Stochastic Simulation
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Lab 14 of Thursday 18th December 2025

Exercice 1.

Consider a simple PERT network problem where

$$z(\theta) = \mathbb{E}[\max\{\theta X_1 + X_2, (1-\theta)X_3\}],$$

with $0 < \theta < 1$. The random variables X_1, X_2, and X_3 follow Erlang(2) distributions with the following properties with means 1,2, and 3, respectively. We consider the optimization problem

$$\min_{\theta \in [0,1]} z(\theta). \tag{1.1}$$

1) Verify that $z'(\theta) = \mathbb{E}[\frac{d}{d\theta} \max\{\theta X_1 + X_2, (1-\theta)X_3\}]$. Write pen and paper the estimators IPA and LR to approximate $z'(\theta)$.

2) Estimate $z'(\theta)$ using IPA and LR with 10^5 samples and for a grid $\theta \in \{0.1, 0.2, \dots, 0.9\}$. For each value of θ estimate the standard deviation of the estimators of $z'(\theta)$ and plot them as a function of θ.

3) Implement the stochastic gradient descent method to minimize $z(\theta)$. Use a step size of your choice and compute the approximated optimal $\widehat{\theta}^*$. Use both

   - SGD with a decreasing step size $\tau_k \propto \frac{1}{k}$ and a fixed sample size to estimate z' at each iteration;
   - SGD with a fixed step size τ and a geometrically increasing sample size to estimate z' at each iteration.
   - Using 0.6253 as a reference solution, plot the error $|\theta_k - \theta^*|$ as a function of the number of gradient evaluations.