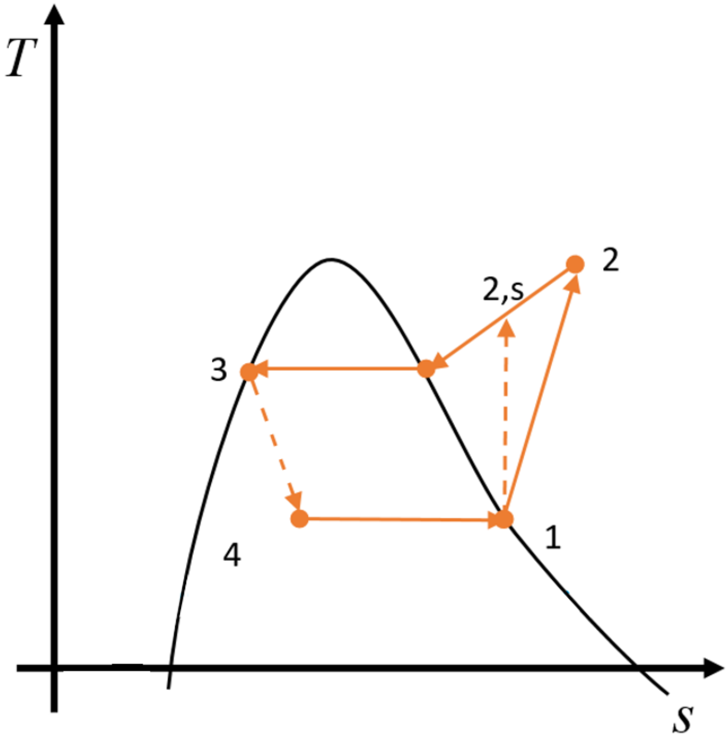
SCHOOL OF ENGINEERING
MECHANICAL ENGINEERING

Thermodynamics and energetics I: Exercise 12 - Solution

1. Refrigeration cycle. Given in the text: $T_\mathrm{C} = 15°\mathrm{C}$, $T_\mathrm{H} = T_0 = 32°\mathrm{C}$, $T_1 = 4°\mathrm{C}$, $p_2 = 1103$ kPa, $\eta_\mathrm{is,c} = 80\%$, $x_3 = 0$, $\dot{m} = 0.07$ kg/s, $p_2 = p_3$, $h_4 = h_3$, $p_4 = p_1$. Hypothesis of steady state.

   (a) T-s diagram



   (b) The enthalpy and entropy at each point **1-4**

   Point 1: From CoolProp
   $h_1 = h(T_1, x_1 = 1) = 400.92$ kJ/kg
   $s_1 = s(T_1, x_1 = 1) = 1.72\,\mathrm{kJ/(kg \cdot K)}$

   Point 2: $s_{2s} = s_1 = 1.72\,\mathrm{kJ/(kg \cdot K)}$, $p_{2s} = p_2 = 11.03\,\mathrm{bar}$.
   From CoolProp:
   $T_{2s} = 47.21°\mathrm{C}$, $h_{2s} = 425.51$ kJ/kg

$$h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_\mathrm{is,c}} = 431.66 \text{ kJ/kg}$$

   From CoolProp:
   $s_2(p_2, h_2) = 1.74$ kJ/(kg · K), $T_2(p_2, h_2) = 52.69$ °C

Point 3: From CoolProp (with the given pressure)
$T_3 = 43.07°\text{C}$
$h_3 = 261.02$ kJ/kg
$s_3 = 1.20$ kJ/(kg K)

Point 4: Valve is assumed isenthalpic $h_4 = h_3 = 261.02$ kJ/kg
Evaporator is assumed isobaric $p_4 = p_1 = 3.377$ bar
Quality from CoolProp: $x_4(h_4, p_4) = 0.28$
Also from the CoolProp: $s_4(T_4 = 4°\text{C}, x_4) = 1.22\text{kJ}/(\text{kg}\cdot\text{K})$

(c) Power required to drive the (adiabatic) compressor

First law of thermodynamics for open systems:

$$\underbrace{\frac{\mathrm{d}E_{\mathrm{CV}}}{\mathrm{d}t}}_{0} = \underbrace{\dot{Q}_{\mathrm{CV}}}_{0} - \dot{W}_{\mathrm{CV}} + \sum_i \dot{m}_{\mathrm{i}} \cdot \left(h_{\mathrm{i}} + \cancel{\frac{w_{\mathrm{i}}^2}{2} + gz_{\mathrm{i}}}\right) - \sum_e \dot{m}_{\mathrm{e}} \cdot \left(h_{\mathrm{e}} + \cancel{\frac{w_{\mathrm{e}}^2}{2} + gz_{\mathrm{e}}}\right)$$

$$\dot{W}_{12} = -\dot{m} \cdot (h_2 - h_1)\ \text{kW}$$

$$\dot{W}_{12} = -0.07 \cdot (431.66 - 400.92) = -2.15\ \text{kW}$$

(d) The coefficient of performance of the cooling cycle

$$\text{COP} = \frac{\dot{Q}_{41}}{-\dot{W}_{12}} = \frac{\dot{m}(h_1 - h_4)}{\dot{m}(h_2 - h_1)} = \frac{(400.92 - 261.02)}{(431.66 - 400.92)} = 4.55$$

(e) Exergy balance at steady state neglecting kinetic and potential energy:

$$\underbrace{\frac{\mathrm{d}B}{\mathrm{d}t}}_{=0} = \sum_j \left(1 - \frac{T_0}{T_{\mathrm{b,j}}}\right) \cdot \dot{Q}_j - \left(\dot{W} - \underbrace{p_0 \frac{\mathrm{d}V_{CV}}{\mathrm{d}t}}_{=0}\right) + \sum_i \dot{m}_i \cdot b_i - \sum_e \dot{m}_e b_e - \dot{\sigma}_{\mathrm{d}}$$

Flow exergy: $b_{f1} = (h_1 - h_0) - T_0 \cdot (s_1 - s_0) + \cancel{KE + PE}$

Exergy destruction rate in the compressor:

$$\dot{B}_{12} = \underbrace{\sum_j \left(1 - \frac{T_0}{T_{\mathrm{b,j}}}\right) \cdot \dot{Q}_j}_{=0} - \dot{W}_{12} + \dot{m} \cdot (b_{f1} - b_{f2})$$

$$b_{f1} - b_{f2} = (h_1 - h_2) - T_0 \cdot (s_1 - s_2) = (400.92 - 431.66) - 305.15 \cdot (1.72 - 1.74) = -24.64\ \text{kJ/kg}$$

$$\dot{B}_{12} = 0.4152\ \text{kW}$$

Alternatively, using the entropy balance:

$$0 = \sum_j \frac{\dot{Q}_j}{T_{\mathrm{b,j}}} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{12}$$

$$\dot{\sigma}_{12} = -\dot{m}(s_1 - s_2) = 1.4\ \mathrm{W/K}$$

$$\dot{B}_{12} = T_0\dot{\sigma}_{12} = 0.427\ \mathrm{kW}$$

For the valve:

$$\dot{B}_{34} = \underbrace{\sum_j \left(1 - \frac{T_0}{T_{\mathrm{b,j}}}\right) \cdot \dot{Q}_j}_{\text{=0,well insulated}} - \underbrace{\dot{W}_{34}}_{\text{=0,no work in the valve}} + \dot{m} \cdot (b_{f3} - b_{f4})$$

$$b_{f3} - b_{f4} = (h_3 - h_4) - T_0 \cdot (s_3 - s_4) = (261.02 - 261.02) - 305.15 \cdot (1.20 - 1.22) = 6.103\ \mathrm{kJ/kg}$$

$$\dot{B}_{34} = 0.427\ \mathrm{kW}$$

Alternatively, using the entropy balance:

$$0 = \sum_j \frac{\dot{Q}_j}{T_{\mathrm{b,j}}} + \dot{m}(s_3 - s_4) + \dot{\sigma}_{34}$$

$$\dot{\sigma}_{34} = -\dot{m}(s_3 - s_4) = 1.4\ \mathrm{W/K}$$

$$\dot{B}_{34} = T_0\dot{\sigma}_{34} = 0.427\ \mathrm{kW}$$

(f) Exergy destruction rate in the control volume around the condenser illustrated in the graphic. The temperature at which the heat transfer takes place can be considered the temperature of the surrounding: $T_b = T_H$

$$\dot{B}_{23} = \cancel{\left(1 - \frac{T_0}{T_{\mathrm{H}}}\right) \cdot \dot{Q}_{23}} - \underbrace{\dot{W}_{23}}_{0} + \dot{m} \cdot (b_{f2} - b_{f3})$$

Note: Observe that heat is transferred from the refrigerant to the environment, but exergy transfer by heat is zero. This is due to the fact that $T_0 = T_{\mathrm{H}}$ and thus $\left(1 - \frac{T_0}{T_{\mathrm{H}}}\right) = 0$. Put into words, this means that heat transfer occurring at environment temperature does not lead to transfer of exergy.

$$b_{f2} - b_{f3} = (h_2 - h_3) - T_0 \cdot (s_2 - s_3) = (431.66 - 261.02) - 305.15 \cdot (1.74 - 1.20) = 5.859\ \mathrm{kJ/kg}$$

$$\dot{B}_{23} = \dot{m} \cdot (b_{f2} - b_{f3}) = 0.41013\ \mathrm{kW}$$

Alternatively, using the entropy balance:

$$0 = \frac{\dot{Q}_{23}}{T_{\mathrm{H}}} + \dot{m}(s_2 - s_3) + \dot{\sigma}_{23}$$

$$\dot{Q}_{23} = \dot{m} \cdot (h_3 - h_2) = -11.94 \text{ kW}$$

$$\dot{\sigma}_{23} = -\frac{\dot{Q}_{23}}{T_{\mathrm{H}}} - \dot{m}(s_2 - s_3) = 1.34 \text{ W/K} \quad \text{and} \quad \dot{B}_{23} = T_0\dot{\sigma}_{23} = 0.41 \text{ kW}$$

(g) Exergy destruction rate in the control volume around the evaporator illustrated in the graphic.

The temperature at which the heat transfer takes place can be considered the temperature of the cool region: $T_b = T_C$

$$\dot{B}_{\mathrm{d},41} = \left(1 - \frac{T_0}{T_{\mathrm{C}}}\right) \cdot \dot{Q}_{41} - \underbrace{\dot{W}_{41}}_{0} + \dot{m} \cdot (b_{f4} - b_{f1})$$

$$\dot{Q}_{41} = \dot{m} \cdot (h_1 - h_4) = 0.07 \cdot (400.92 - 261.02) = 9.79 \text{ kW}$$

Note: Observe that heat is transferred from the cool region to the environment, but exergy transfer by heat is from the refrigerant to the cool region. This is due to the fact that $T_0 > T_{\mathrm{C}}$ and thus $\left(1 - \frac{T_0}{T_{\mathrm{C}}}\right) < 0$, but $\dot{Q}_{41} > 0$. Thus, $\left(1 - \frac{T_0}{T_{\mathrm{C}}}\right) \dot{Q}_{41} < 0$.

Put into words, this means that heat transfer to the system occurring at temperatures lower than environmental temperature leads to transfer of exergy by heat to the environment.

$$b_{f4} - b_{f1} = (h_4 - h_1) - T_0 \cdot (s_4 - s_1) = (261.02 - 400.92) - 305.15 \cdot (1.22 - 1.72) = 12.675 \text{ kJ/kg}$$

$$\dot{B}_{41} = 0.31 \text{ kW}$$

Alternatively, using the entropy balance:

$$0 = \frac{\dot{Q}_{41}}{T_{\mathrm{c}}} + \dot{m}(s_4 - s_1) + \dot{\sigma}_{41}$$

$$\dot{\sigma}_{41} = -\frac{\dot{Q}_{41}}{T_{\mathrm{c}}} - \dot{m}(s_4 - s_1) = 1.025 \text{ W/K}$$

$$\dot{B}_{41} = T_0 \cdot \dot{\sigma}_{41} = 0.31 \text{ kW}$$

(h) Considering the entire cycle, the exergy balance becomes

$$\sum_j \left(1-\frac{T_0}{T_\mathrm{j}}\right)\cdot \dot{Q}_j - \dot{W} + \cancel{\sum_i \dot{m}_i \cdot b_{f,i} - \sum_e \dot{m}_e b_{f,e}} - \dot{B}_\mathrm{tot} = 0$$

$$\dot{W}_{12} = \left(1-\frac{T_0}{T_\mathrm{C}}\right)\cdot \dot{Q}_{41} - \dot{B}_{12} - \dot{B}_{23} - \dot{B}_{34} - \dot{B}_{41}$$

$$-2.15\ \mathrm{kW} = \underbrace{-0.578\ \mathrm{kW} - 0.427\ \mathrm{kW} - 0.41\ \mathrm{kW} - 0.427\ \mathrm{kW} - 0.31\ \mathrm{kW}}_{=-2.15\ \mathrm{kW}}$$

2. MATLAB implementation of exercise 10, question 1.

   (a) Code

```matlab
%% Clean up
clc
clear all
close all

%% Constants
CelsiusToKelvin = 273.15;

%% Given (in bar and C)
p1 = 100;
p2 = 7;
p4 = 0.06;
T1 = 480;
T3 = 480;

p3 = p2;
p5 = p4;
p6 = p1;

eta12 = 0.8;
eta34 = 0.8;
eta56 = 0.8;

%% Point 1
% This is already done to give you an idea of how to proceed.
h1 = XSteam('h_pt',p1,T1);
s1 = XSteam('s_pt',p1,T1);

%% Process 1-2
h2is = XSteam('h_ps',p2,s1);
h2 = h1 + (h2is-h1)*eta12;
s2 = XSteam('s_ph',p2,h2);
T2 = XSteam('T_ph',p2,h2);

%% Point 3
h3 = XSteam('h_pt',p3,T3);
```

```
s3 = XSteam('s_pt',p3,T3);

%% Process 3-4
h4is = XSteam('h_ps',p4,s3);
h4 = h3 + (h4is-h3)*eta34;
s4 = XSteam('s_ph',p4,h4);
T4 = XSteam('T_ph',p4,h4);

%% Point 5
h5 = XSteam('hL_p',p5);
s5 = XSteam('sL_p',p5);
T5 = XSteam('Tsat_p',p5);
v5 = XSteam('vL_p',p5);

%% Process 5-6
h6is = h5 + v5*(p6-p5)*10^5/1000;
h6 = h5 + (h6is-h5)/eta56;

%% Thermal Efficiency
eta_th = ((h1-h2)+(h3-h4)+(h5-h6))/((h1-h6)+(h3-h2))
```

(b) Thermal efficiency increases form 0.345 to 0.372.

Note: Increasing the isentropic efficiency, point 4 is in the two-phase region but it is not affecting the calculations.