \Delta U_{41} = Q_{41}$$
$$W_{\text{cycle}} = W_{12} + W_{34} = 831.83 \text{ kJ}$$

2. The mass flow rate is constant, the system is adiabatic, the system is in steady state and potential energy variations are neglected. From the first law of thermodynamics for open systems,

$$\underbrace{\frac{\mathrm{d}E}{\mathrm{d}t}}_{0\text{ (steady state)}} = \underbrace{\dot{Q}}_{0\text{ (adiabatic)}} - \underbrace{\dot{W}_{cv}}_{0} + \dot{m}\left(h_1 - h_2 + \frac{w_1^2}{2} - \underbrace{\frac{w_2^2}{2}}_{0} + \underbrace{g(z_1 - z_2)}_{0\text{ (neglected)}}\right)$$

we obtain: $h_2 = h_1 + \frac{w_1^2}{2}$

Therefore

$$T_2 = T_1 + \frac{h_2 - h_1}{c_p} = T_1 + \frac{w_1^2}{2c_p} = 319.39\text{ K}$$

3. The volume varies linearly with the pressure and it is possible to assume that $V = 0$ when $p = 0$. Therefore we obtain:

$V = a \cdot p$ with $a = V_1/p_1 = 0.650\text{ m}^3/\text{kPa}$
$p_2 = 150\text{ kPa}$ and $V_2 = 97.500\text{ m}^3$
$W_{12} = \int_1^2 p\mathrm{d}V = \frac{V_2^2 - V_1^2}{2a} = 4.063\text{ MJ}$

For the energy conversion we use the first law of thermodynamics for open systems:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \underbrace{\dot{Q}}_{0\text{ (adiabatic)}} - \dot{W} + \dot{m}_{\mathrm{i}}\left(h_i + \underbrace{\frac{w_{\mathrm{i}}^2}{2}}_{0(neglected)} + \underbrace{gz_{\mathrm{i}}}_{0\text{ (neglected)}}\right) - \underbrace{\dot{m}_{\mathrm{e}}}_{0}\left(h_e + \frac{w_{\mathrm{e}}^2}{2} + gz_{\mathrm{e}}\right)$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\dot{W} + \dot{m}_{\mathrm{i}} \cdot h_i$$

$$\Delta E = -W_{12} + m_{\mathrm{i}} \cdot h_{\mathrm{i}}$$

The process is adiabatic and we neglect kinetic and potential energy variation ($\Delta E = \Delta U = m_2u_2 - m_1u_1$). The mass entering the balloon is the difference between state 2 and 1 ($m_{\mathrm{i}} = m_2 - m_1$) and it has the enthalpy of the gas reservoir ($h_i = h_{res}$):

$m_2u_2 - m_1u_1 = -W_{12} + (m_2 - m_1)h_{\mathrm{res}}$
$m_2(u_2 - h_{\mathrm{res}}) - m_1(u_1 - h_{\mathrm{res}}) = -W_{12}$ (Equation 1)

Assuming perfect gas:

$$c_{\mathrm{v}} = c_{\mathrm{p}} - \frac{\widetilde{R}}{M_{\mathrm{He}}} = 3.114\ \mathrm{kJ/(kg \cdot K)}\ \ (M_{\mathrm{He}} = 4\ \mathrm{g/mol})$$

$$m_1 = \frac{p_1 V_1 M_{\mathrm{He}}}{\widetilde{R} T_1} = 10.601\ \mathrm{kg}$$

$$h_{\mathrm{res}} = u_{\mathrm{res}} + p_{\mathrm{res}} v_{\mathrm{res}} = u_{\mathrm{res}} + \frac{\widetilde{R} T_{\mathrm{res}}}{M_{\mathrm{He}}}$$

We can rewrite Equation 1:

$$\frac{p_2 V_2 M_{\mathrm{He}}}{\widetilde{R} T_2} \left(c_{\mathrm{v}}(T_2 - T_{\mathrm{res}}) - \frac{\widetilde{R} T_{\mathrm{res}}}{M_{\mathrm{He}}} \right) - m_1 \left(c_{\mathrm{v}}(T_1 - T_{\mathrm{res}}) - \frac{\widetilde{R} T_{\mathrm{res}}}{M_{\mathrm{He}}} \right) = -W_{12}$$

By solving this equation for T_2, the numerical result is: $T_2 = 333.58\ \mathrm{K}$