SCHOOL OF ENGINEERING
MECHANICAL ENGINEERING
LRESE - Laboratory of Renewable Energy Sciences and Engineering

EPFL

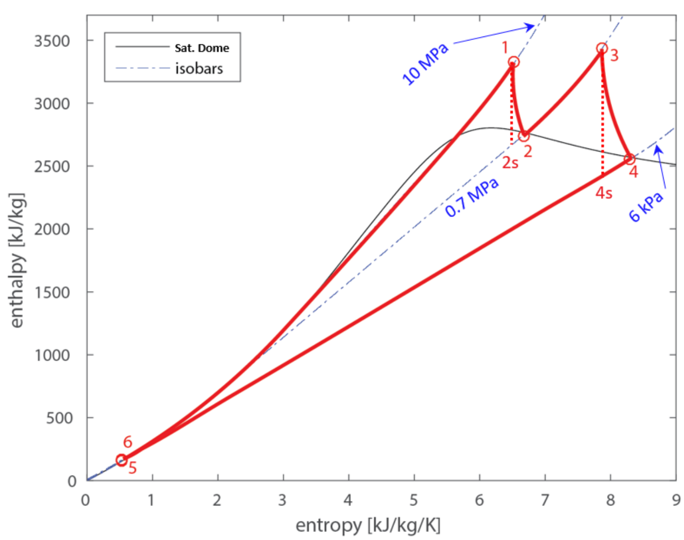
Thermodynamics and energetics I: Exercise 11

1. Rankine cycle with reheat

	p [bar]	T [°C]	x[-]
1	100	480	
2	7		
3	7	480	
4	0.06		
5	0.06		0
6	100		

	Process
1-2	$\eta_{\text{is,hpt}} = 0.8$
2-3	isobaric
3-4	$\eta_{\text{is,lpt}} = 0.8$
4-5	isobaric
5-6	$\eta_{\text{is,comp}} = 0.8$
6-1	isobaric

(a) h-s diagram



(b) specific enthalpy at each point **1 - 6**.

Point 1: From CoolProp database:

$h_1(T_1, p_1) = 3323.0$ kJ/kg

$s_1(T_1, p_1) = 6.531$ kJ/(kg · K)

Point 2: $s_{2s} = s_1 = 6.531$ kJ/(kg · K)

From CoolProp database: $h_{2s}(p_2, x_{2s}) = 2685.7$ kJ/kg

$$\eta_{\text{is,hpt}} = \frac{h_1 - h_2}{h_1 - h_{2s}} = 0.8$$
$$h_2 = h_1 - \eta_{\text{is,hpt}}(h_1 - h_{2s}) = 2813.1 \text{ kJ/kg}$$

Point 3: From CoolProp database:

$$h_3(T_3, p_3) = 3439.3kJ/kg$$
$$s_3(T_3, p_3) = 7.875 \text{ kJ/(kg} \cdot \text{K)}$$

Point 4: $s_{4s} = s_3 = 7.875$ kJ/(kg · K)

$$h_{4s}(p_4, x_{4s}) = 2426.3 \text{ kJ/kg}$$
$$\eta_{\text{is,lpt}} = \frac{h_3 - h_4}{h_3 - h_{4s}} = 0.8$$
$$h_4 = h_3 - \eta_{\text{is,lpt}}(h_3 - h_{4s}) = 2628.9 \text{ kJ/kg}$$

Point 5: From CoolProp database:

$$h_5(p_5, x_5) = 151.48 \text{ kJ/kg}$$
$$s_5(p_5, x_5) = 0.5208 \text{ kJ/(kg} \cdot \text{K)}$$

Point 6: $s_{6s} = s_5 = 0.5208$ kJ/(kg K)

From CoolProp database: $h_{6s}(p_6, s_{6s}) = 161.51$ kJ/kg.

$$\eta_{\text{is,comp}} = \frac{h_{6s} - h_5}{h_6 - h_5} = 0.8$$
$$h_6 = h_5 + \frac{h_{6s} - h_5}{\eta_{\text{is,comp}}} = 164.02 \text{ kJ/kg}$$

Summary:

	h [kJ/kg]	p [bar]
1	3323.0	100
2	2813.1	7
3	3439.3	7
4	2628.9	0.06
5	151.48	0.06
6	164.02	100

(c) Thermal efficiency has the definition: $\eta_{th} = \dfrac{\text{useful output}}{\text{input}}$

If the pump is driven by the turbine output (or the pump/compressor is on the same shaft as the turbine which is often the case in gas cycles) - means the pump work would be a part of the output. Then, thermal efficiency formula is:

Then, $\eta_{\text{th}} = \dfrac{W_{\text{hpt}} + W_{\text{lpt}} + W_{\text{pump}}}{Q_{\text{in}}} = \dfrac{(h_1 - h_2) + (h_3 - h_4) + (h_5 - h_6)}{(h_1 - h_6) + (h_3 - h_2)} = 0.345$

Note that here the pump work subtracts from the turbine output ($W_{pump} < 0$ and $W_{turbines} > 0$)

If the pump is driven by external electricity - which means the pump work would be a part of the input, thermal efficiency formula is:

$$\eta_{\text{th}} = \frac{W_{\text{hpt}} + W_{\text{lpt}}}{Q_{\text{in}} - W_{\text{pump}}} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_6) + (h_3 - h_2) - (h_5 - h_6)} = 0.348$$

Note that the pump work adds to the heat input ($W_{pump} < 0$ and $Q_{in} > 0$) which is why the negative sign in the denominator

Note: Be careful of the signs. Make sure you are subtracting from the turbine output and adding to the heat input in the two cases. If nothing is specified, both definitions are fine unless the pump/compressor is on the same shaft as the turbine in the question/diagram. Also, the two thermal efficiencies are very close as the pump work is very small compared to the heat in and turbine output

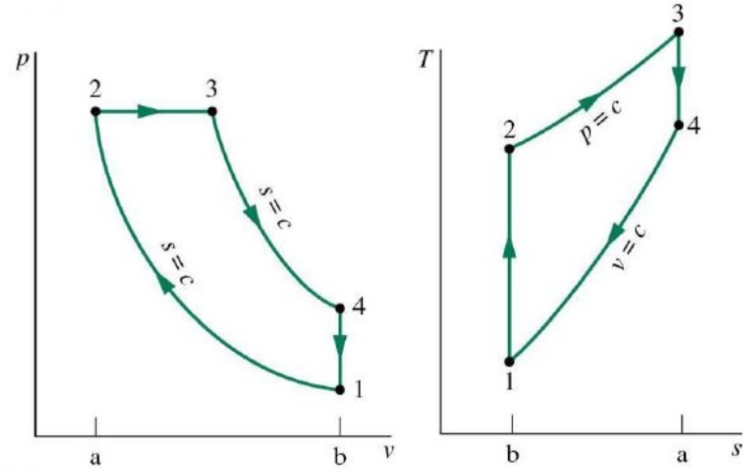
(d) Specific heat extracted during the condensation.

$$\frac{\dot{Q}_{\text{out}}}{\dot{m}} = h_5 - h_4 = -2.48 \text{ MJ/kg}$$

2. Diesel cycle

	p [bar]	T [K]
1	0.95	300
2		
3	72	2150
4		

(a) Draw the Diesel cycle in the p-v and T-s diagram and label the processes either isochoric, isobaric or isentropic.



	Process
1-2	isentropic
2-3	isobaric
3-4	isentropic
4-1	isochoric

(b) Compression factor

$$r = \frac{V_1}{V_2}$$

Process **1-2** is isentropic: $P_1 V_1^\gamma = P_2 V_2^\gamma$

Defining the volume ratio as the compression ratio $r = \frac{V_1}{V_2}$,

we get: $r^\gamma = \dfrac{P_2}{P_1}$

$P_2 = P_3$

Taking the $\frac{1}{\gamma}$ power of both sides, we obtain the compression ratio:

$$r = \left(\frac{P_3}{P_1}\right)^{\frac{1}{\gamma}}$$

Because $c_v = 2.5R, \quad c_p = c_v + R, \quad \gamma = \dfrac{c_p}{c_v} = 1.4$

$$r = \frac{V_1}{V_2} = \left(\frac{P_3}{P_1}\right)^{\frac{1}{\gamma}} = \left(\frac{7.2 \times 10^6}{95 \times 10^3}\right)^{\frac{1}{1.4}} = (75.79)^{0.714} = 22.01$$

(c) Cutoff ratio

$r_c = \frac{V_3}{V_2} = \frac{RT_3p_2}{RT_2p_3} = \frac{T_3}{T_2}$

$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1}$

$T_2 = T_1 \cdot r^{\gamma-1} = 300 \cdot 22.01^{1.4-1} = 1033.16 \text{ K}$

$r_c = \frac{T_3}{T_2} == \frac{2150}{1033.16} = 2.08$

(d) Thermal efficiency of the cycle.

$\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_{\text{in}}} = \frac{Q_{23} - Q_{41}}{Q_{23}} = 1 - \frac{Q_{\text{in}}}{Q_{\text{out}}}$

Here we use:

$\eta_{th} = 1 - \frac{1}{\gamma}\frac{1}{r^{\gamma-1}}\left(\frac{r_c^\gamma - 1}{r_c - 1}\right)$

$r^{\gamma-1} = 22.01^{0.4} = 3.44$

$r_c^\gamma = 2.08^{1.4} = 2.79$

Substitute the values:

$\eta_{th} = 1 - \frac{1}{3.44} \cdot \frac{2.79 - 1}{1.4 \cdot (2.08 - 1)} = 0.66$

$\eta_{th} = 66\%$

3. Solar heat exchanger

(a)

$$\eta_{\text{HX}} = \frac{\dot{Q}}{\dot{Q}_{\text{ideal}}} = \frac{\dot{Q}_{\text{solar}} - \dot{Q}_{\text{lost}}}{\dot{Q}_{\text{solar}}} = 0.5\frac{\dot{Q}_{\text{solar}}}{\dot{Q}_{\text{solar}}} = 0.5$$

(b) Flow exergy between inlet and outlet neglecting kinetic and potential energy changes:

$b_{\text{out}} - b_{\text{in}} = h_{\text{out}} - h_{\text{in}} - T_0 \cdot (s_{\text{out}} - s_{\text{in}})$

We use : $h = C_P T$

The change in enthalpy is given by:

$\Delta h = h_{\text{out}} - h_{\text{in}} = C_p(T_{\text{out}} - T_{\text{in}})$

Given:

$C_p = C_v + R = 2.5R + R = 3.5R$

$\Delta h = 3.5R \cdot (T_{\text{out}} - T_{\text{in}})$

$$\underbrace{\frac{\mathrm{d}E_{\text{CV}}}{\mathrm{d}t}}_{0} = \dot{Q}_{\text{CV}} - \underbrace{\dot{W}_{\text{CV}}}_{0} + \sum \dot{m}_{\text{in}} \cdot \left(h_{\text{in}} + \cancel{\frac{w_{\text{in}}^2}{2} + gz_{\text{in}}}\right) - \sum \dot{m}_{\text{out}} \cdot \left(h_{\text{out}} + \cancel{\frac{w_{\text{out}}^2}{2} + gz_{\text{out}}}\right)$$

$\dot{Q} = \dot{m} \cdot (h_{\text{out}} - h_{\text{in}}) = 0.5 \cdot \frac{\dot{Q}_{\text{solar}}}{A} \cdot \text{A}_{\text{receiver}}$

$h_{\text{out}} = h_{\text{in}} + \frac{0.5 \cdot \frac{\dot{Q}_{\text{solar}}}{A} \cdot \text{A}_{\text{receiver}}}{\dot{m}}$

$\rho_{\text{in}} = \frac{p_{\text{in}}}{RT_{\text{in}}} = 1.245 \text{ kg/m}^3$

$\dot{m}_{\text{out}} = \dot{m}_{\text{in}} = \dot{m} = \rho_{\text{in}} \cdot \text{A}_{\text{in}} \cdot w_{\text{in}} = 0.1245 \text{ kg/s}$

$h_{\text{out}} = 300.209 \text{ kJ/kg}$

$T_{\text{out}} = T_{\text{in}} + \frac{\dot{Q}}{\dot{m} \cdot C_P} = 280 + \frac{2.5}{0.1245 \cdot 8.314/29 \cdot 3.5} = 300.01 \text{ kJ/kg}$

$\Delta s = C_p \ln \frac{T_{\text{out}}}{T_{\text{in}}} - R \ln \frac{P_{\text{out}}}{P_{\text{in}}}$

Assumption: No pressure loss

$P_{\text{out}} = P_{\text{in}} \implies \ln \frac{P_{\text{out}}}{P_{\text{in}}} = 0$

Thus: $\Delta s = C_p \ln \frac{T_{\text{out}}}{T_{\text{in}}} = (8.314/29) \cdot 3.5) \cdot \ln \frac{300.01}{280} = 0.069 kJ/(kg \cdot K)$

$\Delta B = b_{\text{out}} - b_{\text{in}} = \Delta h - T_0 \Delta s = C_p(T_{\text{out}} - T_{\text{in}}) - T_0 \Delta s = (8.314/29) \cdot 3.5 \cdot (300.01 - 280) - 280 \cdot 0.069 = 676 \text{ J/kg}$

(c) Exergy destruction on the heat exchanger.

From an exergy balance ($T_b = 300$ K):

$$\dot{B} = \left(1 - \frac{T_0}{T_b}\right) \cdot 0.5 \cdot \frac{\dot{Q}_{\text{solar}}}{A} \cdot \text{A}_{\text{receiver}} - \cancel{\dot{W}} + \dot{m}(b_{\text{in}} - b_{\text{out}}) = 80.57 \text{ W}$$

Alternatively you can use:

$$\dot{B} = T_0 \cdot \dot{\sigma} = T_0 \left(-\frac{0.5 \cdot \frac{\dot{Q}_{\text{solar}}}{A} \cdot \text{A}_{\text{receiver}}}{T_b} + \dot{m} \cdot \Delta s \right) = 80.57 \text{ W}$$

(d) Exergetic efficiency of the heat exchanger.

When calculating the exergetic efficiency we have to analyze our system. In the present case we have the following exergy flows:

Exergy available: $\left(1 - \frac{T_0}{T_b}\right) \cdot \frac{\dot{Q}_{\text{solar}}}{A} \cdot A_{receiver}$

Useful exergy: $\dot{m} \cdot (b_{\text{out}} - b_{\text{in}})$

Exergy destroyed within the system: $\dot{B}_d$

Exergy lost due to heat transfer $0.5 \cdot \left(1 - \frac{T_0}{T_b}\right) \cdot \frac{\dot{Q}_{\text{solar}}}{A} \cdot A_{receiver}$

$$\varepsilon_b = \frac{\text{useful exergy}}{\text{available exergy}} = \frac{\dot{m} \cdot (b_{\text{out}} - b_{\text{in}})}{\left(1 - \frac{T_0}{T_b}\right) \cdot \frac{\dot{Q}_{\text{solar}}}{A} \cdot \mathrm{A}_{\text{receiver}}} = 0.256$$