Exercise Set #6

Combinatorial Number Theory (2025)

E1. Provide a proof for the following properties that are mentioned in the lecture notes without a proof.

(i) $(A \cap B) - q = (A - q) \cap (B - q)$;

(ii) $(A \cup B) - q = (A - q) \cup (B - q)$;

(iii) $A^c - q = (A - q)^c$;

(iv) $A \subset B \implies A - q \subseteq B - q$.

E2. Let $(x_n)_{n\in\mathbb{N}}$ be a sequence in a compact Hausdorff space X. Recall that $p\text{-}\lim_{n\in\mathbb{N}} x_n$ is defined by

$$x = p\text{-}\lim_{n\in\mathbb{N}} x_n \iff \forall\, U \subseteq X \text{ open neighborhood of } x, \{n \in \mathbb{N} : x_n \in U\} \in p.$$

Show that for any $p, q \in \beta\mathbb{N}$,

$$(p+q)\text{-}\lim_{n\in\mathbb{N}} x_n = p\text{-}\lim_{n\in\mathbb{N}} \big(q\text{-}\lim_{m\in\mathbb{N}} x_{n+m}\big).$$

In particular, if p is idempotent, then

$$p\text{-}\lim_{n\in\mathbb{N}} x_n = p\text{-}\lim_{n\in\mathbb{N}} \big(p\text{-}\lim_{m\in\mathbb{N}} x_{n+m}\big).$$

E3. If A is an IP-set then for every $m \in \mathbb{N}$ the set $\{n \in A : m \mid n\}$ is an IP-set.