

## Thermodynamics and energetics I: Exercise 10, Solution

1. Given:  $p_1 = 4 \text{ bar}$ ,  $T_1 = 35^\circ\text{C}$ ,  $p_2 = 18 \text{ bar}$ ,  $\dot{V} = 1 \text{ m}^3/\text{min}$ , steady state, internally reversible and isothermal process, neglect changes in kinetic and potential energy, ideal gas.  $R_{\text{air}} = \tilde{R}/M_{\text{air}}$

(a)

First law of thermodynamics for open systems:

$$\underbrace{\frac{dE_{\text{CV}}}{dt}}_0 = \dot{Q}_{\text{CV}} - \dot{W}_{\text{CV}} + \sum_i \dot{m}_i \cdot \left( h_i + \frac{w_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \cdot \left( h_e + \frac{w_e^2}{2} + gz_e \right)$$

$$0 = \dot{Q}_{12} - \dot{W}_{12} + \dot{m}(h_1 - h_2)$$

Entropy rate balance:

$$\underbrace{\frac{dS_{\text{CV}}}{dt}}_0 = \sum_j \frac{\dot{Q}_j}{T_b} + \sum_i \dot{m}_i \cdot s_i - \sum_e \dot{m}_e \cdot s_e + \underbrace{\dot{\sigma}_{\text{CV}}}_0$$

$$0 = \frac{\dot{Q}_{12}}{T_b} + \dot{m}(s_1 - s_2)$$

**Note:** As compression is isothermal and only the working fluid is considered to be in the control volume:  $T_b = T_1 = T_2 = 35^\circ\text{C}$ .

Entropy change for an ideal gas:

$$s_2 - s_1 = \underbrace{s^0(T_2) - s^0(T_1)}_0 - R_{\text{air}} \cdot \ln \left( \frac{p_2}{p_1} \right) = -431.203 \text{ J}/(\text{kg} \cdot \text{K})$$

$$v_1 = \frac{R_{\text{air}} T_1}{p_1} = 0.2209 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{\dot{V}}{v_1} = 0.0755 \text{ kg/s}$$

$$\dot{Q}_{12} = \dot{m} T_b (s_2 - s_1) = -10.03 \text{ kW}$$

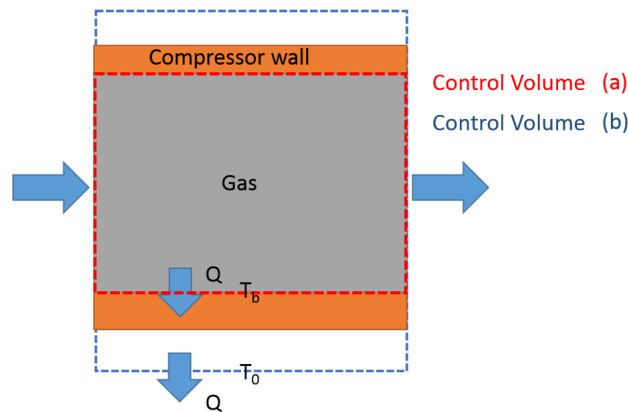
The enthalpy of an ideal gas is only a function of temperature,  $h_2 = h_1$

$$\dot{W}_{12} = \dot{Q}_{12} = -10.03 \text{ kW}$$

Alternatively:

$$\dot{W}_{12} = -\dot{m} \int_1^2 v dp = -\dot{m} \int_1^2 \frac{R_{air} T_1}{p} dp = -\dot{m} R_{air} T_1 \int_1^2 \frac{dp}{p} = \dot{m} R_{air} T_1 \ln \left( \frac{p_1}{p_2} \right) = -10.03 \text{ kW}$$

(b)



Entropy is produced in the enlarged control volume, therefore the entropy balance becomes:

$$0 = \frac{\dot{Q}_{12}}{T_0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{12}$$

Assuming that  $\dot{Q}_{12}$  and  $s_1 - s_2$  are unchanged compared to (a), it follows that:

$$\dot{\sigma}_{12} = -\frac{\dot{Q}_{12}}{T_0} + \dot{m}(s_2 - s_1) = \frac{\dot{Q}_{12}}{T_b} - \frac{\dot{Q}_{12}}{T_0} = 2.02 \text{ W/K}$$

(c) Although the process is internally reversible, there are external irreversibilities in the vicinity of the compressor due to the heat transfer at a temperature different than air temperature.

2. Given:  $p_1 = 38$  bar,  $T_1 = 650$  K,  $w_1 = 10$  m/s,  $p_2 = 1.5$  bar,  $T_2 = 310$  K,  $w_2 = 750$  m/s, steady state, perfect gas ( $c_v = 2.5R$ ), negligible potential energy.

Additionally:  $M_{O_2} = 32$  kg/kmol,  $R_{O_2} = \tilde{R}/M_{O_2} = 259.813$  J/(kg · K)

- (a) First law of thermodynamics for open systems:

$$\underbrace{\frac{dE_{CV}}{dt}}_0 = \dot{Q}_{CV} - \underbrace{\dot{W}_{CV}}_0 + \sum_i \dot{m}_i \cdot \left( h_i + \frac{w_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \cdot \left( h_e + \frac{w_e^2}{2} + gz_e \right)$$

Therefore, the specific heat transfer  $\dot{q}_{12} = \dot{Q}_{12}/\dot{m}$  is:

$$\begin{aligned} \dot{q}_{12} &= -(h_1 - h_2) - \left( \frac{w_1^2}{2} - \frac{w_2^2}{2} \right) \\ &= -c_p(T_1 - T_2) - \left( \frac{w_1^2}{2} - \frac{w_2^2}{2} \right) \\ &= -3.5 \cdot 259.813 \cdot (650 - 310) - 0.5 \cdot (10^2 - 750^2) \\ &= -27.98 \text{ kJ/kg.} \end{aligned}$$

Specific entropy change:

$$\begin{aligned} s_2 - s_1 &= c_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{p_2}{p_1} \right) \\ &= 3.5 \cdot 259.813 \cdot \ln \left( \frac{310}{650} \right) - 259.813 \cdot \ln \left( \frac{150 \times 10^3}{3.8 \times 10^6} \right) \\ &= 166.47 \text{ J/(kg · K).} \end{aligned}$$

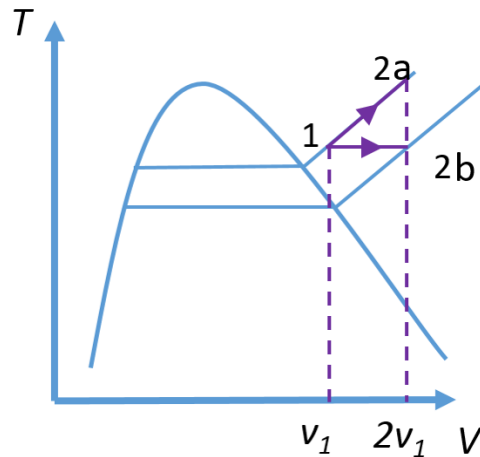
- (b) Would need to know at what temperature the heat transfer occurs.  
(c) Entropy rate balance:

$$\underbrace{\frac{dS_{CV}}{dt}}_0 = \sum_j \frac{\dot{Q}_j}{T_b} + \sum_i \dot{m}_i \cdot s_i - \sum_e \dot{m}_e \cdot s_e + \dot{\sigma}_{CV}$$

Therefore, using the results of (a), specific entropy generation is obtained as:

$$\frac{\dot{\sigma}_{12}}{\dot{m}} = -\frac{\dot{q}_{12}}{T_b} + (s_2 - s_1) = 262.89 \text{ J/(kg · K).}$$

3. Given:  $m = 1$  kg,  $T_1 = 240$  °C,  $p_1 = 20$  bar,  $T_0 = 290$  K,  $p_0 = 1$  bar; ignore kinetic and potential energy.



(a)

Exergy change neglecting kinetic and potential energy changes

$$B_2 - B_1 = (U_2 - U_1) + p_0(V_2 - V_1) - T_0(S_2 - S_1)$$

From CoolProp Data:  $u_1 = 2582.862$  kJ/kg,  $s_1 = 7.609$  kJ/(kg · K),  $v_1 = 0.191$  m<sup>3</sup>/kg

From exercise description:  $p = \text{const} = 20$  bar. and  $v_2 = 2v_1 = 0.382$  m<sup>3</sup>/kg

Using CoolProp Data:  $u_{2a} = 2611.528$  kJ/kg,  $s_{2a} = 7.696$  kJ/(kg · K)

$$B_2 - B_1 = 1 \cdot (2611.528 - 2582.862) + 100 \times (0.382 - 0.191) - 1 \cdot 290 \times (7.696 - 7.609)$$

$$B_2 - B_1 = 22.6 \text{ kJ}$$

(b)

At constant temperature

Exergy change neglecting kinetic and potential energy changes

$$B_2 - B_1 = (U_2 - U_1) + p_0(V_2 - V_1) - T_0(S_2 - S_1)$$

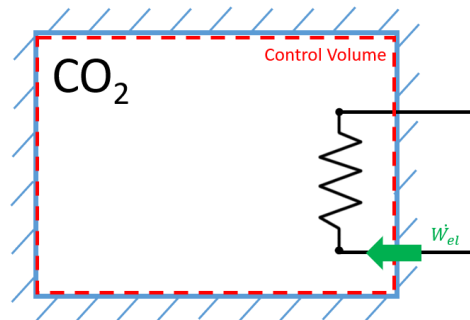
using CoolProp, the final state data is:  $u_{2b} = 2583.115$  kJ/kg,  $s_{2b} = 7.630$  kJ/kg/K,

From exercise description:  $T = \text{const.}$  and  $v_2 = 2v_1 = 0.382$  m<sup>3</sup>/kg

$$B_2 - B_1 = 1 \cdot (2583.115 - 2582.862) + 100 \times (0.382 - 0.191) - 1 \cdot 290 \times (7.63 - 7.609)$$

$$B_2 - B_1 = 13.3 \text{ kJ}$$

4. Given:  $n = 1000$  mol CO<sub>2</sub>,  $V = 7$  m<sup>3</sup> = const.,  $p_1 = 4$  bar,  $|\dot{W}_{\text{elec}}| = 12$  kW,  $t = 1$  min, resistor has negligible mass, ideal gas model, neglect kinetic and potential energy changes,  $T_0 = 20$  °C,  $p_0 = 1$  bar



(a)

$$W_{\text{elec}} = - \int_0^t |\dot{W}_{\text{elec}}|(t) dt = -|\dot{W}_{\text{elec}}| \cdot t = -720 \text{ kJ}$$

**Note:** Electricity supplied to the control volume is considered as work. The minus sign is due to the convention that when work is done to the system it is negative.

(b)

Exergy change neglecting kinetic and potential energy changes:

$$B_2 - B_1 = (U_2 - U_1) + p_0 \underbrace{(V_2 - V_1)}_0 - T_0(S_2 - S_1)$$

First principle of thermodynamics:

$$\Delta U_{12} = U_2 - U_1 = \underbrace{Q_{12}}_0 - W_{12} = -W_{\text{elec}} = 720 \text{ kJ}$$

**Note:** The work provided to the resistor is converted to heat inside the control volume ( $Q_{12} = |W_{\text{elec}}|$ ) and this heat is provided to the system, therefore it is positive. Considering a control volume without the resistor and considering a term of heat provided to the system instead of work provided is also correct. With this hypothesis,

$$\Delta U_{12} = U_2 - U_1 = Q_{12} - \underbrace{W_{12}}_0 = |W_{\text{elec}}| = 720 \text{ kJ} \quad (\text{same result of before})$$

$$T_1 = \frac{p_1 V_1}{n \tilde{R}} = 336.8 \text{ K}$$

For the perfect gas:

$$\Delta U_{12} = n c_v (T_2 - T_1) = W_{elec}$$

$$T_2 = T_1 + \frac{W_{elec}}{n c_v} \approx 361.5 \text{ K}$$

$$p_2 = \frac{n \tilde{R} T_2}{V} = 4.3 \text{ bar}$$

Entropy change for ideal gas:

$$S_2 - S_1 = n \cdot M_{CO_2} \cdot c_p \ln \left( \frac{T_2}{T_1} \right) - n \cdot M_{CO_2} \cdot \tilde{R} \ln \left( \frac{P_2}{P_1} \right)$$

$$c_p = c_v + \tilde{R}$$

$$S_2 - S_1 = 1000 \cdot 44 \cdot \left( 4.5 \cdot \frac{8.314}{44} \right) \cdot \ln \left( \frac{361.5}{336.8} \right) - 1000 \cdot 44 \cdot \frac{8.314}{44} \cdot \ln \left( \frac{4.3}{1} \right)$$

$$\Delta S \approx 2.05 \text{ kJ/K}$$

$$B_2 - B_1 = (U_2 - U_1) - T_0 (S_2 - S_1) = \Delta U_{12} - T_0 \Delta S = 119.6 \text{ kJ}$$

(c)

Closed system exergy balance:

$$B_2 - B_1 = \underbrace{\int_1^2 \left( 1 - \frac{T_0}{T_b} \right) \delta Q}_0 - \left( W - \underbrace{p_0 (V_2 - V_1)}_0 \right) - \underbrace{T_0 \sigma}_{\text{Exergy destruction}}$$

$$T_0 \sigma = -(B_2 - B_1) - W_{elec} = 600.4 \text{ kJ}$$