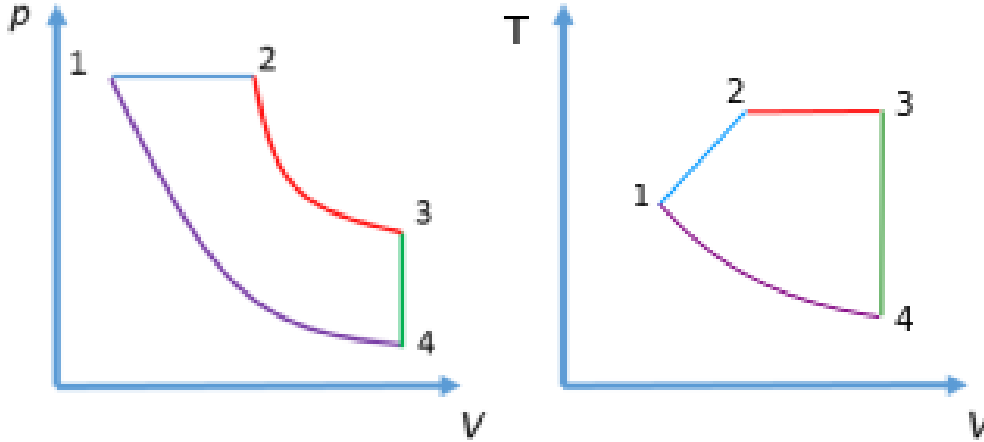


Thermodynamics and energetics I: Exercise 3

1. Cycle with air as ideal gas

(a) Draw the process in a p - v - and a T - v -diagram.



(b) Calculate temperature, pressure and specific volume of the air in every state.

state	1	2	3	4
T [K]	400	550	550	295
p [bar]	10.0	10.0	7.50	4.03
V [m ³]	0.344	0.473	0.631	0.631
v [m ³ /kg]	0.115	0.158	0.210	0.210

- To obtain the volumes, one can use the ideal gas law:

$$pv = RT, \text{ with } v = \frac{V}{m} \text{ and } R = \frac{8.3145 \text{ J}/(\text{mol}\cdot\text{K})}{29 \text{ g/mol}}$$

- For 4-1: $p_4 = p_1 \frac{V_1^{1.5}}{V_4^{1.5}} = 4.028 \text{ bar}$

- For 3-4: Ideal gas and isochoric process $\frac{p}{T} = \text{const.} \rightarrow \frac{p_4}{T_4} = \frac{p_3}{T_3}$

(c) Calculate the work and heat transfer for all of the above processes.

$$\mathbf{1-2:} \quad W_{12} = \int_1^2 p dV = p_1 \int_1^2 dV = p_1(V_2 - V_1) = 129.0 \text{ kJ}$$

$$\text{with } \Delta U = c_v m \Delta T \text{ (neglecting kinetic and potential energies)}$$

$$\text{and } \bar{c}_v = 0.5(c_v(T = 400\text{K}) + c_v(T = 550\text{K})) = 0.7395 \text{ kJ}/(\text{kg}\cdot\text{K})$$

$$Q_{12} = \Delta U_{12} + W_{12} = \bar{c}_v \cdot 3\text{kg} \cdot (T_2 - T_1) + W_{12} = 461.775 \text{ kJ}$$

$$\mathbf{2-3:} \quad W_{23} = \int_2^3 p dV$$

$$= \int_2^3 \frac{p_2 V_2}{V} dV = p_2 V_2 \ln\left(\frac{V_3}{V_2}\right) = 136.1 \text{ kJ}$$

$$\text{With } \Delta U = \bar{c}_v m \Delta T = 0 \rightarrow Q_{23} = 136.1 \text{ kJ}$$

$$\mathbf{3-4:} \quad W_{34} = 0 \rightarrow Q_{34} = \Delta U = \bar{c}_v m \Delta T = \frac{0.753+0.718}{2} \cdot 3 \cdot (295 - 550) = -562.6575 \text{ kJ}$$

$$\mathbf{4-1:} \quad W_{41} = \int_4^1 p dV, \text{ with } pV^n = \text{const. and } n = 1.5$$

$$W_{41} = \frac{p_4 V_4 - p_1 V_1}{n-1} = -179.9 \text{ kJ}$$

$$\text{with } \Delta U = \bar{c}_v m \Delta T = \frac{0.726+0.718}{2} \cdot 3 \cdot (400 - 295) = 227.43 \rightarrow Q_{41} = 47.53 \text{ kJ}$$

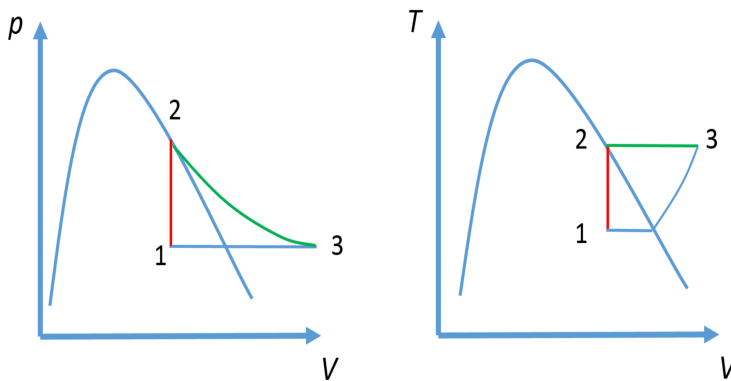
$$\text{(d) } \Delta U_{\text{cycle}} = \sum_{\text{cycle}} Q - \sum_{\text{cycle}} W = 0$$

	W [kJ]	Q [kJ]	ΔU [kJ]
1-2	129.0	461.8	332.8
2-3	136.1	136.1	0
3-4	0	-561.7	-561.7
4-1	-179.9	46.6	226.5
	85.2	82.8	-2.40

Rounding errors lead to $\Delta U_{\text{cycle}} \approx 0$ and $\sum_{\text{cycle}} Q - \sum_{\text{cycle}} W \approx 0$.

2. A system of 1.5 kg ammonia runs through the following cycle:

(a) Draw the process in the p - v -diagram and T - v -diagram.



(b) Net work for the cycle and the work and heat transfer of each process.

1-2: $W_{12} = 0$ (isochoric process)

In state 1, the gas is in equilibrium with its liquid phase. During a change of state, the temperature depends only on the pressure (example: water boils at 100 °C at 1 bar, but on the summit of Everest, the pressure is lower and it boils at

72 °C). T_1 can thus be determined with p_1 and the thermodynamic tables. Using CoolProp:

$$T_1 = T_{\text{sat}}(10 \text{ bar}) = 24.91 \text{ °C}$$

Specific volume for quality = 0.6 mixture at 10 bar:

$$v_1 = 0.077779 \text{ m}^3/\text{kg}$$

Mixture internal energy:

$$u_1 = 1084.69 \text{ kJ/kg}$$

In state 2, $x_2 = 1$ (saturation) and $v_2 = v_1$, using CoolProp

$$u_2 = 1507.09 \text{ kJ/kg}, T_2 = 42.43 \text{ °C}$$

As the work is equal to zero, we can calculate the heat transfer using the first law:

$$Q_{12} = m(u_2 - u_1) = 633.6 \text{ kJ}$$

2-3: First law: $W_{23} = Q_{23} - \Delta U_{23} = Q_{23} - m(u_3 - u_2)$

Due to the isothermal process $T_3 = T_2 = 42.43^\circ \text{ C}$ and $p_3 = p_1 = 10 \text{ bars}$. Using CoolProp:

$$u_3 = 1540.53 \text{ kJ/kg}$$

$$v_3 = 0.140226 \text{ m}^3/\text{kg}$$

As a result

$$W_{23} = 177.84 \text{ kJ}$$

3-1: Isobaric process $p_3 = p_1$

$$W_{31} = mp_1(v_1 - v_3) = -93.67 \text{ kJ}$$

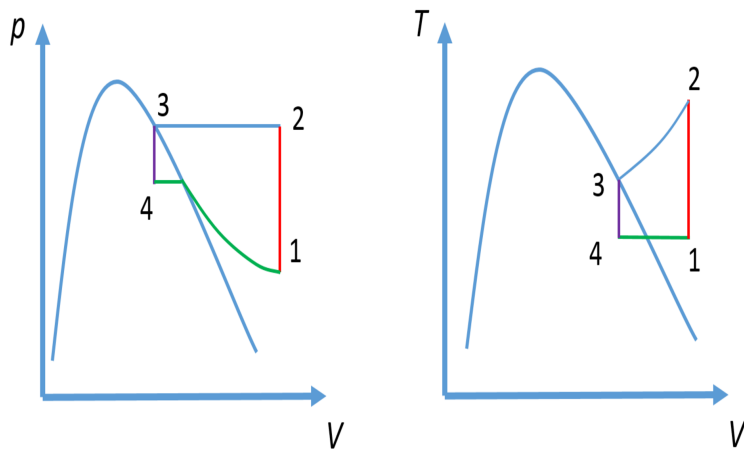
$$Q_{31} = m(u_1 - u_3) + W_{31} = -777.43 \text{ kJ}$$

Your can also verify via first law

$$W_{\text{cycle}} = Q_{12} + Q_{23} + Q_{31}$$

3. Cycle with water

(a) p - v -diagram and T - v -diagram.



(b) Net work for the cycle and the work and heat transfer of each process. Using CoolProp, we obtain superheated water vapor properties for state 1 and 2 and saturated water properties for state 3 and 4

State 1	$p_1 = 5 \text{ bar}$	$T_1 = 160^\circ \text{ C}$
	$v_1 = 0.0.383664 \text{ m}^3/\text{kg}$	$u_1 = 2575.55 \text{ kJ/kg}$
State 2	$p_2 = 10 \text{ bar}$	
	$v_2 = v_1 = 0.383664 \text{ m}^3/\text{kg}$	$u_2 = 3232.59 \text{ kJ/kg}$
State 3	$p_3 = 10 \text{ bar}$	
	$v_3 = v_{3,g} = 0.194362 \text{ m}^3/\text{kg}$	$u_3 = u_{3,g} = 2582.75 \text{ kJ/kg}$
State 4	$v_4 = v_3$	$T_4 = T_1 = 160^\circ \text{ C}$
	$x_4 = 0.6322$	$u_4 = 1871.59 \text{ kJ/kg}$

Process 1-2

$$W_{12} = 0$$

$$Q_{12} = m(u_2 - u_1) = 985.56 \text{ kJ}$$

Process 2-3

$$W_{23} = mp_2(v_3 - v_2) = -283.95 \text{ kJ}$$

$$Q_{23} = m(u_3 - u_2) + W_{23} = -1258.713 \text{ kJ}$$

Process 3-4

$$W_{34} = 0$$

$$Q_{34} = m(u_4 - u_3) = -1066.74 \text{ kJ}$$

As process 4-1 is isothermal, we know that $T_4 = T_1$. With this temperature one can calculate the quality x (=ratio of the mass of vapor to the total mass of the mixture) based on the table.

$$x_4 = \frac{v_4 - v_{4,f}}{v_{4,g} - v_{4,f}} = 0.6322$$

$$u_4 = u_{4,f} + x(u_{4,g} - u_{4,f}) = 1871.59 \text{ kJ/kg}$$

Process 4-1

$$W_{41} = Q_{41} - m(u_1 - u_4) = 159.86 \text{ kJ}$$

Check if cycle is fulfilled: $\sum_{\text{cycle}} Q \approx \sum_{\text{cycle}} W$

$$W_{\text{cycle}} = W_{12} + W_{23} + W_{34} + W_{41} \text{ kJ}$$

$$Q_{\text{cycle}} = Q_{12} + Q_{23} + Q_{34} + Q_{41} \text{ kJ}$$