Exercise Set #13
Combinatorial Number Theory (2025)

E1. Show that for any infinite set $E \subseteq \mathbb{N}$, the set of differences $E - E$ is intersective.

E2. Show that if $R \subset \mathbb{N}$ is intersective, then for any $A \subset \mathbb{N}$ with $\overline{d}(A) > 0$ there is $n \in R$ such that $\overline{d}(A \cap (A - n)) > 0$.
Hint: Let $\delta = \overline{d}(A)$. Consider $N = N(\frac{\delta^2}{4}) + 1$, where $N(\frac{\delta^2}{4})$ satisfies (ii) of Corollary 84. Then by partitioning the set A into sets A_i each of length N, show that $|A_i| \geq \frac{\delta^2}{4} N$ for "many" $i \in \mathbb{N}$.

E3. Prove the family of intersective sets is partition regular.
Hint: Prove first the following: if R is intersective, then for every $\delta > 0$, there exists $N(\delta) \in \mathbb{N}$ such that if $N_1, N_2 \geq N(\delta)$ and $A_i \subseteq \{1, \ldots, N_i\}$ with $|A_1| \cdot |A_2| \geq \delta N_1 N_2$, then $(A_1 - A_1) \cap (A_2 - A_2) \cap R \neq \emptyset$.