

Ergodic Theory

Problem Sheet 12

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- P1.** Let (X, \mathcal{A}, μ, T) be a measure-preserving system, and let ξ and η be countable partitions of the system with finite entropy.
- (a) Show that $h(T, \xi) \leq H(\xi)$.
 - (b) Show that $h(T, \xi \vee \eta) \leq h(T, \xi) + h(T, \eta)$.
 - (c) Show that $h(T, \eta) \leq h(T, \xi) + H(\eta | \xi)$.
 - (d) Show that $h(T, \xi) = h(T, \bigvee_{i=0}^k T^{-i}\xi)$, $\forall k \geq 1$.
 - (e) For T invertible show that $h(T, \xi) = h(T^{-1}, \xi) = h(T, \bigvee_{i=-k}^k T^{-i}\xi)$, $\forall k \geq 1$.
 - (f) Show that $h(T^k) = kh(T)$, $\forall k \geq 1$.
 - (g) Let (Y, \mathcal{B}, ν, S) be a measure-preserving system and suppose that (Y, \mathcal{B}, ν, S) is a factor of (X, \mathcal{A}, μ, T) . Show that $h(S) \leq h(T)$.
- P2.** Let (X, \mathcal{B}, μ, T) be an invertible measure-preserving system, such that ξ is a generating partition of the system. Show that the entropy of T is 0.
- P3.** Given $\alpha \in \mathbb{R}$, calculate the entropy of the circle rotation $R_\alpha : \mathbb{T} \mapsto \mathbb{T}$.
- P4.** Calculate the entropy of the map $T_d : x \rightarrow dx \pmod{1}$ on \mathbb{T} , for any $d \geq 2$.