# Exercise Set #4

## Combinatorial Number Theory (2025)

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**E1.** For any finite coloring of $\mathbb{N}$ and any $k \in \mathbb{N}$, show that there is a monochromatic $k$-term geometric progression (i.e., a monochromatic configuration of the form $\{a, ar, \ldots, ar^{k-1}\}$ for some $a, r \in \mathbb{N}$, $r \geq 2$).

**E2.** Give an example of a finite coloring of $\mathbb{N}$ without any monochromatic infinite arithmetic progression.

**E3.** Show that in any finite coloring of $\mathbb{N}^d$, there are arbitrarily large monochromatic $d$-dimensional rectangular grids. That is, for any $k \in \mathbb{N}$, there is a monochromatic configuration of the form

$$\{(a_1 + i_1 n_1, \ldots, a_d + i_d n_d) : 0 \leq i_1, \ldots, i_d \leq k\}$$

for some $(a_1, \ldots, a_d), (n_1, \ldots, n_d) \in \mathbb{N}^d$.

**E4.** A set is *AP-rich* if it contains a $k$-term arithmetic progression for every $k \in \mathbb{N}$. Show that the family $\mathcal{P}_{AP}$ of AP-rich subsets of $\mathbb{N}$ is partition regular.