SCHOOL OF ENGINEERING
MECHANICAL ENGINEERING

Thermodynamics and energetics I: Exercise 4, Solution

1. (a) The system is closed, therefore $m = \text{const}$. Using ideal gas law:

$$m = \frac{pVM}{\widetilde{R}T} = 396.92 \text{ g}$$

(b)

$$p_2 = p_1 \left(\frac{V_1}{V_2}\right)^{1.3} = 1.22 \text{ bar}$$

(c)

$$T_2 = \frac{p_2 V_2 M}{m\widetilde{R}} = 243.68 \text{ K}$$

(d) With $pV^{1.3} = \text{const}$

$$W_{12} = \int_1^2 p\mathrm{d}V = p_1 V_1^{1.3} \int_1^2 \frac{\mathrm{d}V}{V^{1.3}} = \frac{p_1 V_1^{1.3}}{0.3}\left(V_1^{-0.3} - V_2^{-0.3}\right) = 18.77 \text{ kJ}$$

$$\Delta U_{12} = \int_1^2 m c_\mathrm{v} \mathrm{d}T = m\left[0.6\left(T_2 - T_1\right) + 1.25 \cdot 10^{-4}\left(T_2^2 - T_1^2\right)\right] = -14.93 \text{ kJ}$$

$$Q_{12} = \Delta U_{12} + W_{12} = 3.84 \text{ kJ}$$

2. (a) The system is closed, well insulated (no heat transfer) and the container is rigid (no work exchanged). Therefore, the internal energy of the system, the mass of the gases and the total volume are constant.
The variation of the internal energy of the system is equal to the sum of the changes of the inner energies of its components and equal to zero.

$$\underbrace{\Delta U}_{0} = \Delta U_\mathrm{CO} + \Delta U_\mathrm{air} + \underbrace{\Delta U_\mathrm{partition}}_{0}$$

In equilibrium, the temperatures and the pressures in the two chambers are equal.
Rearranging $\widetilde{c}_\mathrm{p} - \widetilde{c}_\mathrm{v} = \widetilde{R}$, $c_\mathrm{v} = \frac{\widetilde{c}_\mathrm{v}}{M}$ and $k = \frac{\widetilde{c}_\mathrm{p}}{\widetilde{c}_\mathrm{v}}$, we obtain:

$$c_\mathrm{v} = \frac{\widetilde{R}}{M(k-1)}$$

The previous equation becomes thus:

$$\Delta U_\mathrm{CO} + \Delta U_\mathrm{air} = \left[m_\mathrm{CO} c_\mathrm{v,CO}\left(T_2 - T_\mathrm{1,CO}\right) + m_\mathrm{air} c_\mathrm{v,air}\left(T_2 - T_\mathrm{1,air}\right)\right] = 0$$

Considering $M_{\text{air}} = 29$ g/mol and $M_{\text{CO}} = 28$ g/mol

$$T_2 = \frac{m_{\text{air}} c_{\text{v,air}} T_{1,\text{air}} + m_{\text{CO}} c_{\text{v,CO}} T_{1,\text{CO}}}{m_{\text{air}} c_{\text{v,air}} + m_{\text{CO}} c_{\text{v,CO}}} = 417.32 \text{ K}$$

(b) At equilibrium, the pressure is the same in both chambers (the partition moves until the pressure forces equalize). The total volume of the system is the sum of both volumes, and does not change (rigid container):

$$V_{\text{tot}} = V_{1,\text{air}} + V_{1,\text{CO}} = \frac{m_{\text{air}} \widetilde{R} T_{1,\text{air}}}{M_{\text{air}} p_{1,\text{air}}} + \frac{m_{\text{CO}} \widetilde{R} T_{1,\text{CO}}}{M_{\text{CO}} p_{1,\text{CO}}} = 3.076 \text{ m}^3$$

$$V_{\text{tot}} = V_{2,\text{air}} + V_{2,\text{CO}} = \frac{m_{\text{air}} \widetilde{R} T_2}{M_{\text{air}} p_2} + \frac{m_{\text{CO}} \widetilde{R} T_2}{M_{\text{CO}} p_2}$$

$$p_2 = \left(\frac{\frac{m_{\text{air}}}{M_{\text{air}}} + \frac{m_{\text{CO}}}{M_{\text{CO}}}}{V_{\text{tot}}} \right) \widetilde{R} T_2 = 2.39 \text{ bar}$$

(c) Using the ideal gas law, we obtain:

$$V_{2,\text{air}} = \frac{m_{\text{air}} \widetilde{R} T_2}{M_{\text{air}} p_2} = 1.01 \text{ m}^3 \text{ and } V_{2,\text{CO}} = \frac{m_{\text{CO}} \widetilde{R} T_2}{M_{\text{CO}} p_2} = 2.07 \text{ m}^3$$

3. (a) The system is closed:

$$m_1 = m_2 = \frac{p_1 V_1 M_{H_2}}{\widetilde{R} T_1} = 80.19 \text{ g}$$

After the expansion, the volume is doubled: $V_2 = 2$ m^3
The pressure in the cylinder is determined by the atmospheric pressure plus the pressure induced by the force of the spring:

$$p(x) = p_{\text{atm}} + \frac{kx}{A_{\text{piston}}} \text{ with } x = \frac{V - V_1}{A_{\text{piston}}}$$

$$p_2 = p_{\text{atm}} + \frac{k(V_2 - V_1)}{A^2_{\text{piston}}} = 1.47 \text{ bar;}$$

$$x_2 = 1.25 \text{ m; } x_1 = 0 \text{ m}$$

x_1 is chosen to be zero as a reference.
The temperature is calculated using the ideal gas law: $T_2 = 881.25$ K

(b)

$$W_{12} = \int_1^2 p\mathrm{d}V = A_{\text{piston}} \int_{x_1}^{x_2} p(x)\mathrm{d}x = A_{\text{piston}} \cdot p_{\text{atm}} x_2 + \frac{k x_2^2}{2} = 123.44 \text{ kJ}$$

(c)

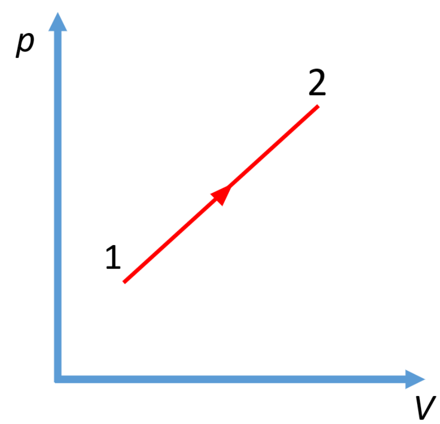
$$W_{\text{spring}} = \int_{x_1}^{x_2} -kx\mathrm{d}x = -\frac{kx_2^2}{2} = -23.44\ \text{kJ}$$

The work transferred is negative because the spring receives work from the cylinder.

(d)

$$Q_{12} = \Delta U_{12} + W_{12} = mc_{\text{v}}(T_2 - T_1) + W_{12} = 607.04\text{kJ}$$

4. The p-V diagram is characterized by $p(V) = p_{\text{atm}} + \dfrac{k(V - V_1)}{A_{\text{piston}^2}}$



Air in the tank and cylinder is a closed system. Taking air in both the tank and the cylinder, the first law of thermodynamics applied for closed systems is:

$$\underbrace{\Delta U_{12}}_{0} = Q_{12} - W_{12}$$

The specific internal energy of an ideal gas only depends on the temperature ($u = u(T)$) and the system composed by cylinder and tank has constant mass. Therefore, $\Delta U_{12} = 0$. The first law of thermodynamics becomes:

$$Q_{12} = W_{12}$$

The work is given by:

$$W_{12} = \int_1^2 p\mathrm{d}V = p_{\text{cyl}}\left(V_{2,\text{cyl}} - V_{1,\text{cyl}}\right)$$

Note: The volume of air in the tank is constant ($V_{\text{tank},1} = V_{\text{tank},2} = V_{\text{tank}}$) and is deleted ($V_{\text{tank}} - V_{\text{tank}} = 0$), so only the volume of gas in the cylinder appears in the formula although the system is composed by both tank and cylinder.

$V_{\text{tank}} = \dfrac{m_{1,\text{tank}} \widetilde{R} T_{\text{tank}}}{M_{CO_2} p_{1,\text{tank}}} = 0.329 \text{ m}^3$

Mass remaining in tank: $m_{2,\text{tank}} = M_{CO_2} \dfrac{p_{2,\text{tank}} V_{\text{tank}}}{\widetilde{R} T_{\text{tank}}}$

Mass entering the cylinder: $\Delta m = m_{1,\text{tank}} - m_{2,\text{tank}} = 1.800$ kg, with $M_{CO_2} = 44$ g/mol

$V_{2,\text{cyl}} = \dfrac{(m_{1,\text{cyl}} + \Delta m) \widetilde{R} T_{\text{cyl}}}{M_{CO_2} p_{\text{cyl}}} = 0.543 \text{ m}^3$

$W_{12} = \displaystyle\int_1^2 p \text{d}V = p_{\text{cyl}} \left(V_{2,\text{cyl}} - V_{1,\text{cyl}} \right) = 98.63 \text{ kJ}$

$Q_{12} = W_{12} = 98.63 \text{ kJ}$