

Thermodynamics and energetics I: Exercise 13

1. (a) Energy conservation for a closed system:

$$Q - W = \Delta U = U_2 - U_1.$$

The tank is rigid and well insulated, so

$$Q = 0 \text{ and } W = 0, \text{ hence } U_2 = U_1.$$

For ideal gases with constant \tilde{c}_v we have $U = \sum_i n_i u_i$ with $u_i = \tilde{c}_{v,i} T$.

Therefore

$$U_1 = n_{H_2} \tilde{c}_{v,H_2} T_{1,H_2} + n_{N_2} \tilde{c}_{v,N_2} T_{1,N_2} \text{ and } U_2 = (n_{H_2} \tilde{c}_{v,H_2} + n_{N_2} \tilde{c}_{v,N_2}) T_2.$$

Setting $U_1 = U_2$ and solving for T_2 :

$$n_{H_2} \tilde{c}_{v,H_2} T_{1,H_2} + n_{N_2} \tilde{c}_{v,N_2} T_{1,N_2} = (n_{H_2} \tilde{c}_{v,H_2} + n_{N_2} \tilde{c}_{v,N_2}) T_2$$

$$\Rightarrow T_2 = \frac{n_{H_2} \tilde{c}_{v,H_2} T_{1,H_2} + n_{N_2} \tilde{c}_{v,N_2} T_{1,N_2}}{n_{H_2} \tilde{c}_{v,H_2} + n_{N_2} \tilde{c}_{v,N_2}}.$$

$$\text{Using } n_{H_2} = \frac{m_{H_2}}{M_{H_2}}, \quad n_{N_2} = \frac{m_{N_2}}{M_{N_2}}$$

$$T_{1,H_2} = 300 \text{ K}, \quad T_{1,N_2} = 600 \text{ K}, \quad m_{H_2} = 1 \text{ kg}, \quad m_{N_2} = 4.667 \text{ kg}$$

$$M_{H_2} = 0.002018 \text{ kg/mol}, \quad M_{N_2} = 0.02801 \text{ kg/mol}$$

$$n_{H_2} = 495.5 \text{ mol}, \quad n_{N_2} = 166.6 \text{ mol}$$

$$\Rightarrow T_2 = 375.8 \text{ K}$$

You can alternatively solve this by iteratively solving the following equation with Python by using CoolProp to calculate u_2 (see Problem1.py file):

$$m_{H_2}u_{1,H_2} + m_{N_2}u_{1,N_2} = (m_{H_2}u_{2,H_2}(T_2) + m_{N_2}u_{2,N_2}(T_2))$$

In this case, the result will be: $T_2 = 377.0$ K

(b) Using the ideal gas relation $pV = nRT$.

Compute the total volume of the container as the sum of the volumes of the two compartments:

$$V = V_{1,H_2} + V_{1,N_2} = \tilde{R} \left(\frac{n_{H_2}T_{1,H_2}}{p_{1,H_2}} + \frac{n_{N_2}T_{1,N_2}}{p_{1,N_2}} \right) = 3.4456 \text{ m}^3.$$

The final pressure is then

$$p_2 = \frac{n\tilde{R}T_2}{V} = 6.005 \text{ bar.}$$

(c) $y_{H_2} = 0.7484$ $y_{N_2} = 0.2516$

$$\tilde{c}_{p,H_2} = \tilde{c}_{v,H_2} + \tilde{R} = 29.255 \text{ J/mol K}$$

$$\tilde{c}_{p,N_2} = \tilde{c}_{v,N_2} + \tilde{R} = 29.375 \text{ J/mol K}$$

$$\Delta s_{H_2} = \tilde{c}_{p,H_2} \ln \left(\frac{T_2}{T_{1,H_2}} \right) - \tilde{R} \ln \left(\frac{P_{2,H_2}}{P_{1,H_2}} \right) = \tilde{c}_{p,H_2} \ln \left(\frac{T_2}{T_{1,H_2}} \right) - \tilde{R} \ln \left(\frac{y_{H_2}P_2}{P_{1,H_2}} \right) = 8.9944 \text{ J/mol K}$$

$$\Delta s_{N_2} = \tilde{c}_{p,N_2} \ln \left(\frac{T_2}{T_{1,N_2}} \right) - \tilde{R} \ln \left(\frac{P_{2,N_2}}{P_{1,N_2}} \right) = \tilde{c}_{p,N_2} \ln \left(\frac{T_2}{T_{1,N_2}} \right) - \tilde{R} \ln \left(\frac{y_{N_2}P_2}{P_{1,N_2}} \right) = -2.2770 \text{ J/mol K}$$

$$\sigma = n_{N_2}\Delta s_{N_2} + n_{H_2}\Delta s_{H_2} = 4077.7 \text{ J/K}$$

2. Considering the mixing of the two streams (reverse project of the one shown), the temperature is the same (20°C) for all streams. Since we are dealing with ideal gases, the enthalpy of the outlet (mixed) gases will be equal to:

$$H_{out} = \dot{m}_{CH_4,out}h_{CH_4,out} + \dot{m}_{C_2H_6,out}h_{C_2H_6,out}$$

Which is the same as the sum of the inlet enthalpies, because $h_{CH_4,out} = h_{CH_4,in}$ (same for C_2H_6). This is because, for an ideal gas, h depends only on temperature (which is constant).

From the first law for open systems (steady state):

$$0 = Q - W + \dot{m}(h_{in} - h_{out})$$

$\dot{m}(h_{in} - h_{out})$ is zero, there is no work delivered by the mixing process, therefore Q must be zero (adiabatic).

For adiabatic mixing of an ideal solution, from the entropy balance for an open, steady-state system, the entropy generation during mixing can be calculated from:

$$\dot{\sigma} = \tilde{s}_{out}\dot{N} - \sum_i \dot{N}_i \tilde{s}_{in,i} = -\tilde{R} \sum_i \dot{N}_i \ln y_i = -\tilde{R}\dot{N} \sum_i y_i \ln y_i$$

The exergy destroyed during this mixing is then

$$T_0\dot{\sigma} = -\tilde{R}T_0\dot{N} \sum_i y_i \ln y_i$$

This corresponds to the work that has been "wasted" in the mixing process, since in natural mixing, no actual work is usefully extracted from the system. Theoretically, to reverse the mixing process (to perform the separation we describe in this problem), the minimum amount of work required would correspond to this destroyed exergy. The reversible (minimum) work for the corresponding separation is thus

$$\dot{W}_{\min,in} = T_0\dot{\sigma} = -\tilde{R}T_0\dot{N} \sum_i y_i \ln y_i$$

For our binary mixture:

- Component 1 (CH_4): $y_1 = 1 - y$
- Component 2 (C_2H_6): $y_2 = y$

Using the ideal gas relation for a flowing gas:

$$\dot{N} = \frac{p\dot{V}}{\tilde{R}T}$$

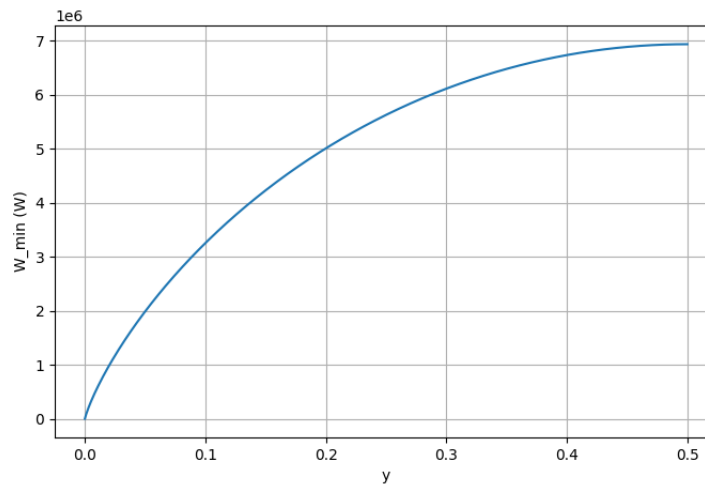
Thus,

$$\dot{W}_{\min}(y) = -\tilde{R}T_0 \frac{p\dot{V}}{\tilde{R}T_{gas}} [(1 - y) \ln(1 - y) + y \ln y] = -T_0 \frac{p\dot{V}}{T_{gas}} [(1 - y) \ln(1 - y) + y \ln y]$$

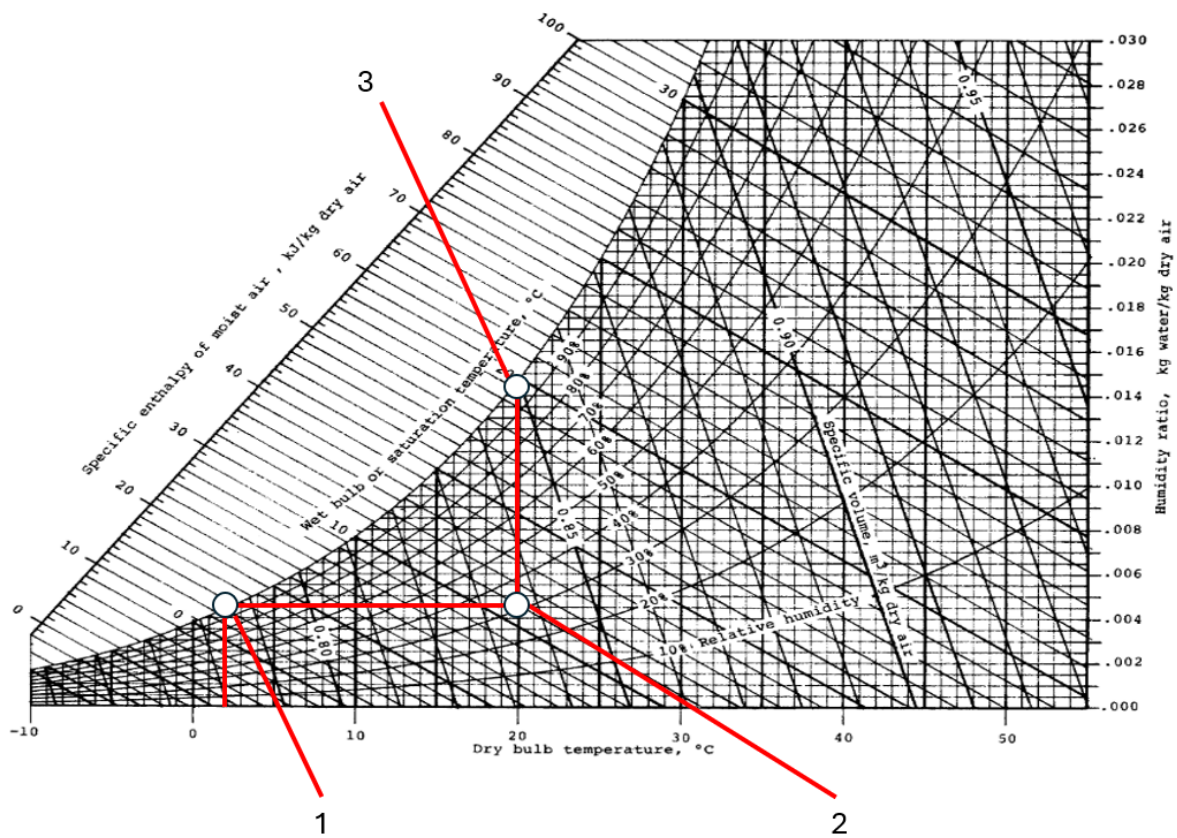
Which results in

$$\dot{W}_{\min}(y) = -1e7 [(1 - y) \ln(1 - y) + y \ln y]$$

Then, plotting \dot{W}_{\min} as a function of y :



3. (a) No. It is however possible to say that the steam content of the air in the room is such that, if it is cooled down to 2°C under constant pressure, the moist air is *at least saturated*.
- (b) State 1: using saturated moist air at 2°C , $\omega_{\min} = 0.0048$ kg water/kg dry air.
State 2: for 20°C and at this humidity ratio, $\phi_{\min}(20^{\circ}\text{C}) = 32\%$.
State 3: $\phi_{\max}(20^{\circ}\text{C}) = 100\%$, with $\omega_{\max} = 0.015$ kg water/kg dry air.



4. (a) Mass balance of water for the open system at steady state yields:

$$\dot{m}_{w,1} + \dot{m}_a(\omega_3 - \omega_4) - \dot{m}_{w,2} = 0$$

- 1 - Water inlet
- 2 - Water outlet
- 3 - Air inlet
- 4 - Air outlet

State 3:

$$\phi_3 = 0.6$$

$$p_g(T_3) = 0.01938 \text{ bar (CoolProp)}$$

$$p_{v,3} = p_g(T_3) \phi_3 = 0.6 \times 0.01938 = 0.011628 \text{ bar}$$

$$\omega_3 = \frac{M_w}{M_a} \frac{p_{v,3}}{p_3 - p_{v,3}} = 0.622 \frac{p_{v,3}}{p_3 - p_{v,3}} = 0.0073177$$

$$\dot{m}_a = \frac{\dot{V}_3}{v_{a,3}}, \quad v_{a,3} = \frac{RT_3}{M_a p_{a,3}}, \quad p_{a,3} = p_3 - p_{v,3} = 0.98837 \text{ bar}$$

$$v_{a,3} = \frac{8.314 \text{ J/molK} \cdot 290.15 \text{ K}}{0.02897 \text{ kg/mol} \cdot 98937 \text{ Pa}} = 0.84249 \text{ m}^3/\text{kg}$$

$$\dot{m}_a = \frac{1.6833 \text{ m}^3/\text{s}}{0.84249 \text{ m}^3/\text{kg}} = 1.998 \text{ kg/s}$$

State 4:

$$p_v(T_4) = 0.04246 \text{ bar (CoolProp)}$$

$$\phi_4 = 1 \Rightarrow \omega_4 = 0.622 \frac{p_v(T_4)}{p_4 - p_v(T_4)} = 0.622 \frac{0.04246 \text{ bar}}{1 \text{ bar} - 0.04246 \text{ bar}} = 0.027581$$

$$\text{Mass balance of water: } \dot{m}_{w,1} = \dot{m}_{w,2} - \dot{m}_a(\omega_3 - \omega_4) = 1.3738 \text{ kg/s}$$

(b) Option 1: Using enthalpies of humid air from psychrometric chart:

Energy balance for open system:

$$\frac{\dot{m}_{w,1} h_{w,1} + \dot{m}_a (h_{airmixture,3} - h_{airmixture,4}) + W_{fan}}{\dot{m}_{w,2}} = h_f(T_2)$$

T_2 is unknown. The other values are either known or can be obtained from the psychrometric chart:

$$\dot{m}_{w,1} = 1.3738 \text{ kg/s}$$

$$h_{w,1} = h_f(T_1) = h_f(50^\circ\text{C}) = 209.42 \text{ kJ/kg (CoolProp)}$$

$$\dot{m}_a = 1.9981 \text{ kg/s}$$

$$h_{airmixture,3} = 37 \text{ kJ/kg (from psychrometric chart)}$$

$$\omega_3 = 0.0073177$$

$$h_{airmixture,4} = 100 \text{ kJ/kg (from psychrometric chart)}$$

$$\omega_4 = 0.027581$$

$$P_{fan} = 8 \text{ kW}$$

$$\dot{m}_{w,2} = 1.333 \text{ kg/s}$$

Thus,

$$h_f(T_2) = 127.4 \text{ kJ/kg}$$

From CoolProp we find that:

$$T_2 = 30.4^\circ\text{C}$$

Option 2: Using enthalpies of dry air and water vapour from CoolProp:

Energy balance:

$$\frac{\dot{m}_{w,1}h_{w,1} + \dot{m}_a (h_{a,3} + \omega_3h_{v,3} - h_{a,4} - \omega_4h_{v,4}) + W_{fan}}{\dot{m}_{w,2}} = h_{w,2}(T_2)$$

The following properties can be obtained from CoolProp. Between parenthesis are the values for enthalpy if we change CoolProp's reference state to match the psychrometric chart's reference state (dry air at 1 atm and 0°C, saturated liquid water at 0°C), refer to the file Problem4.py. Note that the final result will not change as we look at differences (i.e. the reference state cancels out).

$$h_{a,3} = 416.4 \text{ (17.1) kJ/kg}$$

$$h_{a,4} = 429.5 \text{ (30.2) kJ/kg}$$

$$h_{v,3} = 2532.4 \text{ (2532.4) kJ/kg}$$

$$h_{v,4} = 2555.5 \text{ (2555.5) kJ/kg}$$

Thus,

$$h_{w,2}(T_2) = 124.3 \text{ kJ/kg}$$

From CoolProp we find that:

$$T_2 = 29.6^\circ\text{C}$$