Exercise Set #8

Combinatorial Number Theory (2025)

E1. Show that Rado's theorem implies van der Waerden's theorem.

E2. Show that for any finite coloring of $\mathbb{N}$ there is one color class that contains solutions to all partition regular systems of linear equations.
Hint: Show first that if $\mathbf{A} \cdot \mathbf{x} = \mathbf{0}$ and $\mathbf{B} \cdot \mathbf{y} = \mathbf{0}$ are partition regular, then

$$\begin{pmatrix} \mathbf{A} & 0 \\ 0 & \mathbf{B} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = \mathbf{0}$$

is partition regular.

E3. Given a set $A \subset \mathbb{N}$, we let $\mathrm{FS}(A)$ denote the set of all finite sums of A, that is,

$$\mathrm{FS}(A) = \left\{ \sum_{x \in F} x : \emptyset \neq F \subset A \text{ finite} \right\}.$$

Folkman's theorem: For any coloring of $\mathbb{N}$ and any $m \in \mathbb{N}$, there is a set $A \subset \mathbb{N}$ with $|A| = m$, such that $\mathrm{FS}(A)$ is monochromatic.

Show that Rado's theorem implies Folkman's theorem.