

Introduction

These lecture notes are very much inspired from Kunen's *Set Theory an Introduction to Independence Proofs* [23].

The basic requirements for this course are contained in the "Mathematical Logic" course. Among other things, you should have a clear understanding of each of the following: first order language, signature, terms, formulas, theory, proof theory, models, completeness theorem, compactness theorem, Löwenheim-Skolem theorems.

It **makes no sense** to take this course **without a solid background on first order logic** — see [2, 3, 4, 5, 6, 33].

We use the following notations.

Notation 1 (Formulas). *Given \mathcal{L} any first order language, the set of all \mathcal{L} -formulas is the \subseteq -least $X \subseteq \mathcal{L}^{<\omega}$ that satisfies:*

- all atomic formulas belong to X .
- If φ and ψ belong to X , then

$$\neg\varphi, (\varphi \wedge \psi), (\varphi \vee \psi), (\varphi \rightarrow \psi) \text{ and } (\varphi \leftrightarrow \psi)$$

also belong to X .

- If x is any variable and φ belongs to X , then $\forall x \varphi$ and $\exists x \varphi$ also belong to X .

For better readability, we usually omit the outermost parentheses. For instance we write

$$\exists x x \in y \longrightarrow x \neq y$$

instead of

$$(\exists x x \in y \longrightarrow x \neq y).$$

Notation 2. *Given any first order language \mathcal{L} , and any set of \mathcal{L} -formulas Γ , and any \mathcal{L} -formula φ , we write*

- $\Gamma \models \varphi$ for " φ holds in all \mathcal{L} -structures that satisfy Γ ."
- $\Gamma \vdash_c \varphi$ for " Γ proves φ (in classical logic)".

