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% must be nonzero. Stopping criterion based on the residual.
% [ x, k, res ] = gauss_seidel( A, b, x0, tol, kmax )
% Inputs: A      = matrix (square matrix)
%          b      = vector (right hand side of the linear system)
%          x0     = initial solution (column vector)
%          tol    = tolerance for the stopping criterion based on residual
%          kmax   = maximum number of iterations
% Outputs: x     = solution vector (column vector)
%          k     = number of iterations at convergence
%          res   = value of the norm of the residual at convergence
%
return

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- c) If possible, solve the linear systems $A_p \mathbf{x}_p = \mathbf{b}_p$, for $p = 1, 2, 3$, using the Jacobi and Gauss-Seidel methods. In all cases, define the right-hand-sides \mathbf{b}_p so that the exact solution is $\mathbf{x}_p = (1, 1, \dots, 1)^T \in \mathbb{R}^{n_p}$. Use $tol = 10^{-6}$, $kmax = 1000$, and $\mathbf{x}_p^{(0)} = (0, 0, \dots, 0)^T \in \mathbb{R}^{n_p}$. Report the norm of the errors $e_p^{(k_p)} = \|\mathbf{e}_p^{(k_p)}\| = \|\mathbf{x}_p - \mathbf{x}_p^{(k_p)}\|$, the number of iterations k_p necessary for convergence, and the norm of the residual $r_p^{(k_p)} = \|\mathbf{r}_p^{(k_p)}\|$. Discuss the results in relation to point a).
- d) Consider the family of matrices depending on a parameter $\gamma \in \mathbb{R}$, defined as

$$A_4 = \begin{bmatrix} 8 & \gamma & -2 & -1 \\ -2 & 2 & -\gamma & -3 \\ -1 & -2 & 18 & -18 \\ -1 & -3 & -7 & 25 \end{bmatrix}, \quad \text{with } \gamma \in [-10, 25].$$

Note that the matrix A_4 is non-singular for the given range of γ . Investigate graphically the (speed of) convergence of the Jacobi and Gauss-Seidel methods, when applied to solve linear systems with A_4 as matrix, as γ varies (take small steps of γ from -10 to 25). Which of the two methods would you choose for $\gamma = 0, 9$, and 15 ? Motivate the answer.