Choice models with panel data

Serial correlation and dynamic choices

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Mathematical Modeling of Behavior

Outline

Static model

Serial correlation

Dynamic model

Dynamic model with panel effects

Introduction

Panel data

- Type of data used so far: cross-sectional.
- Cross-sectional: observation of individuals at the same point in time.
- Time series: sequence of observations.
- **Panel data** is a combination of comparable time series.

Introduction

Panel data

Data collected over multiple time periods for the same sample of individuals.

Multidimensional

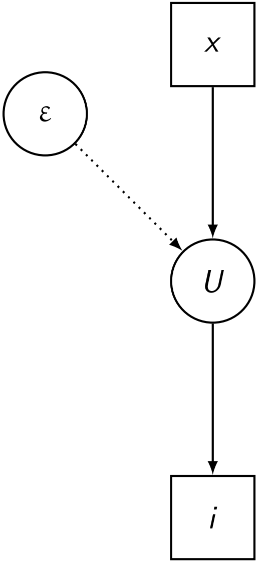
Individual	Day	Price of stock 1	Price of stock 2	Purchase
n	t	x_{1nt}	x_{2nt}	i_{int}
1	1	12.3	15.6	1
1	2	12.1	18.6	2
1	3	11.0	25.3	2
1	4	9.2	25.1	0
2	1	12.3	15.6	2
2	2	12.1	18.6	0
2	3	11.0	25.3	0
2	4	9.2	25.1	1

Introduction

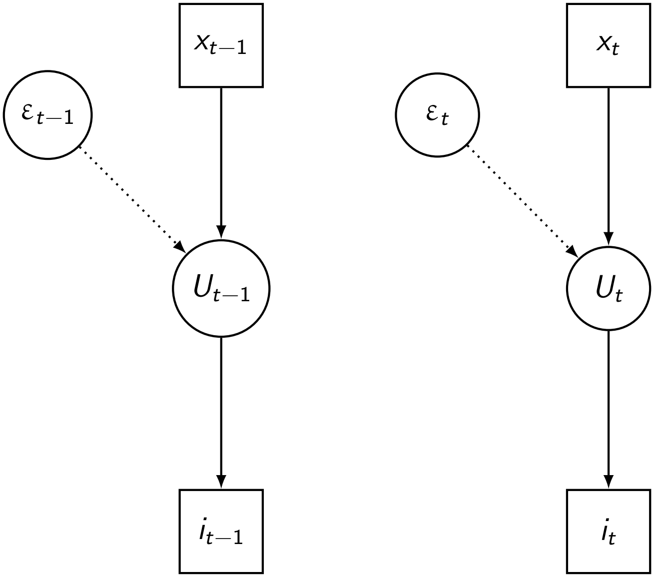
Examples of discrete panel data

- People are interviewed monthly and asked if they are working or unemployed.
- Firms are tracked yearly to determine if they have been acquired or merged.
- Consumers are interviewed yearly and asked if they have acquired a new cell phone.
- Individual's health records are reviewed annually to determine onset of new health problems.

Model: single time period



Static model



Static model

Utility

$$U_{int} = V_{int} + \varepsilon_{int}, \ \ i \in \mathcal{C}_{nt}.$$

Assumption

$$\varepsilon_{int} \text{ i.i.d. } \mathrm{EV}(0,1), \ \text{ across } i, n \text{ and } t.$$

Logit

$$P(i_{nt}) = \frac{e^{V_{int}}}{\sum_{j \in \mathcal{C}_{nt}} e^{V_{jnt}}}.$$

Static model

Estimation: contribution of individual n to the log likelihood

$$P(i_{n1}, i_{n2}, \ldots, i_{nT}) = P(i_{n1})P(i_{n2}) \cdots P(i_{nT}) = \prod_{t=1}^{T} P(i_{nt})$$

$$\ln P(i_{n1}, i_{n2}, \ldots, i_{nT}) = \ln P(i_{n1}) + \ln P(i_{n2}) + \cdots + \ln P(i_{nT}) = \sum_{t=1}^{T} \ln P(i_{nt})$$

Static model

Comments

- Views observations collected through time as supplementary cross sectional observations.
- Standard estimation procedure for cross sectional data may be used directly.
- Simple, but there are two important limitations.

Static model: limitations

Serial correlation

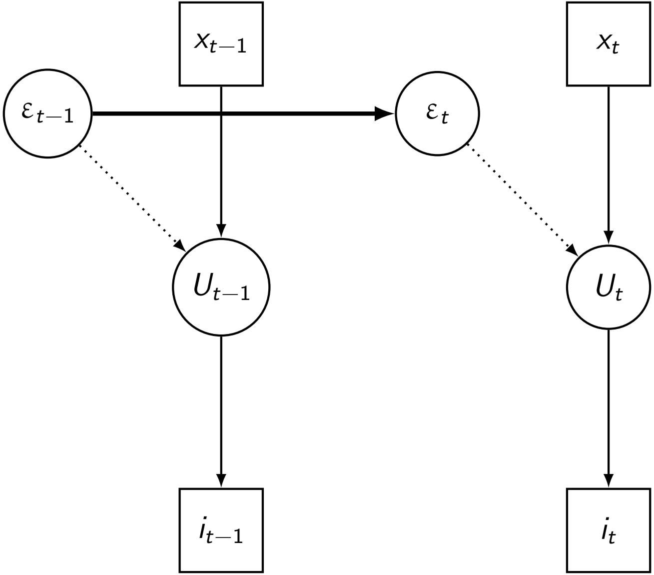
- unobserved factors persist over time,
- in particular, all factors related to individual n,
- $\varepsilon_{in(t-1)}$ cannot be assumed independent from ε_{int}.

Dynamics

- Choice in one period may depend on choices made in the past,
- e.g. learning effect, habits.

Outline

Dealing with serial correlation



Panel effects

Relax the assumption that ε_{int} are independent across t.

Assumption about the source of the correlation

- individual related unobserved factors,
- persistent over time.

The model

$$\varepsilon_{int} = \alpha_{in} + \varepsilon'_{int}$$

It is also known as

- agent effects,
- unobserved heterogeneity.

Panel effects

- Assuming that ε'_{int} are independent across t,
- we can apply the static model.
- Two versions of the model:
  - with fixed effects: α_{in} are unknown parameters to be estimated,
  - with random effects: α_{in} are distributed.

Static model with fixed effects

Utility

$$U_{int} = V_{int} + \alpha_{in} + \varepsilon'_{int}, \; i \in \mathcal{C}_{nt}.$$

Assumptions

- ε'_{int} i.i.d. EV(0, 1), across i, n and t.
- α_{in} unknown parameters to be estimated.
- α_{in} independent from ε'_{int}.

Logit

$$P(i_{nt}) = \frac{e^{V_{int}+\alpha_{in}}}{\sum_{j\in\mathcal{C}_{nt}} e^{V_{jnt}+\alpha_{jn}}}$$

Static model with fixed effects

Estimation: contribution of individual n to the log likelihood

$$P(i_{n1}, i_{n2}, \ldots, i_{nT}) = P(i_{n1}) P(i_{n2}) \cdots P(i_{nT}) = \prod_{t=1}^{T} P(i_{nt})$$

$$\ln P(i_{n1}, i_{n2}, \ldots, i_{nT}) = \ln P(i_{n1}) + \ln P(i_{n2}) + \cdots + \ln P(i_{nT}) = \sum_{t=1}^{T} \ln P(i_{nt})$$

Static model with fixed effects

Comments

- α_{in} capture permanent taste heterogeneity.
- For each n, one α_{in} must be normalized to 0.
- The α's are estimated consistently only if $T \to \infty$.
- This has an effect on the other parameters that will be inconsistently estimated.
- In practice,
  - T is usually too short,
  - the number of α parameters is usually too high,

  for the model to be consistently estimated and practical.

Static model with random effects

- Denote α_n the vector gathering all parameters α_{in}.
- Assumption: α_n is distributed with density $f(\alpha_n)$.
- For instance:

$$\alpha_n \sim N(0, \Sigma).$$

- We have a mixture of static models.
- Given α_n, the model is static, as ε'_{int} are assumed independent across t.

Static model with random effects

Utility

$$U_{int} = V_{int} + \alpha_{in} + \varepsilon'_{int}, \;\; i \in \mathcal{C}_{nt}.$$

Assumptions

- ε'_{int} i.i.d. EV(0, 1), across i, n and t.
- $\alpha_n \sim N(0, \Sigma)$, with pdf f.
- α_n independent from ε'_{int}.

Conditional choice probability

$$P(i_{nt}|\alpha_n) = \frac{e^{V_{int}+\alpha_{in}}}{\sum_{j \in \mathcal{C}_{nt}} e^{V_{jnt}+\alpha_{jn}}}$$

Static model with random effects

Contribution of individual n to the log likelihood, given α_n

$$P(i_{n1}, i_{n2}, \ldots, i_{nT} | \alpha_n) = \prod_{t=1}^{T} P(i_{nt} | \alpha_n).$$

Unconditional choice probability

$$P(i_{n1}, i_{n2}, \ldots, i_{nT}) = \int_{\alpha} \prod_{t=1}^{T} P(i_{nt} | \alpha) f(\alpha) d\alpha.$$

Static model with random effects

Estimation

- Mixture model.
- Usually requires simulation.
- Generate draws $\alpha^1, \ldots, \alpha^R$ from $f(\alpha)$.
- Approximate

$$P(i_{n1}, i_{n2}, \ldots, i_{nT}) = \int_\alpha \prod_{t=1}^{T} P(i_{nt}|\alpha) f(\alpha) d\alpha \approx \frac{1}{R} \sum_{r=1}^{R} \prod_{t=1}^{T} P(i_{nt}|\alpha^r).$$

- The product of probabilities can generate very small numbers.

$$\sum_{r=1}^{R} \prod_{t=1}^{T} P(i_{nt}|\alpha^r) = \sum_{r=1}^{R} \exp\left(\sum_{t=1}^{T} \ln P(i_{nt}|\alpha^r) \right).$$

Static model with random effects

Comments

- Parameters to be estimated: β's and Σ's
- Maximum likelihood estimation leads to consistent and efficient estimators.
- Ignoring the correlation (i.e. assuming that α_n is not present) leads to consistent but not efficient estimators (not the true likelihood function).
- Accounting for serial correlation generates the true likelihood function and, therefore, the estimates are consistent and efficient.

Relax the i.i.d. assumption

i.i.d. assumption

- ✓ Same η for all alternatives i: relaxed.
- ✓ Same η for all observations n: relaxed.
- ✓ Same μ for all alternatives i: relaxed.
- ✓ Same μ for all observations n: relaxed.
- ✓ Independence across alternatives i: relaxed.
- ▶ Independence across observations n: relaxed in this lecture.

Outline

Static model

Serial correlation

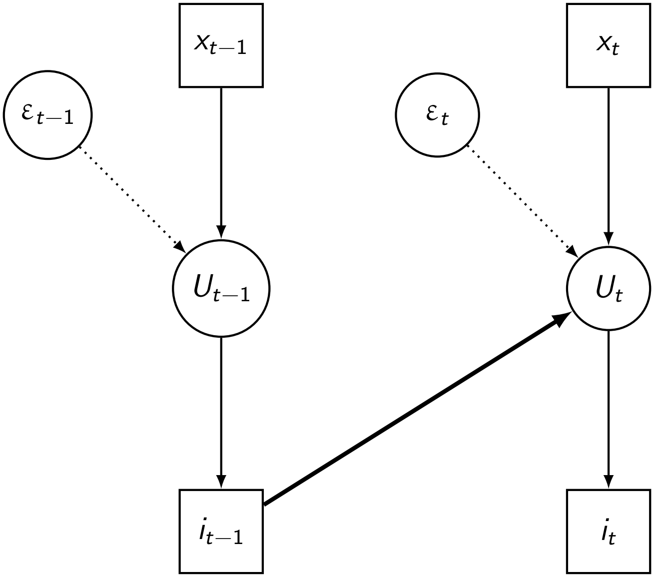
Dynamic model

Dynamic model with panel effects

Dynamics

- Choice in one period may depend on choices made in the past
- e.g. learning effects, habits.
- Simplifying assumption:
  - the utility of an alternative at time t
  - is influenced by the choice made at time $t-1$ only.
- It leads to a dynamic Markov model.

Dynamic Markov model



Notation

$$y_{jnt} = \begin{cases} 1 & \text{if } i_{nt} = j \\ 0 & \text{otherwise.} \end{cases}$$

Example

$$i_{nt} = 2 \Leftrightarrow y_{nt} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Dynamic Markov model

The model

$$U_{int} = V_{int} + \gamma y_{in(t-1)} + \varepsilon_{int}, \;\; i \in \mathcal{C}_{nt}.$$

$$y_{in(t-1)} = \begin{cases} 1 & \text{if alternative } i \text{ was chosen by } n \text{ at time } t-1 \\ 0 & \text{otherwise.} \end{cases}$$

Estimation: same as for the static model

except that observation $t = 0$ is lost

Outline

Static model

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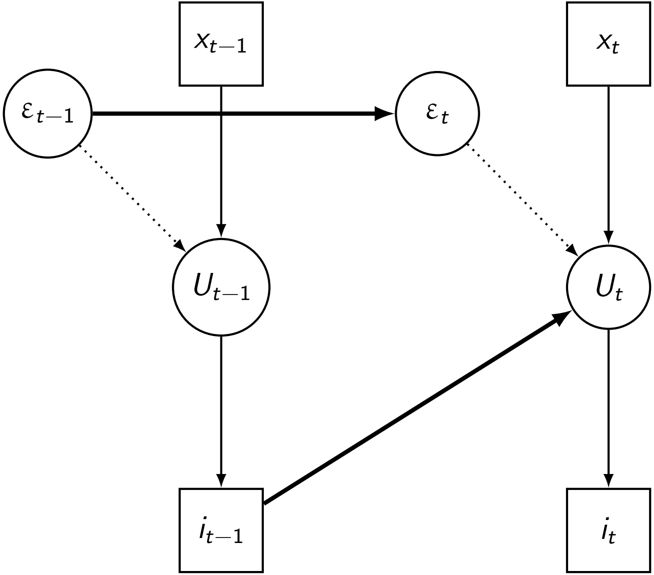
Dynamic Markov model with serial correlation



Dynamic Markov model

Extension: combine Markov with panel effects

$$U_{int} = V_{int} + \alpha_{in} + \gamma y_{in(t-1)} + \varepsilon'_{int}, \; i \in \mathcal{C}_{nt}.$$

Dynamic Markov model with fixed effects

- Similar to the static model with fixed effects.
- Similar limitations.

Dynamic Markov model with random effects

The initial condition problem.

The initial condition problem

History

- The dynamic choice process usually has an history before the sampling period.
- The only information about history is y_{n0}.
- Because of persistence of unobserved factor, y_{n0} is correlated with these unobserved factors.
- Examples: heavy smokers, car lovers, etc.

Endogeneity

- Cause: y_{n0} is correlated with α_n.
- Problem: inconsistent parameter estimates.

Dynamic Markov model with panel effects

Utility function

$$U_{int} = V_{int} + \alpha_{in} + \gamma y_{in(t-1)} + \varepsilon'_{int}, \; i \in \mathcal{C}_{nt}.$$

Contribution of individual n to the log likelihood, given i_{n0} and α_n

$$P(i_{n1}, i_{n2}, \ldots, i_{nT} | i_{n0}, \alpha_n) = \prod_{t=1}^{T} P(i_{nt} | i_{n0}, \alpha_n).$$

Dynamic Markov model with panel effects

Wooldridge's model

Assume a distributions of α, depending on the first choice.

$$f(\alpha_n | i_{n0})$$

We integrate out α_n

$$P(i_{n1}, i_{n2}, \ldots, i_{nT} | i_{n0}) = \int_\alpha \prod_{t=1}^{T} P(i_{nt} | i_{n0}, \alpha) f(\alpha | i_{n0}) d\alpha.$$

[Wooldridge, 2005]

Dynamic Markov model with random effects

- The main difference between static model with RE and dynamic model with RE is the term

$$f(\alpha | i_{n0})$$

- It captures the distribution of the panel effects, knowing the first choice.

Modeling

$$\alpha_n = a + b^T y_{n0} + c^T x_n + \xi_n, \quad \xi_n \sim N(0, \Sigma_\alpha).$$

- a, b and c are vectors and Σ_α a matrix of parameters to be estimated.
- x_n capture the entire observed history $(t = 1, \ldots, T)$ for agent n.
- This addresses the endogeneity issue.

Summary

- Panel data consist in observations of the same individuals over time.
- Static model suffers from two limitations.
- Serial correlation is addressed with the agent effect.
- Dynamic choices are captured by the Markov model.
- Initial condition problem: endogeneity.

Bibliography I

Wooldridge, J. M. (2005).
Simple solutions to the initial conditions problem in dynamic, nonlinear panel data models with unobserved heterogeneity.
Journal of applied econometrics, 20:39–54.