Probability and Statistics | MATH-235
Autumn 2025 | Mats J. Stensrud

Problem Sheet 7 [1]

Based on Chapters 5.1-5.4 of the course book.
Review Analysis I&II with a focus on integration rules if needed.

Optional Revision Problems

Exercise 1. Consider the following simplified scenario based on Who Wants to Be a Millionaire?, a game show in which the contestant answers multiple-choice questions that have 4 choices per question. The contestant (Fred) has answered 9 questions correctly already, and is now being shown the 10th question. He has no idea what the right answers are to the 10th or 11th questions are. He has one "lifeline" available, which he can apply on any question, and which narrows the number of choices from 4 down to 2. Fred has the following options available.

- Walk away with \$16,000.
- Apply his lifeline to the 10th question, and then answer it. If he gets it wrong, he will leave with \$1,000. If he gets it right, he moves on to the 11th question. He then leaves with \$32,000 if he gets the 11th question wrong, and \$64,000 if he gets the 11th question right.
- Same as the previous option, except not using his lifeline on the 10th question, and instead applying it to the 11th question (if he gets the 10th question right).

Find the expected value of each of these options. Which option has the highest expected value? Which option has the lowest variance?

Hint: First derive the PMF-s, i.e. the probabilities of winning the different amounts, following each of the strategies respectively, outlined above.

Exercise 2. For $X \sim Pois(\lambda)$, find $E(2^X)$, if it is finite.

Week 7 Exercises

Exercise 3. Let F be the CDF of a continuous r.v., and $f = F'$ be the PDF.

1. Show that g defined by $g(x) = 2F(x)f(x)$ is also a valid PDF.
2. Show that h defined by $h(x) = \frac{1}{2}f(-x) + \frac{1}{2}f(x)$ is also a valid PDF.

Exercise 4. Let U be a Uniform r.v. on the interval $(-1, 1)$ (be careful about minus signs).

1. Compute $E(U)$, $Var(U)$, and $E(U^4)$.

[1] Exercises are based on the coursebook Statistics 110: Probability by Joe Blitzstein

2. Find the CDF and PDF of U^2. Is the distribution of U^2 Uniform on $(0,1)$?

Exercise 5. A circle with a random radius $R \sim Unif(0,1)$ is generated. Let A be its area.

1. Find the mean and variance of A, without first finding the CDF or PDF of A.

2. Find the CDF and PDF of A.

Hint: For part 2. look up Exercise S5E2.

Exercise 6. Let $U \sim Unif(0,1)$. As a function of U, create an r.v. X with CDF $F(x) = 1 - e^{-x^3}$ for $x > 0$.

Hint: See Example 5.3.4 in the book for some motivation.

Exercise 7. A woman is pregnant, with a due date of January 10, 2020. Of course, the actual date on which she will give birth is not necessarily the due date. On a timeline, define time 0 to be the instant when January 10, 2020 begins. Suppose that the time T when the woman gives birth has a Normal distribution, centered at 0 and with standard deviation 8 days. What is the probability that she gives birth on her due date? (Your answer should be in terms of Φ, and simplified.)

Hint: Exercise S5E2 can be useful here as well.

Exercise 8. Let $Z \sim N(0,1)$. A measuring device is used to observe Z, but the device can only handle positive values, and gives a reading of 0 if $Z \leq 0$; this is an example of censored data. So assume that $X = Z \cdot I_{Z>0}$ is observed rather than Z, where $I_{Z>0}$ is the indicator of $Z > 0$. Find $E(X)$ and $Var(X)$

Hint: Example 5.4.7 can be a good starting point.