EPFL - Advanced Numerical Analysis - Marco Picasso - Spring 2025

# Problem Sheet 11

November 24, 2025

## Question 1

Implement the upwind and Lax-Wendroff schemes for the transport equation :

$$\begin{cases} \dfrac{\partial u}{\partial t}(x,t) + c_0 \dfrac{\partial u}{\partial x}(x,t) = 0, \qquad 0 < x < 1, \qquad 0 < t < 0.48, \qquad c_0 = \sqrt{2}; \\ u(0,t) = 0, \qquad t > 0; \\ u(x,0) = e^{-1000(x-0.2)^2}, \qquad 0 < x < 1. \end{cases} \tag{1}$$

Compare both schemes at time 0.48 when $h = 0.01$, $\Delta t = 0.006$ and check convergence.

## Question 2

## Graded Exercise for Group 2

Consider the Lax-Wendroff scheme for the transport equation :

$$\begin{cases} \dfrac{\partial u}{\partial t}(x,t) + c_0 \dfrac{\partial u}{\partial x}(x,t) = 0, \quad x \in \mathbb{R},\ t > 0,\ c_0 > 0; \\ u(x,0) = e^{imx} \qquad\qquad\qquad\qquad\qquad\qquad x \in \mathbb{R},\ m \in \mathbb{N}^*. \end{cases} \tag{2}$$

**(a)** Check that $u_j^n = u_j^0 \gamma^n$, with $\gamma = 1 - \alpha^2(1 - \cos(mh)) - i\alpha \sin(mh)$, $\alpha = \dfrac{c_0 \tau}{h}$.

**(b)** Check that $|\gamma|^2 = 1 + \alpha^2(\alpha^2 - 1)(1 - \cos(mh))^2$, so that $|\gamma| \leqslant 1$ when $\alpha \leqslant 1$.