MATH562 – Fall 2025

Problem Set: Week 10

1. Let $Y_1, \ldots, Y_n$ be a random sample from the gamma density $\lambda^{\alpha} y^{\alpha-1} e^{-\lambda y}/\Gamma(\alpha)$, for $y > 0$ and $\alpha, \lambda > 0$.

   (a) Find the expected information matrix $\imath(\alpha, \lambda)$, in terms of the digamma function $\Psi(\alpha) = \mathrm{d}\log\Gamma(\alpha)/\mathrm{d}\alpha$ its derivative Ψ' (the trigamma function); note that $\Psi(1) \doteq -0.577$ and $\Psi'(1) \doteq 1.645$..

   (b) Find the score statistic for testing whether $\alpha = 1$ when λ is known, and give its large-sample distribution.

   (c) Find the score statistic for testing whether $\alpha = 1$ when λ is unknown, and give its large-sample distribution.

   (d) Show that the parameter $\mu = \alpha/\lambda$ is orthogonal to α. Find the score statistic corresponding to (b) in this orthogonal parametrisation. Comment.

2. If Y_1 and Y_2 are independent exponential variables with means γ^{-1} and $(\gamma\psi)^{-1}$, show that a parameter $\lambda(\psi, \gamma)$ orthogonal to ψ solves the equation $\partial\gamma/\partial\psi = -\gamma/(2\psi)$, and (without solving this PDE, unless you feel the urge) verify that a possible solution is $\lambda = \gamma\psi^{1/2}$.