EPFL

Thermodynamics and energetics I: Exercise 14 mock exam questions, Solution

Problem 1 [1.5 points]
First law for open systems applied to the valve **(0.5)**:

$$\cancelto{0}{\frac{dE}{dt}} = \dot{Q} - \cancelto{0}{\dot{W}} + \dot{m}_i\big(h_i + \frac{w_i^2}{2} + gz_i\big) - \dot{m}_e\big(h_e + \frac{w_e^2}{2} + gz_e\big)$$

Steady state **(0.25)** and zero work rate **(0.25)**

$$0 = \dot{Q} + \dot{m}_i\big(h_i + \frac{w_i^2}{2} + gz_i\big) - \dot{m}_e\big(h_e + \frac{w_e^2}{2} + gz_e\big)$$

Negligible heat transfer rate and same $\dot{m}$ **(0.25)**

$$h_i + \frac{w_i^2}{2} + gz_i = h_e + \frac{w_e^2}{2} + gz_e$$

Negligible kinetic and potential energy change **(0.25)**

$$h_i = h_e$$

No points are assigned if the answer is simply $h_i = h_e$, the conditions must be clearly specified.

Problem 2 [0.75 points]
In a closed system, air expands from V_1 to $V_2 > V_1$ through an isothermal process. Derive the work transfer in the process supposing that air behaves as:
(a) a perfect gas;

$$W_{\mathrm{PG}} = \int_{V_1}^{V_2} pdV\,\mathbf{(0.125)} = \int_{V_1}^{V_2} \frac{mRT}{V}dV\,\mathbf{(0.125)} = mRT_1\int_{V_1}^{V_2}\frac{1}{V}dV\,\mathbf{(0.125)} = \\ = mRT_1 \ln\left(\frac{V_2}{V_1}\right)\,\mathbf{(0.125)}$$

(b) an ideal gas;

$$W_{\mathrm{IG}} = W_{\mathrm{PG}} = mRT_1 \ln\left(\frac{V_2}{V_1}\right)\,\mathbf{(0.25)}$$

Problem 3 **[11.75 points]**

A cycle is executed within a closed piston-cylinder system, consisting of these processes:

1-2 Polytropic (pv^k = constant, $k = c_p/c_v$) and adiabatic compression from 100 kPa and 27 °C to 700 kPa;

2-3 Isochoric heating;

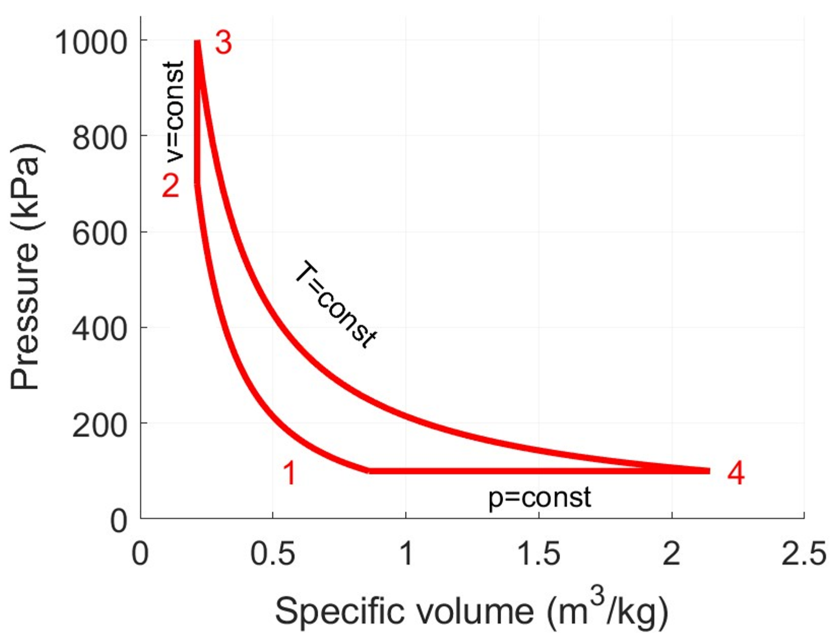
3-4 Isothermal expansion until $v_4 = 10\, v_3$;

4-1 Isobaric heat extraction to the initial state.

Assume a perfect gas with:

$c_v = 0.718$ kJ/(kg · K), $c_p = 1.005$ kJ/(kg · K), $R = 0.287$ kJ/(kg · K)

(a) Sketch the p-v diagram of the cycle. **[1 point]**



$p - v$ diagram

- Curve line 1-2 **(0.25)**
- Vertical line 2-3 **(0.25)**
- Curve line 3-4 **(0.25)**
- Horizontal line 4-1 **(0.25)**

(b) Determine the ratio of the compression work to the expansion work. [7.5]

$$r_{\text{work}} = \frac{|w_{\text{comp}}|}{w_{\text{exp}}}\ \mathbf{(0.25)} = \frac{|w_{1-2} + w_{4-1}|}{w_{3-4}}\ \mathbf{(0.25)}$$

w_{1-2} : Since the process is polytropic:

$$pv^k = \text{constant},\ k = \frac{c_p}{c_v} = 1.4\mathbf{(0.25)}$$

$$w_{1-2} = \int_1^2 p dv\ \mathbf{(0.25)} = \frac{1}{1-k} \cdot (p_2 v_2 - p_1 v_1)\ \mathbf{(0.25)}$$

$$v_1 = \frac{RT_1}{p_1}\ \mathbf{(0.25)} = \frac{0.287 \cdot 300}{100} = 0.861\ \text{m}^3/\text{kg}\ \mathbf{(0.25)}$$

$$p_1 v_1^k = p_2 v_2^k \implies v_2 = v_1 \cdot \left(\frac{p_1}{p_2}\right)^{1/k}\ \mathbf{(0.25)} = 0.214\ \ \text{m}^3/\text{kg}\mathbf{(0.25)}$$

$$T_2 = \frac{p_2 v_2}{R}\mathbf{(0.25)} = 522.95\,K\mathbf{(0.25)}$$

$$w_{1-2} = \frac{1}{1-1.4} \cdot (700 \cdot 0.214 - 100 \cdot 0.861)$$

$$w_{1-2} = -160.08\ \text{kJ/kg}\ \mathbf{(0.25)}$$

w_{2-3} : The process occurs at constant volume

$$w_{2-3} = 0\ \text{kJ/kg}\ \mathbf{(0.5)}$$

w_{3-4} : The process occurs at constant temperature

$$w_{3-4} = \int_3^4 p dv\ \mathbf{(0.25)} = RT_3 \int_3^4 \frac{dv}{v} = RT_3 \ln\left(\frac{v_4}{v_3}\right)\ \mathbf{(0.25)}$$

$$v_3 = v_2\ \mathbf{(0.25)} = 0.214\ \text{m}^3/\text{kg}\ \mathbf{(0.25)}$$

$$v_4 = 10 \cdot v_3 = 2.144\ \text{m}^3/\text{kg}\ \mathbf{(0.25)}$$

$$p_4 = p_1\ \mathbf{(0.25)} = 100\ \text{kPa}\ \mathbf{(0.25)}$$

$$T_4 = \frac{p_4 \cdot v_4}{R}\ \mathbf{(0.25)} = \frac{100 \cdot 2.144}{0.287} = 747.07\ \text{K}\ \mathbf{(0.25)}$$

$$T_3 = T_4\ \mathbf{(0.25)} = 747.07\ \text{K}\ \mathbf{(0.25)}$$

$$w_{3-4} = 0.287 \cdot 747.07 \cdot \ln\left(\frac{2.144}{0.214}\right)$$

$$w_{3-4} = 493.69\ \text{kJ/kg}\ \mathbf{(0.25)}$$

w_{4-1} : The process occurs at constant pressure

$$w_{4-1} = \int_4^1 p dv \ \textbf{(0.25)} = p_1 \int_4^1 dv = p_1(v_1 - v_4) \ \textbf{(0.25)}$$

$$w_{4-1} = 100 \cdot (0.861 - 2.144)$$

$$w_{4-1} = -128.31 \text{ kJ/kg} \ \textbf{(0.25)}$$

Finally:

$$r_{\text{work}} = 0.584 \ \textbf{(0.25)}$$

(c) Determine the thermal efficiency cycle and the heat per unit mass exchanged in every process.**[3.25]**

$$\eta_{\text{th}} = \frac{w_{\text{tot}}}{q_{\text{in}}} \ \textbf{(0.25)} = \frac{w_{1-2} + w_{2-3} + w_{3-4} + w_{4-1}}{q_{2-3} + q_{3-4}} \ \textbf{(0.25)} \qquad \text{or}$$

$$\eta_{\text{th}} = \frac{q_{1-2} + q_{2-3} + q_{3-4} + q_{4-1}}{q_{2-3} + q_{3-4}}$$

When necessary, the first law of thermodynamics for closed systems is applied.

Process 1-2

The process is adiabatic.

$$q_{1-2} = 0 \ \text{ kJ/kg} \ \textbf{(0.25)}$$

Process 2-3

$$\Delta u_{2-3} = q_{2-3} - \cancelto{0}{w_{2-3}} \ \textbf{(0.25)}$$

$$q_{2-3} = c_v(T_3 - T_2) \ \textbf{(0.25)} \rightarrow q_{2-3} = 0.718 \cdot (747.07 - 522.95)$$

$$q_{2-3} = 160.92 \text{ kJ/kg} \ \textbf{(0.25)}$$

Process 3-4

$$\cancelto{0}{\Delta u_{3-4}} \ \textbf{(0.25)} = q_{3-4} - w_{3-4} \ \textbf{(0.25)}$$

$$q_{3-4} = w_{3-4}$$

$$q_{3-4} = 493.69 \text{ kJ/kg} \ \textbf{(0.25)}$$

Process 4-1

$$\Delta u_{4-1} = q_{4-1} - w_{4-1} \ \textbf{(0.25)}$$

$$q_{4-1} = c_v(T_1 - T_4) \ \textbf{(0.25)} + w_{4-1}$$

$$q_{4-1} = 0.718 \cdot (300 - 747.07) - 128.31$$

$$q_{4-1} = -449.30 \text{ kJ/kg} \ \textbf{(0.25)}$$

Replacing the values:

$$\eta_{\text{th}} = 31.36\% \ \textbf{(0.25)}$$