

**INTRO TO DYNAMICAL SYSTEMS FALL 2025,  
PROBLEM SET 2**

- (1) Let  $T$  be a continuous map on a metric space  $X$  without isolated points. Show that if  $x$  has dense orbit under  $T$ , then so does  $T^n x$  for every  $n \geq 1$ .
- (2) Suppose that  $X$  is a metric space with at least one isolated point. Then if  $T : X \rightarrow X$  is continuous and such that for any non-empty open sets  $U, V$  there is  $n \geq 0$  such that  $T^n(U) \cap V \neq \emptyset$ , then  $X$  is a finite set and  $X = \mathcal{O}^+(x)$ .
- (3) By considering the set of  $2^n$ -th roots of unity,  $n \geq 1$ , show that the second part of the Birkhoff transitivity theorem is violated if we drop the extra assumptions on  $(X, d)$ .