Choice models with latent variables

Modeling apparent irrationality

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Mathematical Modeling of Behavior

EPFL

Outline

Beyond rationality

Motivation

- Choice models are derived from a rationality assumption.
- The decision-maker is assumed to solve an optimization problem. The alternative with the highest utility is selected.
- However, there are several evidences that human beings are not necessary rational in the way assumed by random utility models.
- We first review some experiments that illustrate that (apparent) irrationality.

Example: pain lovers

Experiment by [Kahneman et al., 1993]

- Short trial: immerse one hand in water at 14° for 60 sec.
- Long trial: immerse the other hand at 14° for 60 sec, then keep the hand in the water 30 sec. longer as the temperature of the water is gradually raised to 15°.
- Which one do you dislike the less?

Example: pain lovers

Tentative modeling

Variables: time spent in cold water.

$$U_{\text{short}} = \beta_{14} 60 + \varepsilon_{\text{short}},$$
$$U_{\text{long}} = \beta_{14} 60 + \beta_{15} 30 + \varepsilon_{\text{long}},$$

where $\beta_{14}, \beta_{15} < 0$.

$$\Pr(\text{short}) = \Pr(U_{\text{short}} \geqslant U_{\text{long}}) = \Pr(\varepsilon_{\text{short}} \geqslant \beta_{15} 30 + \varepsilon_{\text{long}}).$$

- The time spent in 14° water does not matter, as it cancels out.
- We expect "short" to be chosen more often.

Example: pain lovers

Results of the experiment

- Outcome: most people prefer the long trial.
- Explanation: duration plays a small role, the peak and the final moments matter.

Example: The Economist

[Ariely, 2008]

Subscription to The Economist

Web only	@ $59
Print only	@ $125
Print and web	@ $125

Example: The Economist

[Ariely, 2008]

Subscription to The Economist

Experiment 1	Experiment 2
Web only @ $59	Web only @ $59
Print only @ $125	
Print and web @ $125	Print and web @ $125

Example: The Economist

[Ariely, 2008]

Subscription to The Economist

	Experiment 1	Experiment 2	
16	Web only @ $59	Web only @ $59	68
0	Print only @ $125		
84	Print and web @ $125	Print and web @ $125	32

The Economist: explanations

- Dominated alternative.
- According to utility maximization, should not affect the choice.
- But it affects the perception, which affects the choice.

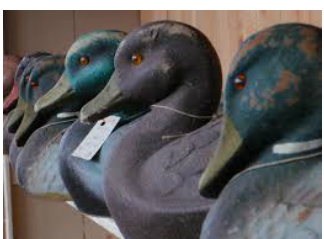
Decoy effect

Decoy

High-price, low-value product compared to other items in the choice set.

Behavior

Consumers shift their choice to more expensive items.



Applications

- Travel and tourism. [Josiam and Hobson, 1995]
- Wine lists in restaurants. [Kimes et al., 2012]
- Tobacco treatment. [Rogers et al., 2020]
- Online diamond retail. [Wu and Cosguner, ta]

Example: good or bad wine?

Choose a bottle of wine...

	Experiment 1	Experiment 2
1	McFadden red at \$10	McFadden red at \$10
2	Nappa red at \$12	Nappa red at \$12
3		McFadden special reserve pinot noir at \$60
	Most would choose 2	Most would choose 1

- Context plays a role on perceptions
- Here, perceived quality is increased

Example: live and let die

Framing of decisions [Kahneman and Tversky, 1986]

Population of 600 is threatened by a disease.
Two alternative treatments to combat the disease have been proposed.

	Experiment 1 # resp. = 152	Experiment 2 # resp. = 155	
72%	Treatment A: 200 people saved	Treatment C: 400 people die	22%
28%	Treatment B: 600 saved with prob. 1/3 0 saved with prob. 2/3	Treatment D: 0 die with prob. 1/3 600 die with prob. 2/3	78%

Example: to be free

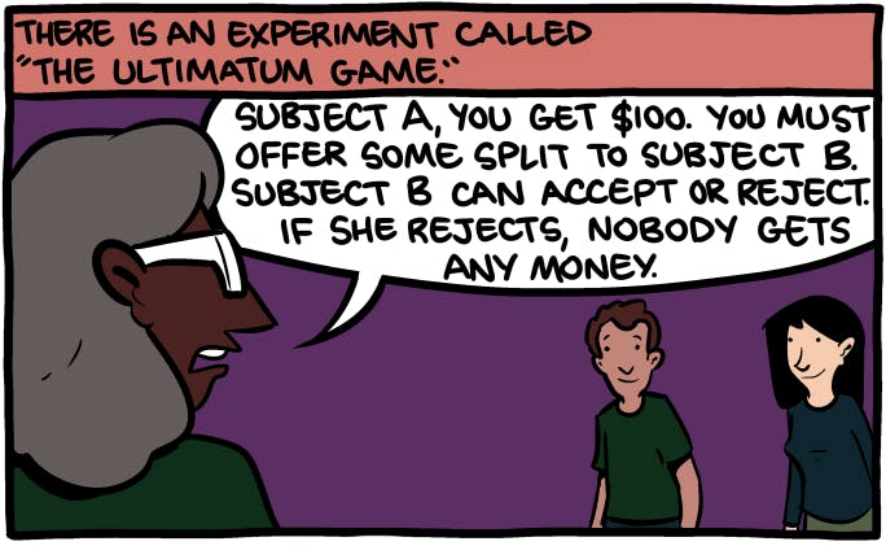
[Ariely, 2008]

Choice between a fine and a regular chocolate

	Experiment 1	Experiment 2
Lindt	\$0.15	\$0.14
Hershey	\$0.01	\$0.00
Lindt chosen	73%	31%
Hershey chosen	27%	69%

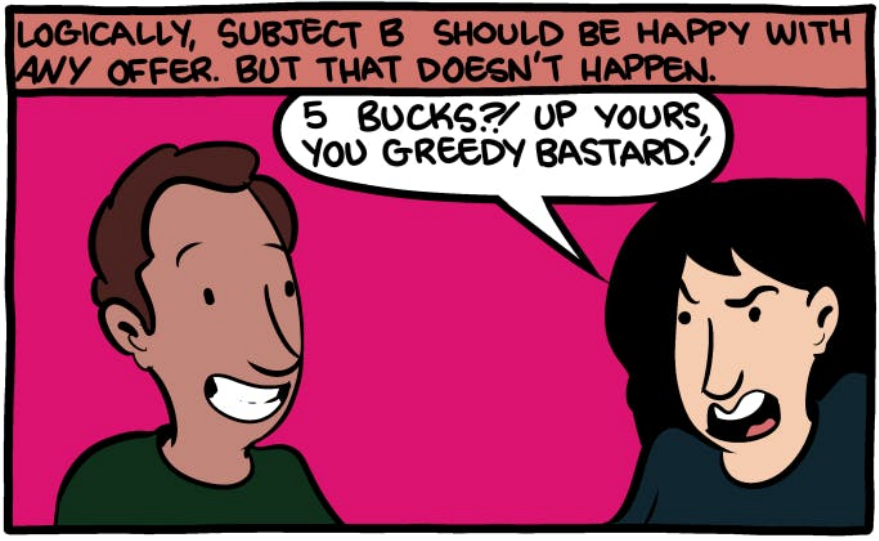
Discontinuity at 0

Ultimatum game



Source: thenib.com

Ultimatum game



Source: thenib.com

Ultimatum game

Source: thenib.com

Ultimatum game

Optimal solution

Subject B should accept any offer.

In practice

Offers of less than 30% are often rejected.

Outline

Indirect measurements of latent concepts

Perception of transportation mode

Please rate the following aspects of your commute by car:

	very poor				very good
Relaxation during the trip	1	2	3	4	5
Reliability of the arrival time	1	2	3	4	5
Flexibility of choosing the departure time	1	2	3	4	5
Ease of traveling with children and/or heavy baggage	1	2	3	4	5
Safety during the trip	1	2	3	4	5
Overall rating of the mode	1	...	...	...	10

Source: [Walker, 2001]

Indirect measurements of latent concepts

Attitude towards the environment

For each question, response on a scale: strongly agree, agree, neutral, disagree, strongly disagree, no idea.

- The price of oil should be increased to reduce congestion and pollution.
- More public transportation is necessary, even if it means additional taxes.
- Ecology is a threat to minorities and small companies.
- People and employment are more important than the environment.
- I feel concerned by the global warming.
- Decisions must be taken to reduce the greenhouse gas emission.

Indirect measurements of latent concepts

Psychometric indicators

- Usually easy to respond.
- Arbitrary units.
- Important to minimize framing.

Data

For each individual, we have

- Vector of independent variables: x.
- Choice: i.
- vector of psychometric indicators: I.

Psychometric indicators

Indicators cannot be used as explanatory variables. Why?

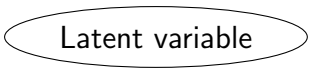
1. Measurement errors
   - Scale is arbitrary and discrete.
   - Interpretation of the scale may vary across individuals.
   - People may overreact.
   - Justification bias may produce exaggerated responses.
2. No forecasting possibility
   - No way to predict the indicators in the future

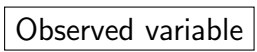
Modeling latent concepts

Latent

- **latent**: potentially existing but not presently evident or realized (from Latin: lateo = lie hidden).
- Here: not directly observed.
- Standard models are already based on a latent concept: utility.

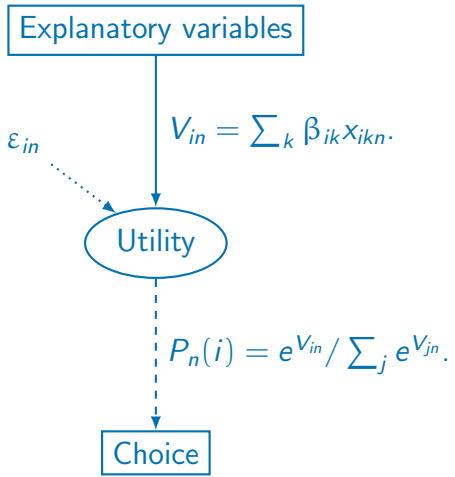
Drawing convention

- 

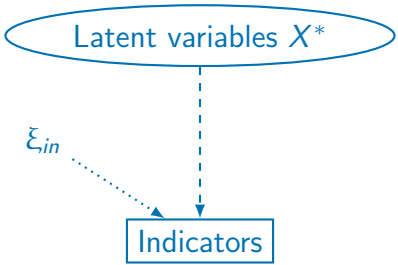
- 

- structural relation:
- measurement:
- errors:

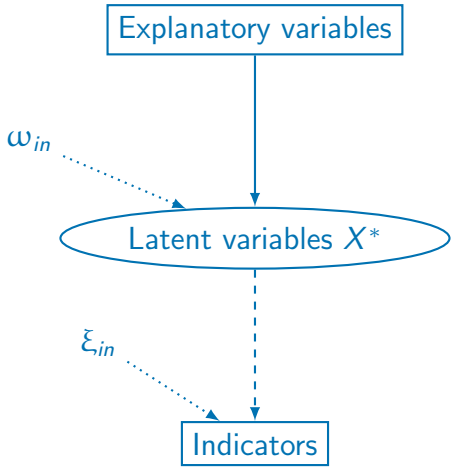
Random utility



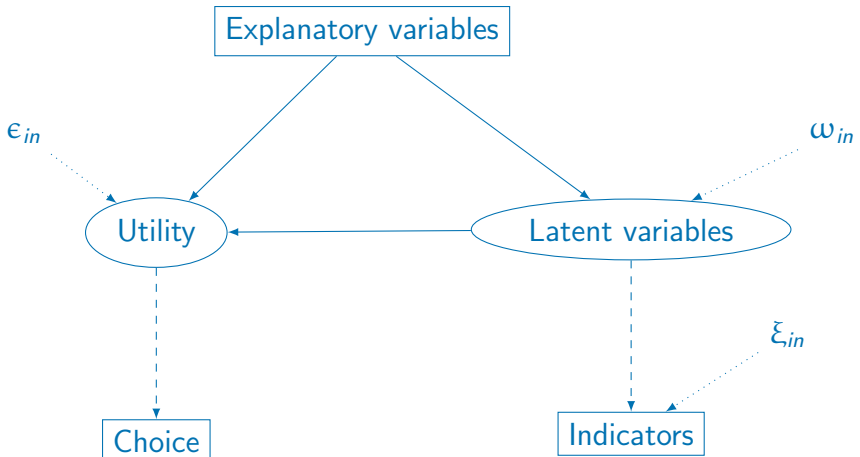
Factor analysis: measurement equation



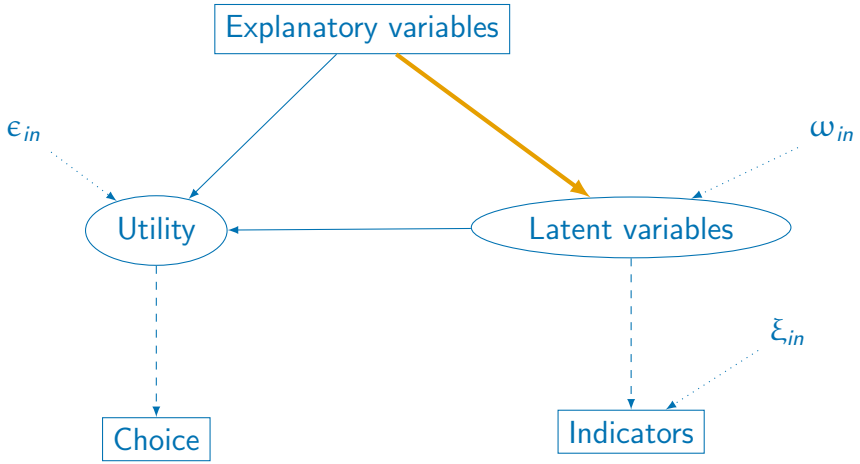
Structural equation



Choice model with latent variables

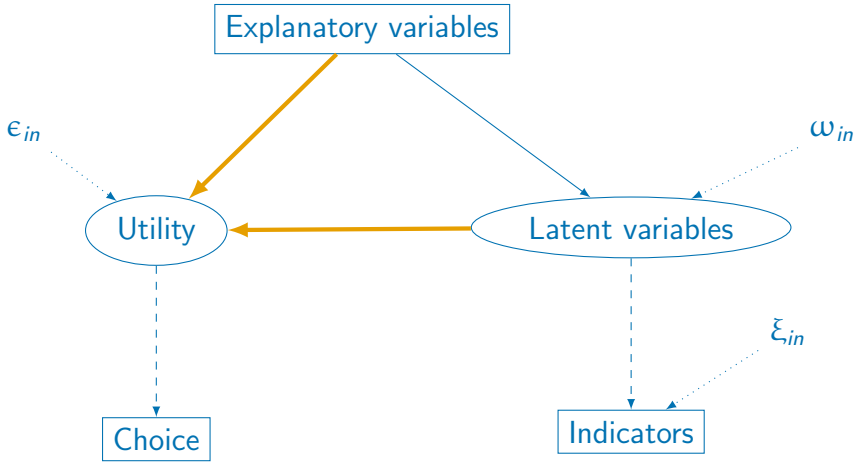


Structural equations: latent variables



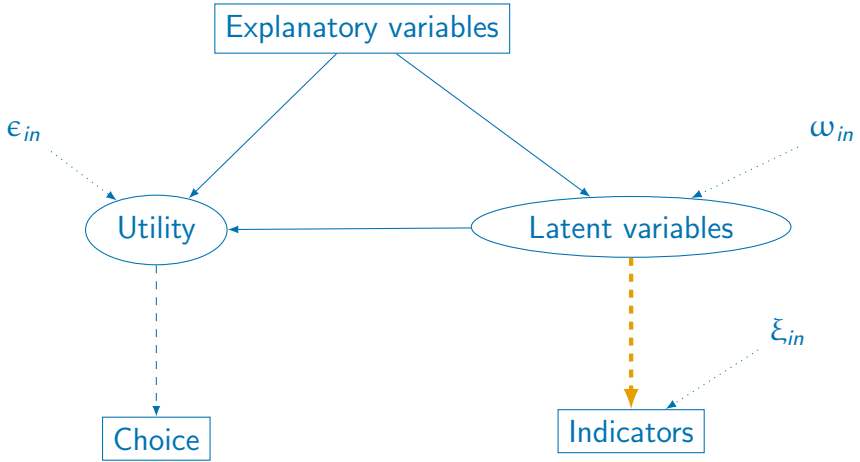
$$X_n^* = h(X_n; \lambda) + \omega_n = \sum_k \lambda_k X_{nk} + \omega_n \quad \omega_n \sim N(0, \Sigma_\omega).$$

Structural equations: utility



$$U_n = V(X_n, X_n^*; \beta) + \varepsilon_n = \sum_k \beta_k X_{nk} + \sum_k \beta_{k+K} X_{nk}^* + \varepsilon_n, \quad \varepsilon_n \sim \mathrm{EV}(0, \mu).$$

Measurement equations: indicators



$$I_i^* = m(X_n^*; \alpha) + \xi_{in} = \alpha_i + \sum_k \alpha_{ik} X_k^* + \xi_{in}, \ \xi_{in} \sim N(0, \Sigma_\xi)$$

Measurement equations: indicators

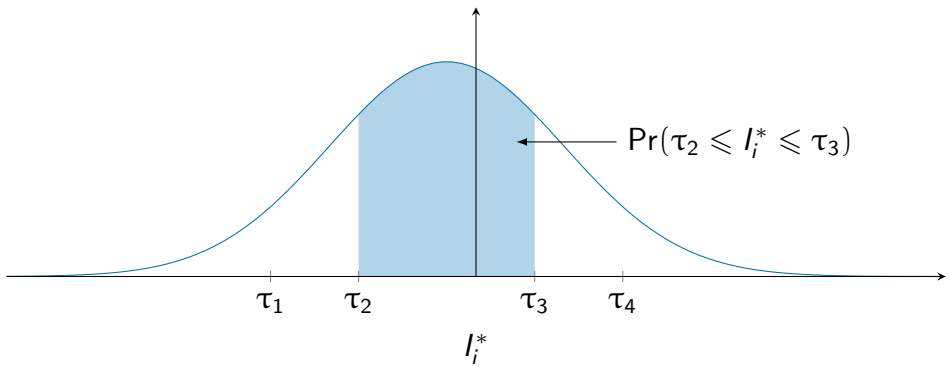
Latent continuous indicator

$$I_i^* = m(X_n^*; \alpha) + \xi_{in} = \alpha_i + \sum_k \alpha_{ik} X_k^* + \xi_{in}$$

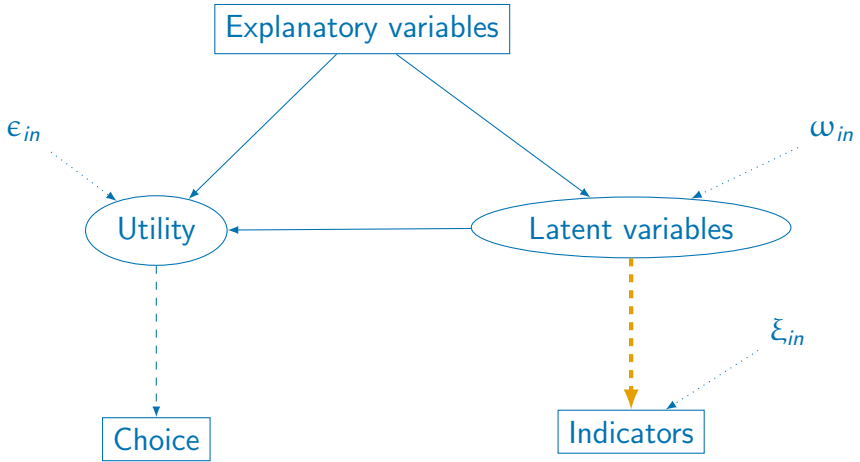
Discrete indicator

$$I_i = \begin{cases} 1 & \text{if } -\infty < I_i^* \leqslant \tau_1 \\ 2 & \text{if } \tau_1 < I_i^* \leqslant \tau_2 \\ 3 & \text{if } \tau_2 < I_i^* \leqslant \tau_3 \\ 4 & \text{if } \tau_3 < I_i^* \leqslant \tau_4 \\ 5 & \text{if } \tau_4 < I_i^* \leqslant +\infty \end{cases}$$

Measurement equations: indicators

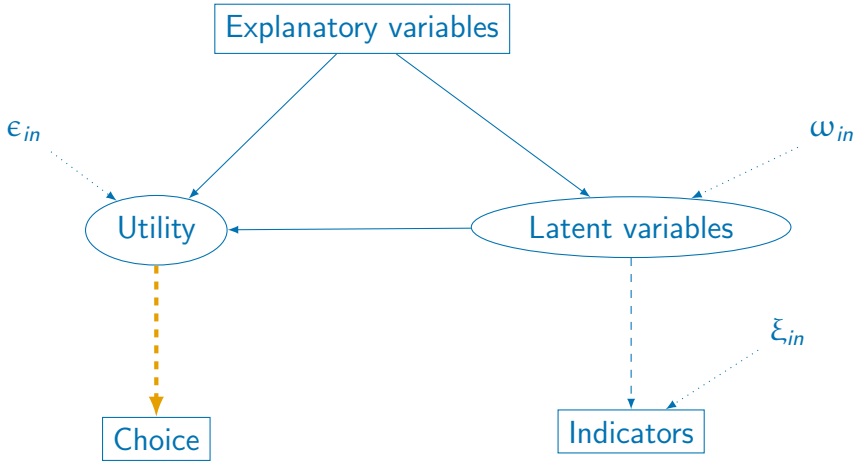


Measurement equations



$$
\begin{aligned}
P(I_n = 1) &= \Pr(I_n^* \leqslant \tau_1) \\
P(I_n = 2) &= \Pr(I_n^* \leqslant \tau_2) - \Pr(I_n^* \leqslant \tau_1) \\
&\vdots \\
P(I_n = 5) &= 1 - \Pr(I_n^* \leqslant \tau_4)
\end{aligned}
$$

Measurement equations



$$P(y_{in} = 1) = \Pr(U_{in} \geqslant U_{jn}, \forall j).$$

Estimation: likelihood

Assumption

ε_n, ω_n, and ξ_n are independent.

Contribution of one individual

Conditional on the latent variables X^*:

- Choice i_n:

$$P(i_n|x_n, X^*; \beta, \mu)$$

- Indicators I_n:

$$\Pr(I_n|x_n, X^*; \alpha, \tau, \Sigma_\xi)$$

Hybrid choice model

Integrate out X^*

$$\mathcal{L}_n(i_n, I_n|x_n; \beta, \mu, \lambda, \Sigma_\omega, \alpha, \tau, \Sigma_\xi) =$$

$$\int_{X^*} P(i_n|x_n, X^*; \beta, \mu) \Pr(I_n|x_n, X^*; \alpha, \tau, \Sigma_\xi) f(X^*|x_n; \lambda, \Sigma_\omega) dX^*.$$

Maximum likelihood estimation

$$\max_{\beta, \mu, \lambda, \Sigma_\omega, \alpha, \tau, \Sigma_\xi} \sum_n \ln \left(\mathcal{L}_n(i_n, I_n|x_n; \beta, \mu, \lambda, \Sigma_\omega, \alpha, \tau, \Sigma_\xi) \right).$$

Source: [Walker, 2001]

Outline

Case study: Optima

Effect of attitude on mode choice

- Switzerland, 2009–2010
- 1124 completed surveys
- 1906 trip chains from home to home

Attitudinal questions

Statements

Envir01	Fuel price should be increased to reduce congestion and air pollution.
Envir02	More public transportation is needed, even if taxes are set to pay the additional costs.
Envir03	Ecology disadvantages minorities and small businesses.
Mobil11	It is difficult to take the public transport when I carry bags or luggage.
Mobil14	When I take the car I know I will be on time.
Mobil16	I do not like changing the mean of transport when I am traveling.
Mobil17	If I use public transportation I have to cancel certain activities I would have done if I had taken the car.

Factor analysis

	Factor1	Factor2	Factor3
Envir01	−0.565		
Envir02	−0.407		
Envir03	0.414		
Mobil11	0.482		
Mobil14	0.477		
Mobil16	0.459		
Mobil17	0.431		
Mobil20			0.412
Mobil26			0.418
ResidCh01		0.565	
ResidCh04		0.414	
ResidCh05		0.606	
ResidCh06		0.441	
LifSty07		0.447	
LifSty10		0.403	

Car lovers

- Latent variable: car loving attitude
- Structural equation:

$$X^* = \lambda_0^s + \sum_{k=1}^{K_s-1} \lambda_k^s x_k + \sigma_s \omega^s$$

Explanatory variables

- age_65_more: the respondent is 65 or older;
- moreThanOneCar: the number of cars in the household > 1;
- moreThanOneBike: the number of bikes in the household > 1; strictly greater than 1;
- individualHouse : the type of house is individual or terraced;
- male: the respondent is a male;

Car lovers

Explanatory variables (ctd)

- haveChildren: the family is a couple or a single with children;
- haveGA: the respondent owns a season ticket;
- highEducation: the respondent has obtained a degree strictly higher than high school.
- ScaledIncome: income, in 1000 CHF;
- ContIncome_0_4: min(ScaledIncome,4)
- ContIncome_4_6: max(0,min(ScaledIncome-4,2))
- ContIncome_6_8: max(0,min(ScaledIncome-6,2))
- ContIncome_8_10: max(0,min(ScaledIncome-8,2))
- ContIncome_10_more: max(0,ScaledIncome-10)

Measurement equations

Indicators

- Likert scale (5 levels)
- 1 — strongly approve $\cdots$ 5 — strongly disapprove

Thresholds

$$I_i^* = \alpha_{0i}^m + \alpha_i^m X^* + \sigma_i^* \xi_i^*$$

$$I_i = \begin{cases} 1 & \text{if } I_i^* < \tau_1 \\ 2 & \text{if } \tau_1 \leqslant I_i^* < \tau_2 \\ 3 & \text{if } \tau_2 \leqslant I_i^* < \tau_3 \\ 4 & \text{if } \tau_3 \leqslant I_i^* < \tau_4 \\ 5 & \text{if } \tau_4 \leqslant I_i^* \end{cases}$$

Symmetry

$$\begin{aligned} \tau_1 &= -\delta_1 - \delta_2 \\ \tau_2 &= -\delta_1 \\ \tau_3 &= \delta_1 \\ \tau_4 &= \delta_1 + \delta_2 \end{aligned}$$

Measurement equations: ordered probit

Contribution to the likelihood

$$
\begin{aligned}
\Pr(I_i = j_i) &= \Pr(\tau_{i-1} \leqslant I_i^* \leqslant \tau_i) \\
&= \Pr(\tau_{i-1} \leqslant \alpha_{0i}^m + \alpha_i^m X^* + \sigma_i^* \xi_i^* \leqslant \tau_i) \\
&= \Pr\left(\frac{\tau_{i-1} - \alpha_{0i}^m - \alpha_i^m X^*}{\sigma_i^*} < \xi_i^* \leqslant \frac{\tau_i - \alpha_{0i}^m - \alpha_i^m X^*}{\sigma_i^*} \right) \\
&= \Phi\left(\frac{\tau_i - \alpha_{0i}^m - \alpha_i^m X^*}{\sigma_i^*} \right) - \Phi\left(\frac{\tau_{i-1} - \alpha_{0i}^m - \alpha_i^m X^*}{\sigma_i^*} \right).
\end{aligned}
$$

Choice model

Specification table

	Public transp.	Car	Slow modes
β_1	0	1	0
β_2	0	0	1
β_3'	Travel time (min)	0	0
β_5'		Travel time (min)	0
β_7	Waiting time (min)	0	0
β_8	Cost if HWH (CHF)	Cost if HWH (CHF)	0
β_9	Cost if not HWH (CHF)	Cost if not HWH (CHF)	0
β_{10}	0	0	Distance
β_{11}	0	CarLovers	0

Travel time coefficients

$$\beta_3' = \beta_3 e^{\beta_4 \text{CarLovers}}$$

$$\beta_5' = \beta_5 e^{\beta_6 \text{CarLovers}}$$

Value of time

Public transportation — HWH

$$\text{VOT} = 60 \frac{10}{\beta_8} \frac{\beta_3 e^{\beta_4 \text{CarLovers}}}{200} CHF/h$$

Car — HWH

$$\text{VOT} = 60 \frac{10}{\beta_8} \frac{\beta_5 e^{\beta_6 \text{CarLovers}}}{200} CHF/h$$

Model estimation

- Simultaneous estimation of all parameters
- with Biogeme.
- Important: both the choice and the indicators reveal something about the attitude.

Measurement equations: estimation results

Envir01 Fuel price should be increased to reduce congestion and air pollution.

$$I_1^* = -X^*$$

Envir02 More public transportation is needed, even if taxes are set to pay the additional costs.

$$I_2^* = 0.456 - 0.449X^* + 0.903\xi_2^*$$

Envir03 Ecology disadvantages minorities and small businesses.

$$I_3^* = -0.367 + 0.494X^* + 0.84\xi_3^*$$

Measurement equations: estimation results

Mobil11 It is difficult to take the public transport when I carry bags or luggage.

$$I_{11}^{*} = 0.414 + 0.56X^{*} + 0.869\xi_{11}^{*}$$

Mobil14 When I take the car I know I will be on time.

$$I_{14}^{*} = -0.164 + 0.574X^{*} + 0.739\xi_{14}^{*}$$

Mobil16 I do not like changing the mean of transport when I am traveling.

$$I_{16}^{*} = 0.144 + 0.51X^{*} + 0.867\xi_{16}^{*}$$

Mobil17 If I use public transportation I have to cancel certain activities I would have done if I had taken the car.

$$I_{17}^{*} = 0.12 + 0.513X^{*} + 0.847\xi_{17}^{*}$$

Measurement equations: estimation results

Thresholds for the measurement equations

$$\delta_1 = 0.318,\ \delta_2 = 0.973.$$

$$\begin{aligned}
\tau_1 &= -\delta_1 - \delta_2 &&= -1.29,\\
\tau_2 &= -\delta_1 &&= -0.318,\\
\tau_3 &= \delta_1 &&= 0.318,\\
\tau_4 &= \delta_1 + \delta_2 &&= 1.29.
\end{aligned}$$

Structural equation: estimation results

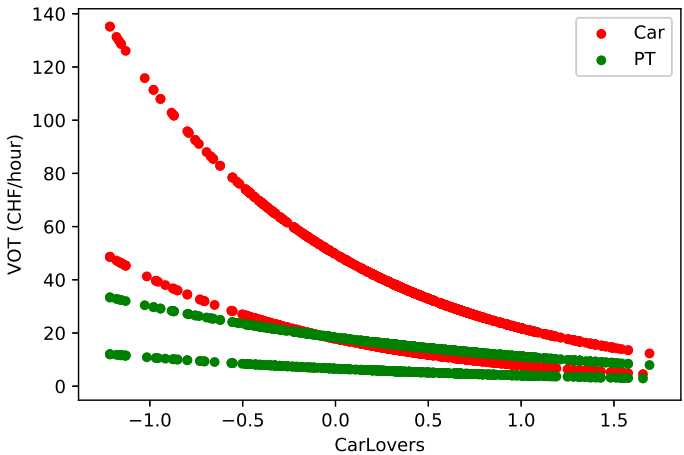
age_65_more	0.0333
moreThanOneCar	0.762
moreThanOneBike	-0.326
individualHouse	-0.117
male	0.0606
haveChildren	-0.0632
haveGA	-0.751
highEducation	-0.294
ContIncome_0_4	-0.0351
ContIncome_4_6	-0.0685
ContIncome_6_8	0.0716
ContIncome_8_10	-0.436
ContIncome_10_more	0.075
σ	0.838

Choice model

Specification table

	Public transp.	Car	Slow modes	
β_1	0	1	0	0.739
β_2	0	0	1	0.305
β_3	Travel time (ref)	0	0	-0.0126
β_4	Travel time (att)	0	0	-0.498
β_5		Travel time (ref)	0	-0.0385
β_6		Travel time (att)	0	-0.812
β_7	Waiting time	0	0	-0.0269
β_8	Cost if HWH	Cost if HWH	0	-0.137
β_9	Cost if not HWH	Cost if not HWH	0	-0.0404
β_{10}	0	0	Distance	-1.28
β_{11}	0	CarLovers		0.588

Value of time



Summary

- Evidences of (apparent) rationality.
- Rationality may be based on unobserved subjective aspects, such as attitudes or perceptions.
- They can be explicity modeled using latent variables.
- Indirectly measured using indicators.
- Integrated in choice models using mixtures.

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