Thermodynamics and energetics I: Exercise 2, solution

1. A gas in a piston-cylinder assembly.

   (a) For a reversible process, the work involved when a gas changes from state 1 to state 2 is defined by $W = \int_1^2 p\mathrm{d}V$ (Attention, the work provided by the system is positive and the work delivered to the system is negative), with $p = \frac{c}{V^n}$ and $c = \text{constant} = p_1 V_1^n = p_2 V_2^n$. After solving the integral we obtain:

$$\text{for } n \neq 1: W_{1,2} = \frac{p_1 V_1 - p_2 V_2}{n-1}$$
$$\text{for } n = 1: W_{1,2} = p_1 V_1 \ln \frac{V_2}{V_1}$$
$$\text{for } n = 0: W_{1,2} = p_1 (V_2 - V_1)$$

   (b) i. isobaric process ($n = 0$): $W_{1,2} = 30.0$ kJ

   ii. isothermal process ($n = 1$): $W_{1,2} = 24.3$ kJ

   iii. isochoric process ($n \to \infty$): No change in volume, $\mathrm{d}V = 0 \to W_{1,2} = 0$kJ

   iv. $n = 1.5$: $p_2 = p_1 \left(\frac{V_1}{V_2}\right)^n = 1.633$ bar, $W_{1,2} = 22.0$ kJ

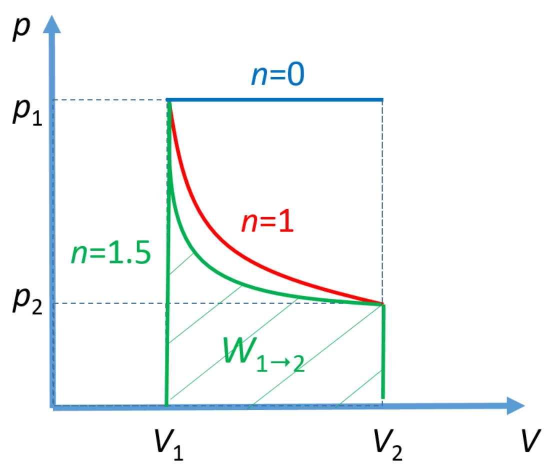
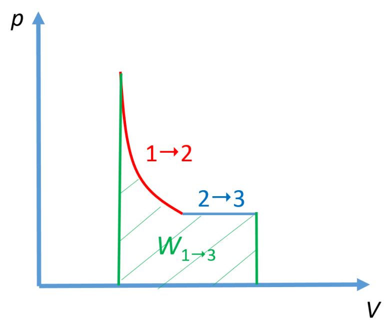


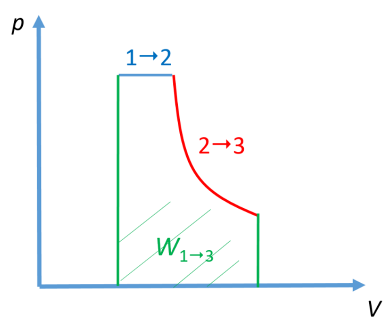
Figure 1: pV-diagram of polytropic process. Isochoric process (iii) corresponds to the vertical line from (V_1, p_1) to (V_1, p_2). The area under the curves corresponds to the energies transfer by work. Area size is consistent with previously calculated results: $W_{1,2}(n = 0) > W_{1,2}(n = 1) > W_{1,2}(n = 1.5)$.

(c) $V_1 = 0.1$ m^3, $p_1 = 3$ bar, $V_2 = 0.15$ m^3, $V_3 = 0.2$ m^3.



$$\text{Isothermal expansion (n=1):}\ W_{1,2} = p_1 V_1 \ln(V_2/V_1) = 12.16 \text{ kJ}, p_2 = p_1 V_1/V_2 = 2.000 \text{ bar}$$
$$\text{Isobaric expansion (n=0):}\ W_{2,3} = p_2(V_3 - V_2) = 10.00 \text{ kJ}$$
$$\text{Total work of two-step process:}\ W_{1,3} = W_{1,2} + W_{2,3} = 22.2 \text{ kJ}$$

(d) $V_1 = 0.1$ m^3, $p_1 = 3$ bar, $V_2 = 0.15$ m^3, $V_3 = 0.2$ m^3.



$$\text{Isobaric expansion (n=0):}\ W_{1,2} = p_1(V_2 - V_1) = 15.00 \text{ kJ}, p_2 = p_1 = 3.000 \text{ bar}$$
$$\text{Isothermal expansion (n=1):}\ W_{2,3} = p_2 V_2 \ln(V_3/V_2) = 12.946 \text{ kJ}$$
$$\text{Total work of two-step process:}\ W_{1,3} = W_{1,2} + W_{2,3} = 27.9 \text{ kJ}$$

(e) See above.

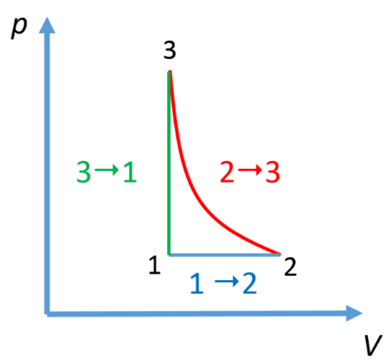
2. Energy balance in a closed system.

$$1^{st} \text{ law:} \quad \Delta U = U_2 - U_1 = Q - W$$

Q[kJ]	W [kJ]	U_1[kJ]	U_2[kJ]	ΔU[kJ]
50	-20	-20	50	70
50	20	20	50	30
-40	-60	40	60	20
-90	-90	50	50	0
5	150	2	-143	-145

3. Thermodynamic cycle with an ideal gas.

(a) pV-diagram:



Process 1–2: constant pressure $p_1 = 1.4$ bar, $V_1 = 0.028$ m^3, $W_{1,2} = 10.5$ kJ
Process 2–3: compression with pV = constant, $U_3 = U_2$
Process 3–1: constant volume, U_1 - U_3 = -26.4 kJ

(b) There is no work performed in step 3-1 ($W_{3,1} = 0$) as the volume stays constant:

$$W_{\text{cycle}} = W_{1,2} + W_{2,3} + W_{3,1}, \text{ with } W_{1,2} \text{ given.}$$

To calculate $W_{2,3}$ we need V_2. Since the first expansion is isobaric, we use

$$V_2 = V_1 + W_{1,2}/p_1 = 0.1030\,\text{m}^3$$

With $p_1 = p_2$ and $V_3 = V_1$ we get

$$W_{2,3} = p_1 V_2 \ln \frac{V_1}{V_2} = -18.78 \text{ kJ}$$

and hence

$$W_{\text{cycle}} = 10.5 \text{ kJ} - 18.78 \text{ kJ} = -8.28 \text{ kJ}$$

(c) In a thermodynamic cycle, the initial and the final states are identical. Therefore, the variation of the internal energy is zero for the complete cycle. We neglect kinetic and potential energies and apply the 1st law:

$$\begin{aligned}
U_{\text{cycle}} = Q_{\text{cycle}} - W_{\text{cycle}} &= 0 \\
Q_{1,2} + Q_{2,3} + Q_{3,1} &= W_{1,2} + W_{2,3} \\
Q_{1,2} &= W_{1,2} + W_{2,3} - Q_{2,3} - Q_{3,1} \\
&\qquad \text{with } W_{2,3} - Q_{2,3} = U_2 - U_3 = 0 \\
&\qquad\qquad \text{and } -Q_{3,1} = U_3 - U_1 = 26.4 \text{ kJ} \\
Q_{1,2} &= W_{1,2} - Q_{3,1} = 10.5 \text{ kJ} + 26.4 \text{ kJ} = 36.9 \text{ kJ}
\end{aligned}$$