MATH-329 Nonlinear optimization
Exercise session 11: Lagrangian Duality

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1. Minimization of quadratic function on affine subspace. Let $H \in \mathbb{R}^{n\times n}, A \in \mathbb{R}^{p\times n}, b \in \mathbb{R}^p, c \in \mathbb{R}^n$ and assume that H is positive definite. Consider the optimization problem

$$\min f(x) \quad \text{subject to} \quad x \in S,$$

where $f(x) = \frac{1}{2}x^\top Hx + c^\top x$ and $S = \{x \in \mathbb{R}^n \mid Ax = b\}$. We assume S is non-empty.

1. Write the Lagrangian function L for this problem. What is its domain?
2. What is the primal function L_P and its domain?
3. Obtain an explicit expression for the dual function L_D. What is its domain? What is the dual problem?
4. Find a characterization for points that solve the dual problem. Hint: You should find that they are defined by a linear system of equations.
5. Argue that strong duality holds by checking the assumptions of the strong duality theorem explicitly. Use your reasoning here to argue that the linear system of equations you found in the previous question has a solution.
6. Use strong duality to solve the primal problem. Check that the solution satisfies the constraints.

As a side note for context: the exercise would become quite a bit harder (and interesting) if we only assume H positive *semi*definite.