

Problem Set 13 Solutions

Exercise 1. Let V_λ denote the Specht module for S_n , where λ is a partition of n .

(a) Show that

$$\operatorname{Res}_{S_{n-1}}^{S_n} V_\lambda \simeq \bigoplus_{\mu \in R(\lambda)} V_\mu,$$

where $R(\lambda)$ is the set of Young diagrams obtained by removing one square from Y_λ .

(b) Show that

$$\operatorname{Ind}_{S_{n-1}}^{S_n} V_\mu \simeq \bigoplus_{\lambda \in A(\mu)} V_\lambda,$$

where $A(\mu)$ is the set of Young diagrams obtained by adding one square from Y_μ .

Hint: Use the formula for the character of V_μ in (a) and the Frobenius reciprocity in (b).

Exercise 2. (Transitivity of the induction) Let $K \subset H \subset G$ be subgroups of a finite group G and V a complex representation of K . Show that

$$\operatorname{Ind}_H^G \operatorname{Ind}_K^H V \simeq \operatorname{Ind}_K^G V.$$

Hint: Use the tensor product form of the induced representations.

Exercise 3. (a) Let G be a finite group and V_R an irreducible representation of G defined over the real numbers. Show that its complexification $V = \mathbb{C} \otimes_{\mathbb{R}} V_R$ is a representation of real type.

(b) Show that all Specht modules V_λ for S_n are of real type.

(c) Use the Frobenius-Schur indicator to find the sum of dimensions of all irreducible representations of S_n .