

Thermodynamics and energetics I: examples lecture, chapter 8

0.

State 1: from Table 3: $h_1 = 2758.0 \text{ kJ/kg}$, $s_1 = 5.7432 \text{ kJ/kg/K}$

State 2: isentropic from state 1, $s_2 = s_1 \rightarrow x_2 = \frac{s_2 - s_{f@0.008 \text{ MPa}}}{s_{g@0.008 \text{ MPa}} - s_{f@0.008 \text{ MPa}}} = \frac{5.7432 - 0.5926}{7.6361} = 0.6745$

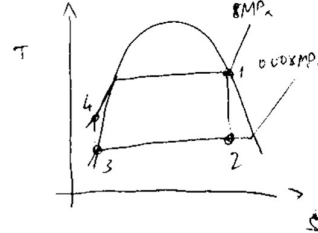
$h_2 = h_{f@0.008 \text{ MPa}} + x_2 h_{fg@0.008 \text{ MPa}} = 1794.8 \text{ kJ/kg}$

State 3: at saturated liquid: $h_3 = h_{f@0.008 \text{ MPa}} = 173.88 \text{ kJ/kg}$

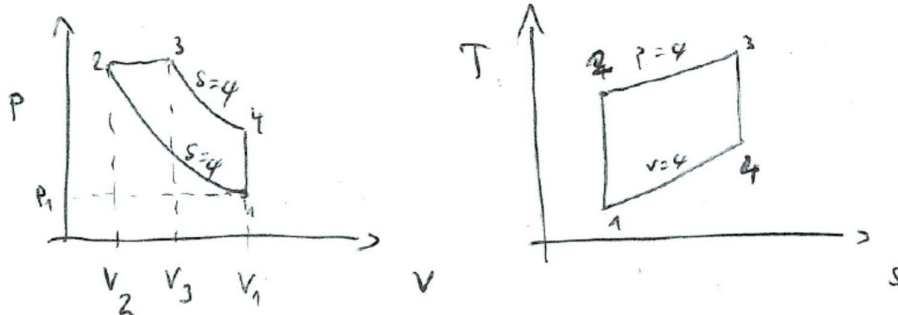
State 4: $h_4 = h_3 - \frac{\dot{W}_p}{\dot{m}} = h_3 + v_3(p_4 - p_3) = 181.9 \text{ kJ/kg}$

Efficiency: $\eta = \frac{h_1 - h_2 + h_3 - h_4}{h_1 - h_4} = 37.1\%$

Back work ratio: $bwr = \frac{|\dot{W}_p|}{\dot{W}_t} = \frac{h_4 - h_3}{h_1 - h_2} = 0.84\%$



1.



State 1: $T_1 = 300 \text{ K}$; $p_1 = 0.1 \text{ MPa}$; $v_1 = \frac{\tilde{R}T_1}{Mp_1} = 0.86 \text{ m}^3/\text{kg}$; $u_1 = 214.07 \text{ kJ/kg}$; $v_{1,r} = 621.2$;

State 2: $r = 18 = \frac{v_1}{v_2} = \frac{v_{1,r}}{v_{2,r}}$ (only for isentropic process) $\rightarrow v_2 = 0.048 \text{ m}^3/\text{kg}$; $v_{2,r} = 34.51 \rightarrow T_2 = 898 \text{ K}$

$p_2 = \frac{\tilde{R}T_2}{Mv_2} = 5.388 \text{ MPa}$; $h_2 = 930.98 \text{ kJ/kg}$;

State 3: $r_c = 2 = \frac{v_3}{v_2} \rightarrow v_3 = 0.095 \text{ m}^3/\text{kg}$; $p_3 = p_2 = 5.388 \text{ MPa}$; $T_3 = \frac{P_3 v_3}{\tilde{R} / M} = 1797 \text{ K}$

$h_3 = 1999.1 \text{ kJ/kg}$; $v_{3,r} = 3.97$;

State 4: $v_4 = v_1 = 0.86 \text{ m}^3/\text{kg}$; $\frac{v_4}{v_3} = \frac{v_{4,r}}{v_{3,r}}$ (only for isentropic process) $\rightarrow v_{4,r} = 35.73 \rightarrow T_4 = 888 \text{ K}$

$p_4 = \frac{\tilde{R}T_4}{Mv_4} = 0.3 \text{ MPa}$; $u_4 = 664.3 \text{ kJ/kg}$;

Efficiency: $\eta = 1 - \frac{u_4 - u_1}{h_3 - h_2} = 58\%$

2.

State 1: $T_1 = 300\text{K}$; $p_1 = 0.1\text{MPa}$; $v_1 = \frac{\tilde{R}T_1}{Mp_1} = 0.86\text{m}^3/\text{kg}$;

$u_1 = 214.07\text{kJ/kg}$; $v_{1,r} = 621.2$;

State 2: $r = 18 = \frac{v_1}{v_2} = \frac{v_{1,r}}{v_{2,r}}$ (only for isentropic process)

$\rightarrow v_{2,r} = 34.51 \rightarrow u_2 = 673.3\text{kJ/kg}$

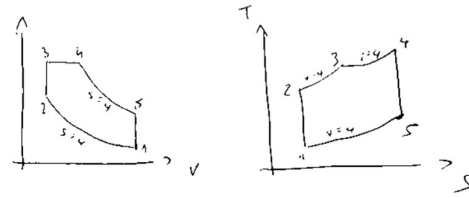
State 3: Ideal gas with $v_3 = v_2 \rightarrow T_3 = \frac{p_3}{p_2} T_2 = 1347.5\text{K} \rightarrow u_3 = 1065.8\text{kJ/kg}$; $h_3 = 1452.6\text{kJ/kg}$

State 4: Ideal gas with $p_4 = p_3 \rightarrow T_4 = \frac{v_4}{v_3} T_3 = 1617\text{K} \rightarrow v_{4,r} = 5.609$; $h_4 = 1778.3\text{kJ/kg}$

State 5: $v_{5,r} = v_{4,r} \frac{v_5}{v_4} = v_{4,r} \frac{v_5}{v_4} \frac{v_3}{v_3} = v_{4,r} \frac{v_1}{v_2} \frac{v_3}{v_4} = 84.135 \rightarrow u_5 = 475.96\text{kJ/kg}$

Efficiency: $\eta = 1 - \frac{u_5 - u_1}{u_3 - u_2 + h_4 - h_3} = 63.5\%$

Mean effective pressure: $\text{mep} = \frac{\text{net work}}{\text{displacement vol.}} = \frac{W_{\text{cyc}} / m}{v_1 - v_2} = \frac{u_3 - u_2 + h_4 - h_3 + u_1 - u_5}{v_1(1 - 1/r)} = 0.56\text{MPa}$



3.

State 1: $h_1 = 300.19\text{kJ/kg}$; $p_{1,r} = 1.386$;

State 2: $p_{2,r} = p_{1,r} \frac{p_2}{p_1} = 13.86 \rightarrow h_2 = 579.9\text{kJ/kg}$

State 3: $T_3 = 1400\text{K} \rightarrow h_3 = 1515.4\text{kJ/kg}$; $p_{3,r} = 450.5$

State 4: $p_{4,r} = p_{3,r} \frac{p_4}{p_3} = 45.05 \rightarrow h_4 = 808.5\text{kJ/kg}$

Efficiency: $\eta = \frac{\dot{W}_t / \dot{m} + \dot{W}_c / \dot{m}}{\dot{Q}_{\text{in}} / \dot{m}} = \frac{h_3 - h_4 + h_1 - h_2}{h_3 - h_2} = 45.7\%$

Back-work-ratio: $\text{bwr} = \frac{|\dot{W}_c / \dot{m}|}{\dot{W}_t / \dot{m}} = 39.6\%$

Net power: $\dot{W}_{\text{net}} = \dot{m}(\dot{W}_t / \dot{m} + \dot{W}_c / \dot{m}) = \frac{\dot{V}}{v_1} (h_3 - h_4 + h_1 - h_2) \stackrel{v_1 = \tilde{R}/M \cdot T_1 / p_1}{=} 2481\text{kW}$

4.

State 1: sat. vapor; $T_1 = -10^\circ\text{C}$; $h_1 = 241.4 \text{ kJ/kg}$; $s_1 = 0.9253 \text{ kJ/kg/K}$

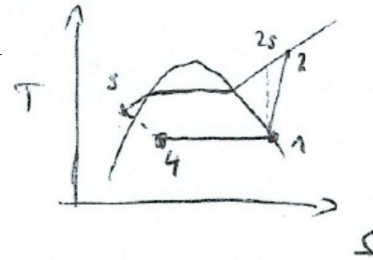
State 2s: $s_{2s} = s_1 = 0.9253 \text{ kJ/kg/K}$; $h_{2s} = 272.4 \text{ kJ/kg}$

State 2: $h_2 = \frac{h_{2s} - h_1}{\eta_{c,s}} + h_1 = 280.2 \text{ kJ/kg}$

State 3: $h_3 \approx h_1(T_{\text{sat}}) = 91.94 \text{ kJ/kg}$; $s_3 \approx s_1(T_{\text{sat}}) = 0.3396 \text{ kJ/kg/K}$

State 4: $h_4 = h_3$; $x = \frac{h_4 - h_1(T_4)}{h_g(T_4) - h_l(T_4)} = 0.2667$; $s_4 = s_1(T_4) + x(s_g(T_4) - s_l(T_4)) = 0.3557 \text{ kJ/kg/K}$

$$|\dot{W}_c| = \dot{m}(h_2 - h_1) = 3.1 \text{ kW}; \quad \dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4) = 11.95 \text{ kW} \rightarrow \text{COP}_{\text{em}} = \frac{\dot{Q}_{\text{in}}}{|\dot{W}_c|} = 3.86$$



Exergy destruction (via 2nd law): $0 = \dot{m}(s_3 - s_4) + \dot{\sigma} \rightarrow \dot{E}x_d = T_0 \dot{\sigma} = T_0 \dot{m}(s_4 - s_3) = 0.39 \text{ kW}$

Alternative: exergy destruction (via exergy balance): $0 = \dot{m}(ex_{f,3} - ex_{f,4}) - \dot{E}x_d$

$$\begin{aligned} \dot{m}(ex_{f,3} - ex_{f,4}) &= \dot{m}(u_3 - u_4 - T_0(s_3 - s_4) + p_0(v_3 - v_4) + (p_3 - p_0)v_3 - (p_4 - p_0)v_4) \\ &= \dot{m} \left(\underbrace{u_3 + p_3 v_3}_{=h_3} - \underbrace{(u_4 + p_4 v_4)}_{=h_4} - T_0(s_3 - s_4) + \underbrace{p_0(v_3 - v_4) + (-p_0)v_3 - (-p_0)v_4}_{=0} \right) \\ &\stackrel{h_3=h_4}{=} \dot{m}(-T_0(s_3 - s_4)) = \dot{E}x_d = 0.39 \text{ kW} \end{aligned}$$

5.

Single cycle:

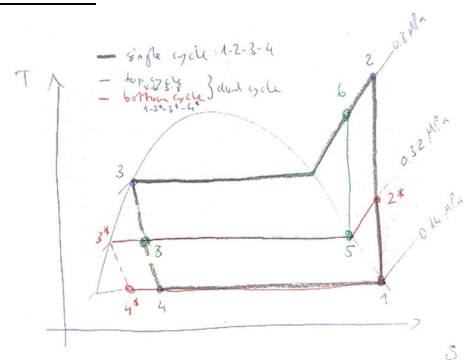
Given: Evaporator (0.14 MPa) and condenser (0.8 MPa) pressures, mass flow rate: 0.05 kg/s
Heat transfer fluid is R-134a, use tables at the end of this document

State 1: $h_1 = h_v = 239.16 \text{ kJ/kg}$; $s_1 = s_v = 0.94456 \text{ kJ/kg/K}$

State 2: $h_2 = \text{table interpolated at } s_1=s_2 = 275.39 \text{ kJ/kg}$

State 3: $h_3 = h_l = 95.74 \text{ kJ/kg}$; State 4: $h_4 = h_3 = 95.74 \text{ kJ/kg}$

$$\text{COP}_{\text{single cycle}} = \frac{h_1 - h_4}{h_2 - h_1} = 3.97$$



Dual cycle with same top condenser and bottom evaporator pressures:

Additionally given: Evaporator pressure of top cycle and condenser pressure of bottom cycle (0.32 MPa), plus mass flow rate stays 0.05 kg/s for top cycle, use tables at the end of this document

Bottom cycle (1-2*-3*-4*):

State 1: as above; State 2*: $h_2^* = 255.93 \text{ kJ/kg}$; table interpolated at $s_1 = s_2$
State 3*: $h_3^* = h_1 = 55.16 \text{ kJ/kg}$; State 4*: $h_4^* = h_3^* = 55.16 \text{ kJ/kg}$;

Top cycle (5-6-3-8):

State 5: $h_5 = h_v = 251.88 \text{ kJ/kg}$; $s_5 = s_v = 0.93 \text{ kJ/kg/K}$; ; State 6: $h_6 = 270.92 \text{ kJ/kg}$; table interpolated at $s_1 = s_2$
State 3: as above; State 8: $h_8 = h_3 = 95.47 \text{ kJ/kg}$;

Mass flow rate of bottom cycle: Energy balance over heat exchanger (2*-3* and 8-5):

$$\dot{m}_{\text{top}} (h_5 - h_8) = \dot{m}_{\text{bot}} (h_2^* - h_3^*) \rightarrow \dot{m}_{\text{bot}} = \frac{\dot{m}_{\text{top}} (h_5 - h_8)}{(h_2^* - h_3^*)} = 0.039 \text{ kg/s}$$

$$\text{COP}_{\text{dual cycle}} = \frac{\dot{m}_{\text{bot}} (h_1 - h_4^*)}{\dot{m}_{\text{top}} (h_6 - h_5) + \dot{m}_{\text{bot}} (h_2^* - h_1)} = 4.46$$

6.

State 1: $T_1 = 270 \text{ K}$; $p_1 = 1 \text{ bar}$; $v_1 = \frac{\tilde{R}T_1}{Mp_1} = 0.77 \text{ m}^3/\text{kg}$

$$\rightarrow \dot{m} = \frac{\dot{V}}{v} = 1.81 \text{ kg/s}; h_1 = 270.11 \text{ kJ/kg}; p_{1,r} = 0.959$$

State 2: $3 = \frac{p_2}{p_1} = \frac{p_{2,r}}{p_{1,r}}$ (only for isentropic process)

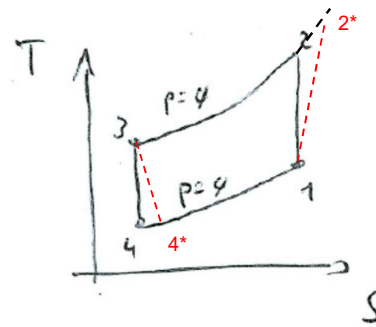
$$p_{2,r} = \frac{p_2}{p_1} p_{1,r} = 2.877 \rightarrow h_2 = 370.1 \text{ kJ/kg}$$

State 3: $T_3 = 300 \text{ K}$; $h_3 = 300.19 \text{ kJ/kg}$; $p_{3,r} = 1.386$

State 4: $p_{4,r} = \frac{p_4}{p_3} p_{3,r} = 0.462$ (only for isentropic process) $\rightarrow h_4 = 219 \text{ kJ/kg}$

$$\dot{W}_{\text{net}} = |\dot{W}_c| - \dot{W}_t = \dot{m}(h_2 - h_1) - \dot{m}(h_3 - h_4) = 33.97 \text{ kW}; \dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4) = 92.36 \text{ kW}$$

$$\text{COP}_{\text{cm}} = \frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{net}}} = 2.72$$



Compressor and turbine are not ideal, with an isentropic efficiency $\eta_{is,t} = \eta_{is,c} = 80\%$

$$\text{Compressor work: } \eta_{is,c} = \frac{(\dot{W}_c / \dot{m})_{is}}{\dot{W}_c / \dot{m}} \rightarrow \dot{W}_c = \dot{m} (\dot{W}_c / \dot{m})_{is} = \dot{m} \frac{(h_2 - h_1)}{\eta_{is,c}} = 225.9 \text{ kW}$$

$$\text{Turbine work: } \eta_{is,t} = \frac{\dot{W}_t / \dot{m}}{(\dot{W}_t / \dot{m})_{is}} \rightarrow \dot{W}_t = \dot{m} (\dot{W}_t / \dot{m})_{is} = \dot{m} (h_3 - h_4) \eta_{is,t} = 117.4 \text{ kW}$$

$$\text{Refrigeration capacity: } \dot{Q}_{in} = \dot{m} (h_1 - h_4^*) = \dot{m} (h_1 - [h_3 - \dot{W}_t / \dot{m}]) = 63.08 \text{ kW}$$

$$\text{COP}_{\text{non ideal}} = \frac{\dot{Q}_{in}}{\dot{W}_{\text{net}}} = 0.581$$

$$\text{Generalized: } \text{COP}_{\text{non ideal}} = \frac{h_1 - h_3 + \overbrace{(\dot{W}_t / \dot{m})_{is}}^{h_3 - h_4} \cdot \eta_{is,t}}{\overbrace{(\dot{W}_c / \dot{m})_{is}}^{h_2 - h_1} / \eta_{is,c} - \overbrace{(\dot{W}_t / \dot{m})_{is}}^{h_3 - h_4} \cdot \eta_{is,t}}$$

Plotted in matlab:

```
% Thermo 1 - COP dependence on
isentropic turbine and compressor
efficiency
```

```
h1=270.11; % in kJ/kg
h2=370.1; % in kJ/kg
h3=300.19; % in kJ/kg
h4=219; % in kJ/kg
eta=[0:0.05:1]; % isentropic
efficiency turbine
eta2=0.8; % isentropic efficiency
compressor
m=1.807; % mass flow rate in kg/s
```

```
Qin=m*h1-m*(h3-(h3-h4).*eta);
% in kW
Wnet=m*((h2-h1)./eta2-(h3-h4).*eta);
% in kW
```

```
%figure;
hold on; plot(eta,Qin./Wnet,'-b')
xlabel('\eta_{is,t}');
ylabel('COP');
axis square;
axis([0 1 0 3]);
```

